Exercise 6

Introduction to Complex Network Analysis

Melanija Kraljevska

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Consider a Barabasi-Albert model with a prefential attachment given by:

$$\Pi\left(k_i\right) = \frac{1}{m_0 + t - 1}$$

1.1 Degree dynamics

Let m be the number of links a new node arrives in the network. The rate at which an existing node i acquires links as a result of new nodes connecting to it is:

$$\frac{dk_i}{dt} = m\Pi\left(k_i\right) = m\frac{1}{m_0 + t - 1}$$

To calculate the degree dynamics $k_i(t)$, we integrate the above equation and use the knowledge that node i joins the network at time t_i with m links, i.e $k_i(t_i) = m$. By this we obtain:

$$k_i(t) = m\ln\left(e\frac{m_0 + t - 1}{m_0 + t_i - 1}\right) = m\ln\left(\frac{t}{t_i}\right) + m$$

1.2 Degree distribution

$$p(k) = \frac{e}{m}\exp\left(-\frac{k}{m}\right)$$

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The closeness centrality of a node is the sum of the lengths of the shortest paths to every other node in the network. A node that has a smaller closeness is relatively more close to the rest of the nodes in the network compared to a node that has a larger closeness.

The betweenness centrality of a node is a measure of the fraction of the shortest paths between any two nodes that go through this node. A node with a larger betweenness centrality lies on the shortest paths between several node pairs. This measure shows which nodes are 'bridges' between nodes in a network. It does this by identifying all the shortest paths and then counting how many times each node falls on one.