$$\begin{aligned}
&\int \prod_{i=1}^{3} \frac{d^3 p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^3 (\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3) \delta (M - E_1 - E_2 - E_3) \\
= &\int \frac{d^3 p_1 d^3 p_3}{(2\pi)^5 8E_1 E_2 E_3} \delta (M - E_1 - E_2 - E_3) \bigg|_{E_2 = \sqrt{(\boldsymbol{p}_1 + \boldsymbol{p}_3)^2 + m_2^2}}
\end{aligned}$$

设

$$d^3 p_1 = p_1^2 dp_1 d\Omega_1, \quad d^3 p_3 = p_3^2 \sin\theta dp_3 d\theta d\varphi$$

其中 θ, φ 为 $\boldsymbol{p}_3$ 相对于 $\boldsymbol{p}_1$ 的角度，则

$$\begin{aligned}
\text{原式} &= \int \frac{(4\pi) p_1^2 dp_1 (2\pi) p_3^2 dp_3 \sin\theta d\theta}{(2\pi)^5 8E_1 E_2 E_3} \delta (M - E_1 - E_2 - E_3) \bigg|_{E_2 = \sqrt{p_1^2 + p_3^2 + 2p_1 p_3 \cos\theta + m_2^2}} \\
&= \int \frac{p_1^2 dp_1 p_3^2 dp_3 \sin\theta d\theta}{32\pi^3 E_1 E_2 E_3} \delta \left(M - E_1 - \sqrt{p_1^2 + p_3^2 + 2p_1 p_3 \cos\theta + m_2^2} - E_3 \right)
\end{aligned}$$

由

$$\delta (f(x)) = \sum_i \frac{1}{|f'(x_i)|} \delta (x - x_i), \quad f(x_i) = 0$$

设

$$\begin{aligned}
f(\theta) &= M - E_1 - \sqrt{p_1^2 + p_3^2 + 2p_1 p_3 \cos\theta + m_2^2} - E_3 \\
f'(\theta) &= -\frac{-2p_1 p_3 \sin\theta}{2\sqrt{p_1^2 + p_3^2 + 2p_1 p_3 \cos\theta + m_2^2}} = \frac{p_1 p_3 \sin\theta}{E_2}
\end{aligned}$$

可得（$0 < \theta < \pi$, $\sin\theta > 0$）

$$\delta \left(M - E_1 - \sqrt{p_1^2 + p_3^2 + 2p_1 p_3 \cos\theta_3 + m_2^2} - E_3 \right) = \frac{E_2}{p_1 p_3 \sin\theta_0} \delta (\theta - \theta_0)$$

所以

$$\begin{aligned}
\text{原式} &= \int \frac{p_1^2 dp_1 p_3^2 dp_3 \sin\theta d\theta}{32\pi^3 E_1 E_2 E_3} \frac{E_2}{p_1 p_3 \sin\theta_0} \delta (\theta - \theta_0) \\
&= \int \frac{p_1 dp_1 p_3 dp_3}{32\pi^3 E_1 E_3}
\end{aligned}$$

$$dp_1 = d\sqrt{E_1^2 - m_1^2} = \frac{E_1 dE_1}{\sqrt{E_1^2 - m_1^2}} = \frac{E_1 dE_1}{p_1}, \quad dp_3 = \frac{E_3 dE_3}{p_3}$$

所以

$$\text{原式} = \frac{1}{32\pi^3} \int dE_1 dE_3$$