2022 Vill. Mat A1 – 8. gyakorlat

Nevezetes határértékek

1.

$$\lim_{n\to\infty}\left(\frac{n+4}{n+6}\right)^{n+2} =? \qquad \textbf{hf.:}\ \lim_{n\to\infty}\left(\frac{n-3}{n-7}\right)^{n+4} =?$$

2.

$$\textbf{a)}\ \lim_{n\to\infty}\sqrt[n]{n^4+5n^3+2} =?, \qquad \textbf{b)}\ \lim_{n\to\infty}\sqrt[n]{3^n-2n^4+1} =?$$

$$\textbf{gy.:}\ \lim_{n\to\infty}\sqrt[n]{\frac{4^n-3n^2}{n^2+n+1}} =?, \qquad \textbf{hf.:}\ \lim_{n\to\infty}\sqrt[n]{\frac{6^n+2n^7-n^5}{2^n-n+4}} =?$$

3.

$$\textbf{a)}\ \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2} =?, \qquad \textbf{b)}\ \lim_{n\to\infty}\left(1+\frac{1}{n^2}\right)^{n} =?$$

hf.: Igazoljuk, hogy $\lim_{n\to\infty}\frac{n^n}{(n!)^2}=0$

4.

$$\textbf{a)}\ \lim_{x\to 0}\frac{\ln(1+x^2)}{\sin^2 x}, \qquad \textbf{b)}\ \lim_{x\to 0}\frac{\ln(1-x^2)}{1-\cos(x^2)}, \qquad \textbf{c)}\ \lim_{x\to 0}\frac{e^{\sin x}-1}{x}$$

$$\textbf{hf)}\ \lim_{x\to 0+}\frac{\ln(\cos x)}{x^3}, \qquad \lim_{x\to 0}\frac{e^{x^2}-1}{1-\cos x}$$

5.

$$\lim_{x\to 0}(1+2x)^x, \qquad \textbf{hf)}\ \lim_{x\to 0}(\cos x)^{x^2}$$

6.

$$\lim_{x\to\infty} x\sin\frac{1}{x}, \qquad \lim_{x\to 0} x\sin\frac{1}{x}$$

$$\textbf{hf*)}\ \lim_{x\to\infty} x\,\mathrm{arctg}\,\frac{1}{x^2}, \qquad \lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{1-\cos x}\right)$$