Lecture 07: Synthetic Controls (SC)

Applied Econometrics

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Outline

Main References: SC Chapter 10 and Abadie (2021, JEL)

Table of Contents

Synthetic Controls

"The synthetic control approach developed by Abadie, Diamond, and Hainmueller (2010, 2015) and Abadie and Gardeazabal (2003) is arguably the most important innovation in the policy evaluation literature in the last 15 years."

– Athey and Imbens (2017)

A Primer on Synthetic Controls I

- The synthetic control model was developed in **Abadie and Gardeazabal (2003)** in a study of terrorism's effect on aggregate income, and then on **Abadie, Diamond, and Hainmueller (2010)** in a study on cigarette tax in California.
- To estimate the effects of **aggregate interventions** affecting **a small number of large units** (such as cities, regions, or countries), on some **aggregate outcome of interest**.
- Synthetic controls models optimally **choose a set of weights** which when applied to a group of corresponding units produce an optimally estimated **counterfactual ("synthetic unit")** to the unit that received the treatment.

A Primer on Synthetic Controls II

- It is a powerful, yet surprisingly simple, generalization of the difference-in-differences strategy.
- The method is based on the observation that, when the units of analysis are a few aggregate units, **a combination of comparison units** (the "synthetic control") often does a **better job** of reproducing characteristics of a treated unit than using **a single comparison unit** alone.
- The comparison unit, therefore, in this method is selected to be the **weighted average of all comparison units** that best resemble the characteristics of the treated unit(s) in the **pre-treatment** period.

Table of Contents

The Setting

- We obtain data for $J+1$ units: $j = 1, 2, ..., J+1$.
- We assume that the first unit ($j = 1$) is the treated unit (affected by the policy intervention).
- The **"donor pool"** is the set of potential comparisons, $j = 2, ..., J+1$.
- Data span T periods and that the first T_0 periods are before the intervention.
- We observe the outcome of interest, Y_{jt} for each j and t, as well as a set of k **predictors** $X_{1j}, ..., X_{kj}$ that which may include pre-intervention values of Y_{jt} which are themselves unaffected by the intervention.
- Let X_1 be a $(k \times 1)$ vector of pre-intervention characteristics for the treated unit. Similarly, let X_0 be a $(k \times J)$ matrix which contains the same variables for the unaffected units.

Treatment Effect

- The effect of the intervention of intertest for $t > T_0$:

$$\tau_{1t} = Y_{1t}^{I} - Y_{1t}^{N}$$

- The great policy evaluation challenge is to estimate Y_{1t}^{N} for $t > T_0$:
- How the outcome of interest would have evolved for the affected unit **in the absence of the intervention**; This is a counterfactual outcome.

Estimation

- It is often difficult to find a single unaffected unit that provides a suitable comparison for the unit affected by the policy intervention of interest.
- A combination of units in the donor pool may approximate the characteristics of the affected unit substantially better than any unaffected unit alone.
- A synthetic control can be represented by a $J \times 1$ vector of optimally chosen weights, $W = (w_2, ..., w_{J+1})'$:

$$\hat{Y}_{1t}^N = \sum_{j=2}^{J+1} w_j Y_{jt}.$$

- The weights are restricted to be non-negative and to sum to one.

SC Weights

- Choose $W^* = (w_2^*, ..., w_{J+1}^*)'$ that minimizes:

$$||X_1 - X_0W|| = \left(\sum_{m=1}^{k} v_m (X_{m1} - \sum_{j=2}^{J+1} w_j X_{mj})^2 \right)^{1/2}$$

- Constants $v_1, ..., v_k$ reflect the relative importance of the synthetic control reproducing the values of each of the k predictors.
- Abadie, Diamond, and Hainmueller (2010) suggests different choices of V, but ultimately it appears from practice that most people choose a that minimizes the **mean squared prediction error (MSPE)**:

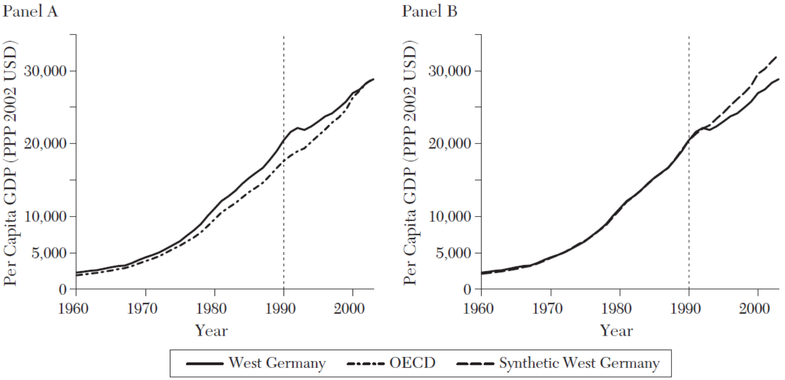
$$\sum_{t=1}^{T_0} \left(Y_{1t} - \sum_{j=2}^{J+1} w_j^*(V) Y_{jt} \right)^2$$

Table of Contents

Example: 1990 German Reunification

- Abadie, Diamond, and Hainmueller (2015)
- Estimates the effect of the 1990 German reunification on per capita GDP in West Germany.
  - The intervention: The 1990 German reunification,
  - Treated unit: The former West Germany,
  - Donor pool: A set of industrialized countries,
  - X_1 and X_0: pre-reunification values of predictors of economic growth.

Synthetic Control Estimation



Predictor Means

TABLE 1
Economic Growth Predictor Means before the German Reunification

	West Germany (1)	Synthetic West Germany (2)	OECD average (3)	Austria (nearest neighbor) (4)
GDP per capita	15,808.9	15,802.2	13,669.4	14,817.0
Trade openness	56.8	56.9	59.8	74.6
Inflation rate	2.6	3.5	7.6	3.5
Industry share	34.5	34.4	33.8	35.5
Schooling	55.5	55.2	38.7	60.9
Investment rate	27.0	27.0	25.9	26.6

Weights

- Austria carries the largest weight.
- Sparsity of the weights is typical of synthetic control estimators.
  - Is a consequence of the geometric characteristics of the solution to the optimization problem that generates synthetic controls.

TABLE 2
SYNTHETIC CONTROL WEIGHTS FOR WEST GERMANY

Australia	—
Austria	0.42
Belgium	—
Denmark	—
France	—
Greece	—
Italy	—
Japan	0.16
Netherlands	0.09
New Zealand	—
Norway	—
Portugal	—
Spain	—
Switzerland	0.11
United Kingdom	—
United States	0.22

Linear Regression Estimator

- A linear regression estimator of the effect of the treatment can easily be constructed using the panel data structure.
- A regression-based estimator of the counterfactual Y_{1t}^N for $t > T_0$ is $\hat{\mathrm{B}}'\bar{\mathrm{X}}_1$ where $\hat{\mathrm{B}} = (\bar{\mathrm{X}}_0\bar{\mathrm{X}}_0')^{-1}\bar{\mathrm{X}}_0\mathrm{Y}_0'$.
- Here Y_0 is $(T - T_0) \times J$ matrix of post-intervention outcomes for the units in the donor pool and X_0 is a $k \times (J+1)$ matrix with columns $[1, X_2, X_3, \ldots, X_{J+1}]$. Here X_j include $X_{1j}, X_{2j}, \ldots, X_{kj}$.
- The regression-based estimator is akin to a synthetic control, as it uses a linear combination, $\mathrm{Y}_0\mathrm{W}^{reg}$, of the outcomes in the donor pool, with $\mathrm{W}^{reg} = \bar{\mathrm{X}}_0'(\bar{\mathrm{X}}_0\bar{\mathrm{X}}_0')^{-1}\bar{\mathrm{X}}_1$.

Linear Regression Weights

- Like their SC counterparts, the regression weights in W^{reg} sum to one.
- Unlike the SC weights, regression weights may be outside the $[0, 1]$ interval, allowing extrapolation outside of the support of the data.

TABLE 3
REGRESSION WEIGHTS FOR WEST GERMANY

Australia	0.12
Austria	0.26
Belgium	0.00
Denmark	0.08
France	0.04
Greece	−0.09
Italy	−0.05
Japan	0.19
Netherlands	0.14
New Zealand	0.12
Norway	0.04
Portugal	−0.08
Spain	−0.01
Switzerland	0.05
United Kingdom	0.06
United States	0.13

Table of Contents

Example: Abadie, Diamond, and Hainmueller (2010)

- In 1988, California passed comprehensive tobacco control legislation called Proposition 99. Proposition 99 increased cigarette taxes by $0.25 a pack, spurred clean-air ordinances throughout the state, etc.
- Other states had similar control programs, and they were dropped from their analysis.

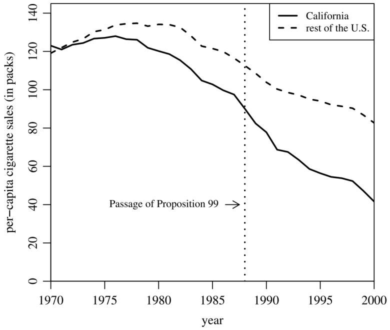


Figure 1. Trends in per-capita cigarette sales: California vs. the rest of the United States.

Synthetic California

- In an ideal world, the average of the other states would work - however, not clear empirically that they are a good counterfactual.
- We need a “synthetic California” as our control.
- Still subject to same caveats from DiD – not invariant to some transformations (e.g. log and linear)

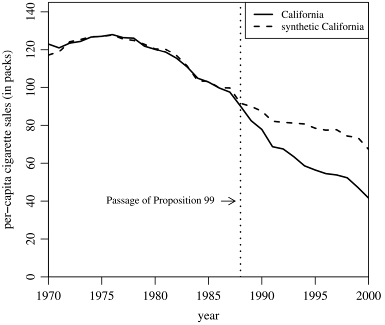


Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

Inference in SC I

- The use of statistical inference in comparative case studies is difficult because of the **small-sample** nature of the data, the **absence of randomization**, and the fact that **probabilistic sampling is not employed** to select sample units.
- Inference for the synthetic control framework is based on permutation methods (**randomization inference**).
- This alternative inference approach is based on the idea that if we observe comparable effects without the real intervention, it undermines the validity of attributing the observed estimate to the intervention being studied.
- A permutation distribution can be obtained by iteratively reassigning the treatment to the units in the donor pool and estimating "**placebo effects**" in each iteration.

Inference in SC II

- Then, the **permutation distribution** is constructed by pooling the effect estimated for the treated unit together with placebo effects estimated for the units in the donor pool.
- The effect of the treatment on the unit affected by the intervention is deemed significant when its magnitude is **extreme relative to the permutation distribution**.

California vs. Synthetic California

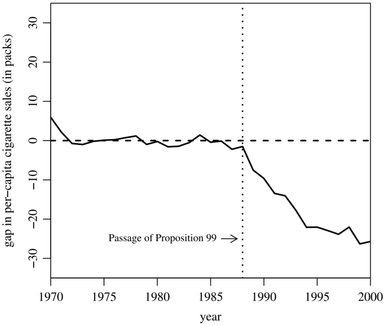


Figure 3. Per-capita cigarette sales gap between California and synthetic California.

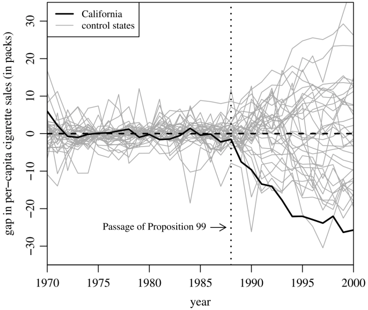


Figure 5. Per-capita cigarette sales gaps in California and placebo gaps in 34 control states (discards states with pre-Proposition 99 MSPE twenty times higher than California's).

Procedure

1. Iteratively apply the synthetic control method to each country/state in the donor pool and obtain a distribution of placebo effects.
2. Calculate the RMSPE for each placebo for the post-treatment period:

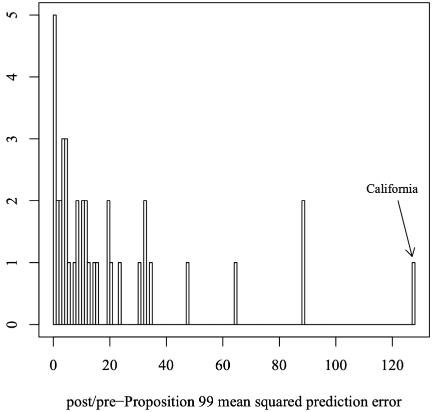
$$RMSPE = \left(\frac{1}{T - T_0} \sum_{t=T_0+t}^{T} \left(Y_{1t} - \sum_{j=2}^{J+1} w_j^* Y_{jt} \right) \right)$$

3. Calculate the RMSPE for each placebo for the pre-treatment period (similar equation but for the pre-treatment period).
4. Compute the ratio of the post- to pre-treatment RMSPE.
5. Sort this ratio in descending order from greatest to highest.
6. Calculate the treatment unit's ratio in the distribution as $p = \frac{RANK}{TOTAL}$.

Extreme Enough?

- California is ranked first out of thirty-eight state units. This gives an exact p-value of 0.026, which is less than the conventional 5% most journals want to (arbitrarily) see for statistical significance.

(All 38 States in Donor Pool)



German Reunification

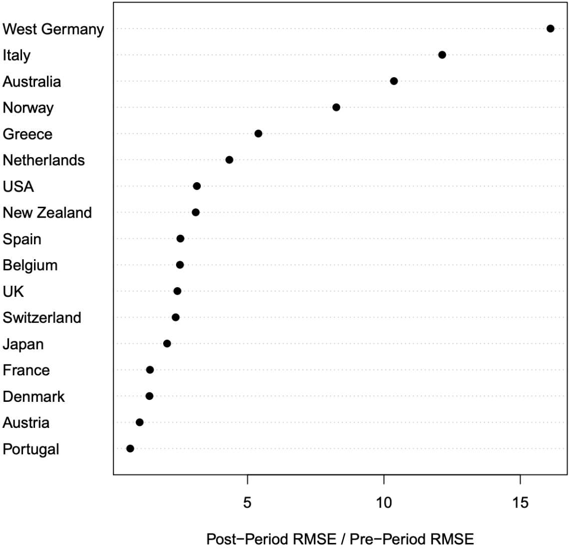
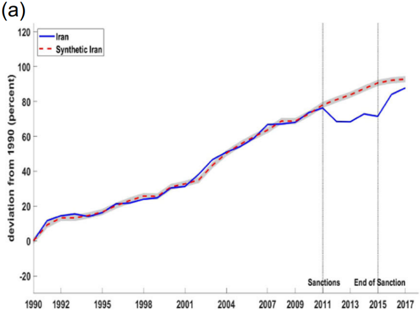


Table of Contents

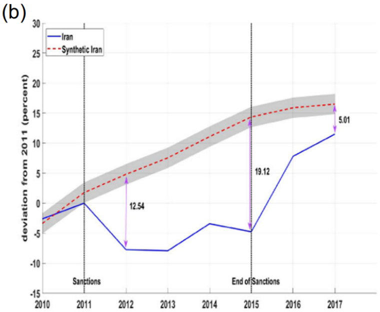
Morteza Ghomi's JMP

- Quantify the aggregate and heterogeneous effects of sanctions imposed on Iran.
- Match the GDP growth path, population, and the presanction averages of the rents from natural resources, international trade, industry, agriculture, and service production—all in GDP share—of the synthetic Iran with the actual Iran.

Synthetic Control Results



Deviation from 1990



Deviation from 2011

Synthetic Iran

TABLE 1 Composition of the synthetic Iran: country weights

Country	Weight	Country	Weight
Saudi Arabia	0.53	Sudan	0.07
Nigeria	0.12	Algeria	0.05
Greece	0.1	China	0.02
Korea, Rep.	0.08		

Note: Other countries have either a weight of zero or less than 10^{-4}.

TABLE 2 Indicators mean matching before the sanctions

Covariate	Iran	Synthetic Iran
Natural resource rent (% GDP)	28	28
Import of goods and services (% GDP)	22	27
Industry added value (% GDP)	45	44
Agriculture added value (% GDP)	7	8
Population (millions)	70.8	70.8

Main Results of this Paper

- Sanction cost reached its maximum of 19.1% of real gross domestic product 4 years after the application of the sanctions, and the economy has not fully recovered after their removal.
- Households working in governmental sectors and educated households are unaffected by the sanctions.
- Instead, the sanctions condemn young, illiterate, rural, or religious minority households to poverty.

Table of Contents

Contextual Requirements I

- **Size of the effect and volatility of the outcome:**
  - Small effects will be indistinguishable from other shocks to the outcome of the affected unit, especially if the outcome variable of interest is highly volatile.

- **Availability of a Comparison Group:**

  Untreated units that
  - Do not adopt interventions similar to the one under investigation during the period of the study.
  - Do not suffer large idiosyncratic shocks to the outcome of interest during the study period.
  - Have characteristics similar to the characteristics of the affected unit.

Contextual Requirements II

- **No Anticipation:**
  - Synthetic control estimators may be biased if forward-looking economic agents react in advance of the policy intervention under investigation.
  - It is advisable to backdate the intervention in the data set to a period before any anticipation effect can be expected.
- **No Interference:**
  - Units' outcomes are invariant to other units' treatments.
- **Convex Hull Condition:**
  - Synthetic control estimates are predicated on the idea that a combination of unaffected units can approximate the pre-intervention characteristics of the affected unit.

Contextual Requirements III

- We need $(X_{11}, X_{21}, ..., X_{k1})$ to fall close to the convex hull of the set of points $\{(X_{12}, X_{22}, ..., X_{k2}), ..., (X_{1J+1}, X_{2J+1}, ..., X_{kJ+1})$.

- **Time Horizon:**
  - The effect of some interventions may take time to emerge or to be of sufficient magnitude to be quantitatively detected in the data.

Data Requirements

- **Aggregate Data on Predictors and Outcomes:**
  - When aggregate data do not exist, aggregates of micro-data are employed in comparative case studies.
- **Sufficient Pre-intervention Information:**
  - The credibility of a synthetic control estimator depends in great part on its ability to steadily track the trajectory of the outcome variable for the affected unit before the intervention.
- **Sufficient Post-intervention Information:**
  - Extensive post-intervention information allows a more complete picture of the effects of the intervention, in time and across the various outcomes of interest.

Table of Contents

Robustness Checks: Backdating

- The result of estimating the effect of the 1990 German reunification with the intervention backdated to 1980.

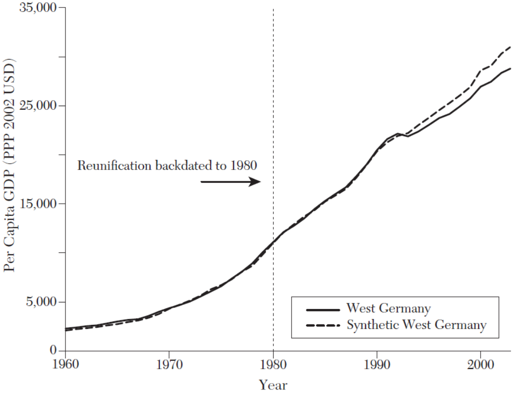


Figure 3. Backdating the 1990 German Reunification Application

Robustness Checks: Backdating

- The synthetic control estimator closely tracks per capita GDP for West Germany in the 1981-90 period, before the start of the actual intervention. This is the "in-time placebo test".

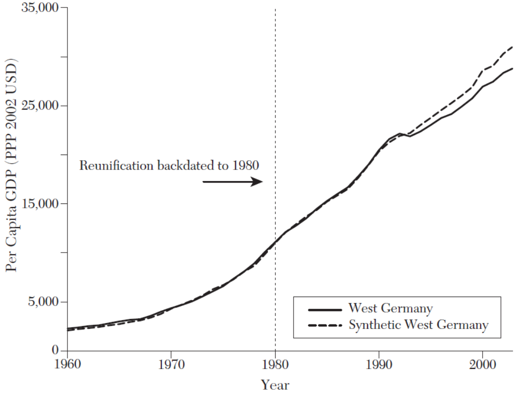


Figure 3. Backdating the 1990 German Reunification Application

Robustness Checks: Backdating

- A gap between per capita GDP for West Germany and its synthetic control counterpart appears around the time of the German reunification.

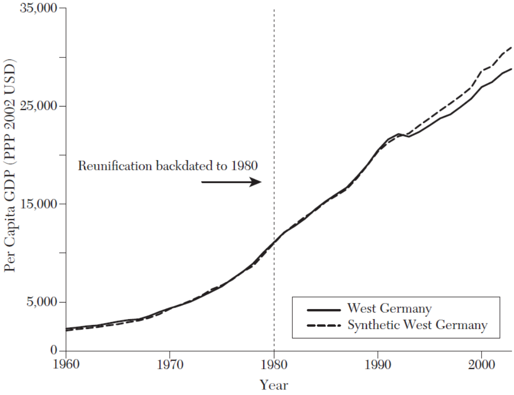


Figure 3. Backdating the 1990 German Reunification Application

Robustness Checks: Leave-one-out Re-analysis

- Taking from the sample one-at-a-time each of the countries that contribute to the synthetic control.

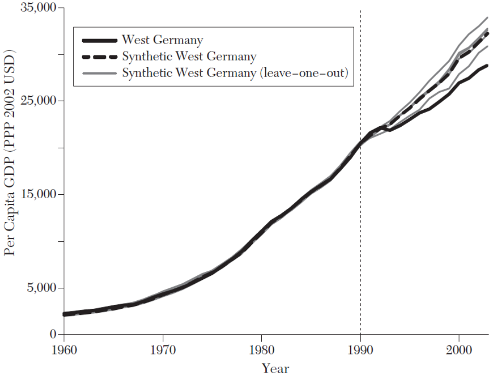


Figure 4. Leave-one-out Estimates of the Effect of the 1990 German Reunification

Robustness Checks: Leave-one-out Re-analysis

- The main conclusion of a negative estimate of the German reunification on per capita GDP is robust to the exclusion of any particular country.

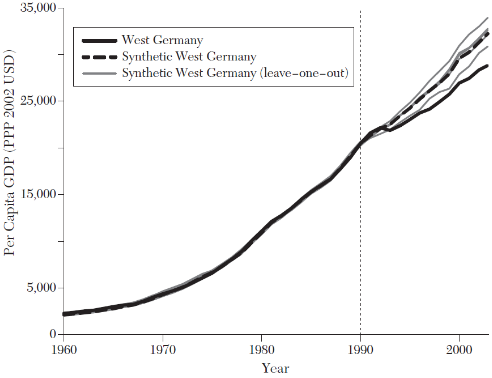


Figure 4. Leave-one-out Estimates of the Effect of the 1990 German Reunification

SC Checklist

- Authors using synthetic control must do more than merely run the synth command when doing comparative case studies. They must:
  - Find the exact p-values through placebo-based inference,
  - Check for the quality of the pre-treatment fit,
  - Investigate the balance of the covariates used for matching, and
  - Check for the validity of the model through placebo estimation (e.g., rolling back the treatment date).