Minimum enclosing ball problem

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Intro

Definition

Given a set of vectors $A = \{a_1, \ldots, a_m\} \in \mathbb{R}^d$ we want to find the smallest ball containing all points of A. I.e.

$$\min_{c,\rho} \ \rho$$
$$s.t. \|a_i - c\| \leq \rho \quad \forall i = 1, \ldots, m$$

Used in clustering, Data classification, facility location, computer graphics.

Rewriting the problem

Square constraints for smoothness

$$\min_{c,\rho} \rho$$

$$s.t. \|a_i\|^2 - 2a_i^T c + c^T c \leq \rho \quad \forall i = 1, \ldots, m$$

Build Lagrangian Dual:

$$L(c, \rho, u) = \rho + \sum_{i=1}^{m} u_i * (\|a_i - c\|^2 - \rho^2)$$

and the dual function:

$$\Phi(u) = \inf_{c,\rho} L(c, \rho, u) = \sum_{i=1}^{m} \|a_i\|^2 u_i - \sum_{i=1}^{m} u_i a_i^T c$$

if $\sum_{i=1}^{m} u_i = 1$ with $u_i \geq 0$. Yields the unit simplex.
sparsity (dual is typically sparse because the dual solution is combination of only a few points)

Solving via Frank-Wolfe