

Calculus 1 for Informaticians, Exam 1 Problem Set

Rules

You have to **work alone**, exchange of solutions are prohibited and easy to detect!

Do not use any kind of human intelligence, except **your own!**

You can use artificial intelligence, however in the problems to elaborate parts you have to explain your solution as detailed, as it is needed; the more you detail your solution the higher it will be scored.

The exam lasts from 14:00 to 16:00 (Monday, 18 December). After that, you should upload your work up to 16:10.

Do not break the rules!

Make the following table on the top of the page. Fill it with the letter of the correct answer.

Name	Neptun code	1.	2.	3.	4.	5.	6.

Elaborate your detailed solution on at best 6 pages and upload them on the following site in pdf format, not larger than 10 MBs

<https://forms.gle/UAGqEq6vysXfTav77>

under the name

your name.pdf

Don't forget to hit .

In the four choice test part, you have to choose the one and only correct answer. Each one of the four choice test problems are scored 0 or 2 points. You don't have to explain your choice.

All the correct and full solutions of the problems to elaborate are scored at best 8 points.

Problem 1

PROBLEM TO ELABORATE – Determine f' and prove that f' is continuous at point 0, where f is

$$f(x) = \begin{cases} x^2 \cdot \arctan\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

FOUR CHOICE TEST – Consider the function f above (that is, exactly the f above)

\boxed{A} f is periodic.	\boxed{B} f is not differentiable at 0.
\boxed{C} $\lim_{0+} f = \infty$	\boxed{D} f is Lipschitz continuous.

Problem 2

PROBLEM TO ELABORATE – Determine the left and right one-sided limits of the following function at points 0 and 1!

$$f(x) = \frac{e^{\frac{1}{x}} + 1}{x - 1}$$

FOUR CHOICE TEST – Classify the discontinuity of f at point 0.

\boxed{A} Jump.	\boxed{B} Removable.
\boxed{C} Essential.	\boxed{D} Infinite.

Problem 3

PROBLEM TO ELABORATE – Determine the indefinite integral $\int (2x - 3) \cdot \cos(x + 4) dx$.

FOUR CHOICE TEST – The improper integral of $f : (1, \infty) \rightarrow \mathbf{R}, x \mapsto \frac{1}{x^2}$

\boxed{A} is non-zero and finite.	\boxed{B} is infinite.
\boxed{C} is zero.	\boxed{D} does not exist.

Problem 4

PROBLEM TO ELABORATE – Determine the following definite integral: $\int_0^1 (x + 1)\sqrt{x^2 + 2x} dx$

FOUR CHOICE TEST – Determine the location of the absolute minima of

$$f : \mathbf{R} \setminus \{0\} \rightarrow \mathbf{R}; x \mapsto x^2 + \frac{1}{x^2}$$

\boxed{A} 2.	\boxed{B} there is no absolute minimum.
\boxed{C} 1.	\boxed{D} -1 and 1.

Problem 5

PROBLEM TO ELABORATE – Determine the following limit $\lim_{n \rightarrow \infty} \sqrt[n]{\frac{2^n + n^2 + 1}{n^n + 3n}}$

FOUR CHOICE TEST – If $a_n > 0$ and $a_n \rightarrow 0$, then

\boxed{A} $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = 1$.	\boxed{B} $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = 0$.
\boxed{C} $\sqrt[n]{a_n}$ is bounded.	\boxed{D} $\sqrt[n]{a_n}$ is monotone.

Problem 6

PROBLEM TO ELABORATE – Determine the following limit: $\lim_{n \rightarrow \infty} \left(\frac{n+4}{n+2} \right)^{2n+1}$

FOUR CHOICE TEST – The sequence $a_n = \left(1 + \frac{1}{n^2}\right)^n$

\boxed{A} is bounded.	\boxed{B} has a limit and it is e .
\boxed{C} is not convergent.	\boxed{D} periodic.