Exercise 2

Introduction to Complex Network Analysis

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1

Cayley tree with k = 3 and h = 6

(a) The total number of nodes in the Cayley tree is given by the sum of 1 (indicating that there is only one central node) and the sum of all the number of nodes with distances d in [1,P] from the central node. Therefore we have:

$1 + k[(k-1)^h - 1/(k-2)]$

which is equal to: $1 + 3[(3-1)^6 - 1/(3-2)] = 190$

(b) The number of nodes at distance d (larger than 0) from the central node can be calculated with the following formula:

$N_d = k(k-1)^{d-1}$

So the number of leaves are the number of nodes at distance h from the central node:

$N_{leaves} = 3(3-1)^{6-1} = 96$

The total number of inner nodes are: $N_{inner} = 190 - 96 = 94$. They can be also calculated using the formula from a) but using h = 5.

The number of inner nodes is bigger than the number of leaves for k = 2 and $h > 1$.

(c) Expression for the diameter of Cayley tree in terms of the total number of nodes N.
It is not hard to see that he diameter D = 2h. We find the expression using D = 2h and the formula for total number of nodes (N):

$$
\begin{aligned}
N &= 1 + \frac{k}{k-2}\left[(k-1)^{D/2} - 1\right] \\
\frac{(N-1)(k-2)}{k} &= (k-1)^{D/2} - 1 \\
\frac{(N-1)(k-2)}{k} + 1 &= (k-1)^{D/2} \\
\frac{D}{2}\ln(k-1) &= \ln\left[1 + \frac{(N-1)(k-2)}{k}\right] \\
D &= \frac{2}{\ln(k-1)}\ln\left[1 + (N-1)\frac{k-2}{k}\right]
\end{aligned} \tag{1}
$$

For $N >> 1$, the diameter D can be expressed as:

$$
D \simeq \frac{\ln N}{\frac{1}{2}\ln(k-1)} \tag{2}
$$

In this case, the diameter equals to 12, but using this formula it approximates it to:

$$
D \simeq \frac{\ln 190}{\frac{1}{2}\ln(3-1)} \simeq 15 \tag{3}
$$

(d) The Cayley network is a tree, i.e. does not contain any loop and therefore does not contain any triangle. The clustering coefficient is C = 0.

2

Erdős–Rényi (ER) network with N=5000 nodes and connection probability of node pairs of p=0.002

(a) Expected number of edges: $p\frac{N(N-1)}{2} = 24995$
Average degree: $p(N-1) = 9.998$

(b) The network is connected (the average degree 9,998 is bigger than ln(N) = 8.517).

The critical value p_c is at average degree of 1.

$p_c(N-1) = 1$
$p_c = \frac{1}{N-1} = 0.00020004$

(c) Expected size of the giant component:
$s = 1 - e^{9.998s}$

(d) We use this formula:

$$\langle k \rangle = p(N-1) \tag{4}$$

1. N = 100, average degree= 5
$p = \frac{5}{99} = 0.05$
2. N = 1000, average degree = 20
$p = \frac{20}{999} = 0.05$

3

In the Tutorial3, by looking at the graphs, we can easily observe that the more trials we compute, the more apparent is the bias that 6 has. For this particular case, choosing a number of trials of 1000, was already enough to notice that the bars do not overlap with 6.