

④

$$\frac{\begin{array}{c} [x] \\ \vdots \\ A(x) \end{array}}{\forall x, A(x)}$$

$$\frac{\forall x, A(x)}{A[x/t]} \quad t: \mathcal{U}$$

⑤

$$\frac{A[x/t]}{\exists x, A(x)}$$

$$\frac{\boxed{\exists x, A(x)} \quad \begin{array}{c} [x, A(x)] \\ \vdots \\ C \end{array} \quad \begin{array}{c} x \rightarrow t \\ [x/t] \end{array}}{C}$$

$$\exists x \forall y A(x, y) \rightarrow \forall y \exists x A(x, y)$$

$$\boxed{\exists x \forall y A(x, y)} \vdash \forall y \exists x A(x, y)$$

$$[x]; [\forall y A(x, y)]$$

$$\frac{\forall y A(x, y)}{A(x, y)}$$

$$\frac{\boxed{\exists x \forall y A(x, y)} \quad \exists x A(x, y)}{\exists x A(x, y)}$$

[y]

$$\frac{\exists x A(x, y)}{\forall y \exists x A(x, y)}$$

$$\forall y \exists x A(x, y)$$

$$\textcircled{2} \quad (A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$$

$$\begin{array}{c}
 \boxed{A \rightarrow B} \quad \boxed{A} \\
 \downarrow \quad \downarrow \\
 B \rightarrow 1 \quad A \rightarrow B \rightarrow A \\
 \hline \quad \quad B \\
 \hline \quad \quad 1 \\
 \hline \quad \quad A \rightarrow 1 \\
 \hline \quad \quad \neg A \\
 \hline \neg B \rightarrow \neg A
 \end{array}
 \quad \begin{array}{l}
 \text{def } \neg B \equiv B \rightarrow 1 \\
 \text{def } \neg A \equiv A \rightarrow 1
 \end{array}$$

$$(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$$

$\textcircled{3}$

$$((\neg A) \vee (\neg B)) \rightarrow \neg(A \wedge B)$$

$$\begin{array}{c}
 \boxed{(A \rightarrow 1) \vee (B \rightarrow 1)} \quad \boxed{A \wedge B} \\
 \downarrow \quad \downarrow \\
 1 \quad A \wedge B \rightarrow 1 \\
 \hline 1 \\
 \hline A \wedge B \rightarrow 1
 \end{array}$$

$$\frac{\begin{array}{c} [A] \\ \vdots \\ B \end{array}}{A \rightarrow B}$$