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Machine Learning II
1. Exercise Sheet
Discussed on Apr 25, 14:00

Exercise 1. *Give a real-world example of a joint distribution $p(x, y)$ where X is discrete and Y is continuous.* **1 points**

Exercise 2. *If random variables A and B are conditionally independent given X and variables A and C are conditionally independent given X, does it follow that variables B and C are independent given X?* **1 points**

Exercise 3. *Suppose that we have three coloured boxes r (red), b (blue), and g (green). Box r contains 3 apples, 4 oranges, and 3 limes, box b contains 1 apple, 1 orange, and no limes, and box g contains 3 apples, 3 oranges, and 4 limes. If a box is chosen at random with probabilities $p(r) = 0.2, p(b) = 0.2, p(g) = 0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple? If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?* **2 points**

Exercise 4. *Consider the following story:*

> *You are the manager of a small company. At the last party about a third of the employees have stolen beverages from the company, i.e. $P(E = thief) = \frac{1}{3}$.*
>
> *To identify the guilty ones each employee has to undertake a lie detector test. The probability that an employee passes the test is $\frac{1}{6}$ if she is guilty and $\frac{5}{6}$ if she is not.*

- *Use Bayes rule to compute the probability that an employee is guilty given that she failed the test.*
- *To obtain more evidence you can test each employee a second time. What is the probability of guilt given two failed tests?*

  Hint: Assume that the two test results are (conditionally) independent.

3 points

Exercise 5. *Consider a biased die where the probabilities of rolling sides $\{1, 2, 3, 4, 5, 6\}$ are $\{1/12, 1/12, 1/12, 1/6, 1/6, 5/12\}$, respectively. What are the expected value and the variance of the die? If I roll the die twice, what is the expected value of the sum of the two rolls?* **3 points**

Exercise 6. *Show that if two random variables X and Y are independent, then their covariance $\mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$ is zero.* **2 points**