Analysis for Informaticians, Exam 1 Problem Set

Rules

You have to work **alone,** exchange of solutions are prohibited and easy to detect!

Do not use any kind of human intelligence, except **your own!**

You can use artificial intelligence, however in the problems to elaborate parts you have to explain your solution as detailed, as it is needed; the more you detail your solution the higher you will be scored.

The exam lasts from 12:00 to 14:00 (Monday Dec. 19). After that, you should upload your work till 14:10.

Do not break the rules!

Make the following table on the top of the page. Fill it with the letter of the correct answer.

Name	**Neptun code**	1.	2.	3.	4.	5.	6.

Elaborate your detailed solution on at best 6 pages and upload them on the following site in pdf format, not larger than 10 MBs

`https://forms.gle/okPGNXuyZnT1LSde7`

under the name

your name.pdf

Don't forget to hit Submit.

In the four choice test part, you have to choose the only correct answer. Each one of the four choice test problems are scored 0 or 2 points. You don't have to explain your choice.

All the correct and full solutions of the problems to elaborate are scored at best 8 points.

Problem 1

PROBLEM TO ELABORATE – Differentiate the following function!

$$f(x) = \begin{cases} x \cdot \arctan\left(\frac{1}{x}\right) & \text{if } x < 0 \\ -1 + x^2 & \text{if } x \geq 0 \end{cases}$$

FOUR CHOICE TEST – Consider the function f above (that is, exactly the f above)

$\boxed{A}$ f is strictly monotone. $\boxed{B}$ The derivative of f is continuous at 0.
$\boxed{C}$ $\lim_0 f = 0$. $\boxed{D}$ f is Riemann integrable on $[-1, 1]$.

Problem 2

PROBLEM TO ELABORATE – Determine the following limit!

$$\lim_{x \to 0} \frac{e^{\sin(x^2)} - 1}{x}$$

FOUR CHOICE TEST – Determine the classification of the discontinuity of function

$$f(x) = \frac{e^{\sin(x^2)} - 1}{x}$$

at point 0.

$\boxed{A}$ Jump. $\boxed{B}$ Removable.
$\boxed{C}$ Essential. $\boxed{D}$ Infinite.

Problem 3

PROBLEM TO ELABORATE – Determine the integral $\int (x+1)\cos(2x+3)\,dx$.

FOUR CHOICE TEST – The improper integral of $f : [0, \infty) \to \mathbf{R}, x \mapsto \begin{cases} \frac{\sin^2 x}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

$\boxed{A}$ is non-zero and finite. $\boxed{B}$ is infinite.
$\boxed{C}$ is zero. $\boxed{D}$ does not exist.

Problem 4

PROBLEM TO ELABORATE – Determine the following integral: $\int_e^{e^2} \frac{1}{x \cdot \ln^3 x}\,dx$

FOUR CHOICE TEST – Where are the local extrema of $f : [0, \infty) \to \mathbf{R}$; $\begin{cases} -x \cdot \ln x, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \end{cases}$

$\boxed{A}$ at 0, $\boxed{B}$ no local extrema,
$\boxed{C}$ at $1/e$, $\boxed{D}$ at 0 and $1/e$.

Problem 5

PROBLEM TO ELABORATE – Determine the following limit $\sqrt[n]{\frac{2^n + n^3 + 5}{2n^4 - \frac{1}{n}}}$

FOUR CHOICE TEST – If $a_n > 0$ and $a_n \to 0$, then

$\boxed{A}$ $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$ $\boxed{B}$ $\lim_{n\to\infty} \sqrt[n]{a_n} = 0$
$\boxed{C}$ $\sqrt[n]{a_n}$ is bounded $\boxed{D}$ $\sqrt[n]{a_n}$ is monotone.

Problem 6

PROBLEM TO ELABORATE – Determine the following limit $\lim_{n\to\infty} \left(\frac{n+4}{n}\right)^{n^2}$

FOUR CHOICE TEST – The sequence $a_n = \left(\frac{n+4}{n}\right)^{n^2}$

$\boxed{A}$ is not bounded $\boxed{B}$ has a limit and it is e^4
$\boxed{C}$ is convergent $\boxed{D}$ periodic