Calculus 1 for Informaticians, Exam 2 Problem Set

Rules

You have to **work alone,** exchange of solutions are prohibited and easy to detect!

Do not use any kind of human intelligence, except **your own!**

You can use artificial intelligence, however in the problems to elaborate parts you have to explain your solution as detailed, as it is needed; the more you detail your solution the higher it will be scored.

The exam lasts from 10:00 to 12:00 (Wednesday, 10 January). After that, you should upload your work up to 16:10.

Do not break the rules!

Make the following table on the top of the page. Fill it with the letter of the correct answer.

Name	**Neptun code**	1.	2.	3.	4.	5.	6.

Elaborate your detailed solution on at best 6 pages and upload them on the following site in pdf format, not larger than 10 MBs

`https://forms.gle/55irqYEhraveXT1P9`

under the name

your name.pdf

Don't forget to hit $\boxed{\text{Submit}}$.

In the four choice test part, you have to choose the one and only correct answer. Each one of the four choice test problems are scored 0 or 2 points. You don't have to explain your choice.

All the correct and full solutions of the problems to elaborate are scored at best 8 points.

Problem 1

PROBLEM TO ELABORATE – Determine f' and prove that f' is continuous at point 0, where f is

$$f(x) = \begin{cases} x^2 \cdot \cos\left(\dfrac{1}{x^2}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

FOUR CHOICE TEST – Consider the function f above (that is, exactly the f above)

$\boxed{A}$ f is periodic.	$\boxed{B}$ f is differentiable at 0.
$\boxed{C}$ $\lim\limits_{0+} f = 1$	$\boxed{D}$ f is not continuous.

Problem 2

PROBLEM TO ELABORATE – Determine the left and right one-sided limits of the following function at points 0 and 1!

$$f(x) = \frac{e^{-\frac{1}{x^2}} + 1}{x^3 - 1}$$

FOUR CHOICE TEST – Classify the discontinuity of f at point 0.

$\boxed{A}$ Jump.	$\boxed{B}$ Removable.
$\boxed{C}$ Essential.	$\boxed{D}$ Infinite.

Problem 3

PROBLEM TO ELABORATE – Determine the indefinite integral $\int (2x-1) \cdot \ln(x+4)\,dx$.

FOUR CHOICE TEST – The improper integral of $f : (1, \infty) \to \mathbf{R}, x \mapsto x \ln x$

$\boxed{A}$ is non-zero and finite.	$\boxed{B}$ is infinite.
$\boxed{C}$ is zero.	$\boxed{D}$ does not exist.

Problem 4

PROBLEM TO ELABORATE – Determine the following definite integral: $\int\limits_0^1 (x^2+x)\sqrt[3]{2x^3+3x^2+1}\,dx$

FOUR CHOICE TEST – Determine the location of the absolute minima of

$$f : \mathbf{R} \setminus \{0\} \to \mathbf{R}\,;\, x \mapsto e^{x^2}(x^2 - 5)$$

$\boxed{A}$ 0.	$\boxed{B}$ there is no absolute minimum.
$\boxed{C}$ -2, 2 and 0.	$\boxed{D}$ -2 and 2.

Problem 5

PROBLEM TO ELABORATE – Determine the following limit $\lim \left(1 + \dfrac{1}{n^2}\right)^n$

FOUR CHOICE TEST – If $a_n > 0$ and $a_n \to 0$, then

$\boxed{A}$ $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = 1$.	$\boxed{B}$ $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = 0$.
$\boxed{C}$ $\sqrt[n]{a_n}$ is bounded.	$\boxed{D}$ $\sqrt[n]{a_n}$ is monotone.

Problem 6

PROBLEM TO ELABORATE – Determine the following limit: $\lim\limits_{n\to\infty} \left(\dfrac{2n+4}{2n+1}\right)^{3n-1}$

FOUR CHOICE TEST – The sequence $a_n = \left(1 + \frac{1}{n}\right)^{n^2}$

$\boxed{A}$ is bounded.	$\boxed{B}$ has a limit and it is e.
$\boxed{C}$ is not convergent.	$\boxed{D}$ periodic.