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ANSWERS TO QUESTIONS

Task1 :

In [34]:

```
from assignment_5 import *
%pylab inline
def train_krr(X_train, Y_train, kwidth, llambda):
    ''' Trains kernel ridge regression (krr)
    Input:      X_train - DxN array of N data points with D features
                Y       - D2xN array of length N with D2 multiple labels
                kwidth  - kernel width
                llambda  - regularization parameter
    Output:     alphas  - NxN array, weighting of training data used for apply_krr
    '''
    # your code here
    K = GaussianKernel(X_train, X_train, kwidth)
    alphas = sp.linalg.inv(K + llambda * sp.eye(X_train.shape[1])).dot(Y_train.T)
    return alphas

def apply_krr(alphas, X_train, X_test, kwidth):
    ''' Applies kernel ridge regression (krr)
    Input:      alphas    - NtrxD2 array trained in train_krr
                X_train   - DxNtr array of Ntr train data points with D features
                X_test    - DxNte array of Nte test data points with D features
                kwidth    - Kernel width
    Output:     Y_test    - D2xNte array
    '''
    # your code here
    K = GaussianKernel(X_test, X_train, kwidth)
    Y = (K.dot(alphas).T)
    return Y
```

Populating the interactive namespace from numpy and matplotlib

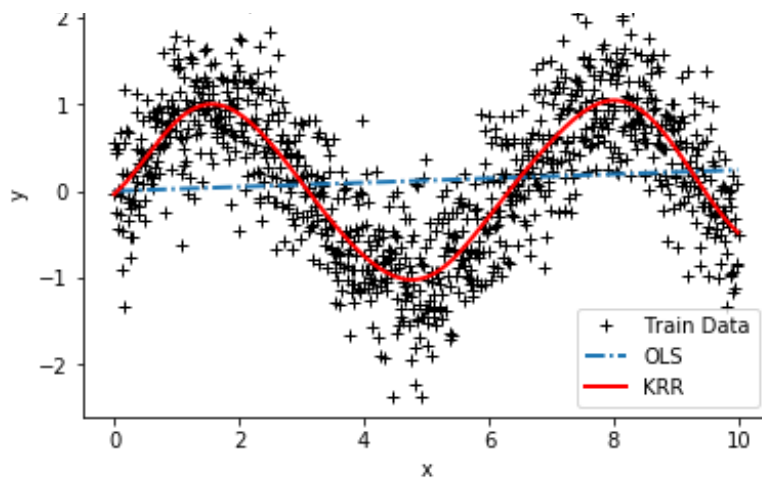
Task2 :

a) When the kernel width σ gets smaller the prediction fits better

In [35]:

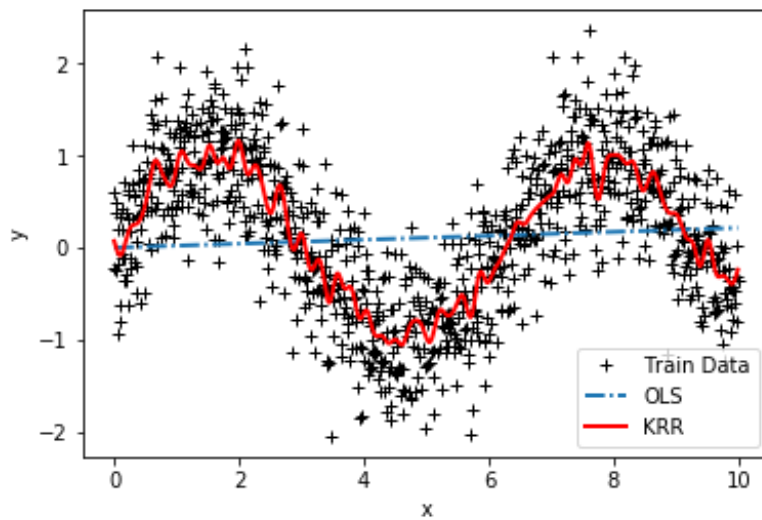
```
test_sine_toydata(kwidth = 1, llambda = 1)
```





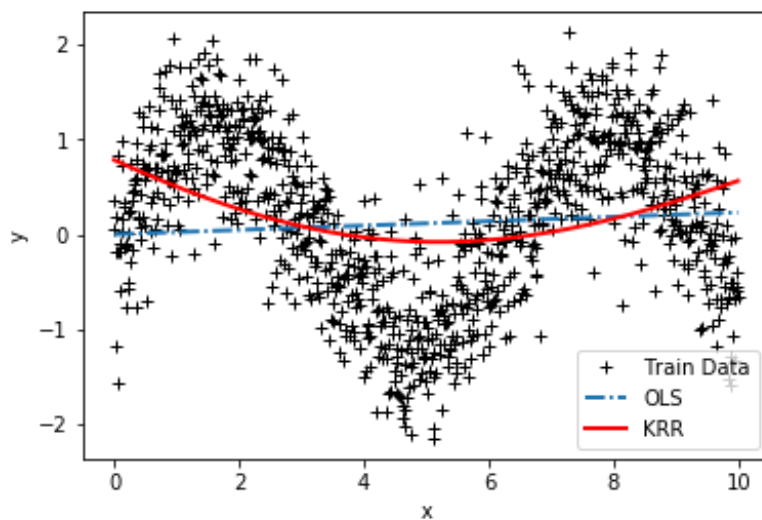
In [36]:

```
test_sine_toydata(kwidth = 0.1, llambda = 1)
```



In [37]:

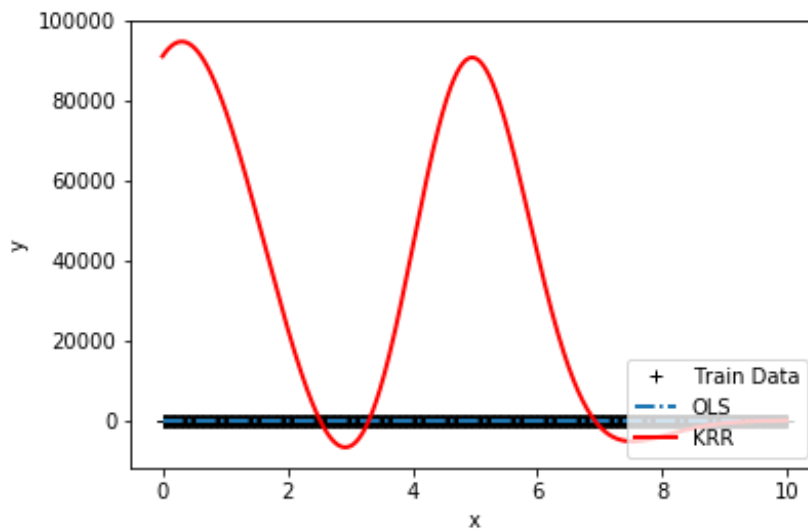
```
test_sine_toydata(kwidth = 10, llambda = 1)
```



b) When the regularization parameter gets smaller the prediction fits better.

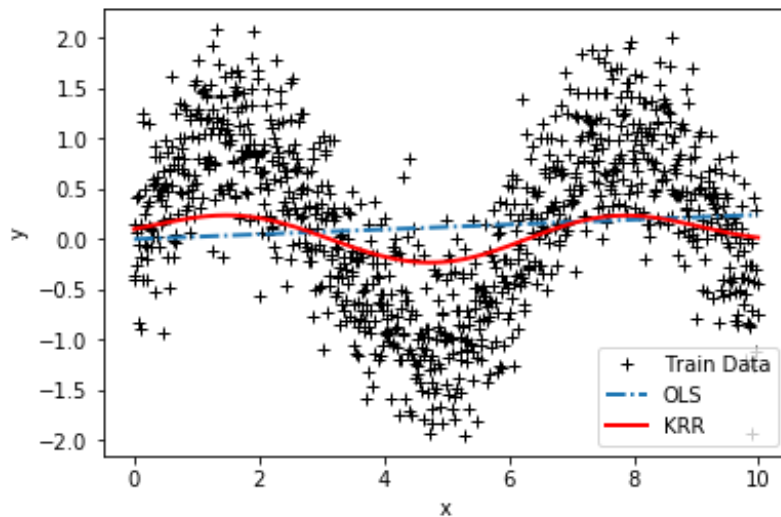
In [38]:

```
test_sine_toydata(kwidth = 1, llambda = 10 ** (-10))
```



In [39]:

```
test_sine_toydata(kwidth = 1, llambda = 500)
```



In [40]:

```
test_sine_toydata(kwidth = 1, llambda = 1)
```

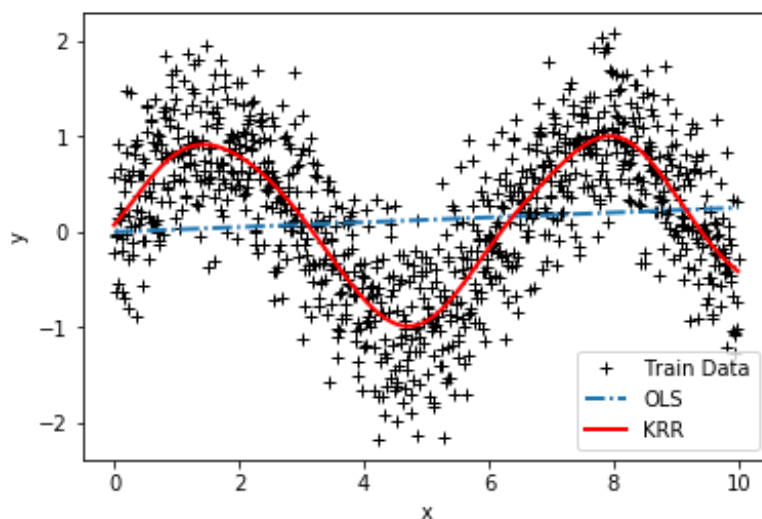


Figure 4 :

The function `crossvalidate_krr` uses the nested crossvalidation technique. The data is split into F disjunct folds. Then two crossvalidations are performed: An inner one and an outer one. In the outer one, we leave one fold out and pass the $F-1$ remaining folds to the inner one. The inner crossvalidation is a normal crossvalidation performed with the data of the $F-1$ remaining folds. The inner cross validation then outputs the best parameter for the $F-1$ remaining folds (the model selection part). The outer cross validation then evaluates the performance of the best parameter returned from the inner cross validation on the remaining f_{Outer} fold.

This technique is used to counter the problem that in a normal cross validation, we use one test set to both select the parameters and evaluated the performance the chosen parameters give us, which might lead to a too optimistic evaluation of our performance.

Task 4 :

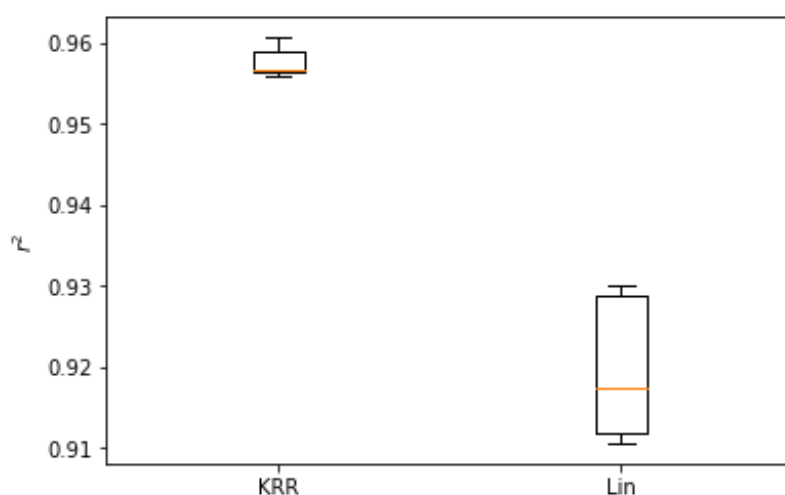
Box plot shows that distribution of R-square values across the 5 folds of cross validation for both Kernel ridge regression and Linear regression. Distributions are changing every time calling the `test_handpositions` function but median value of r-squares for kernel ridge is always bigger than simple linear regression. There could be some outliers for both two methods. Yes, we gain better fitted regression line from Kernel ridge regression as compared to Linear regression since R- square values of Kernel ridge is much bigger than Linear regression .

In [41]:

```
test_handpositions()

/Users/ozgesahin/anaconda/lib/python3.6/site-packages/numpy/core/fromnumeric.py:224: VisibleDeprecationWarning: using a non-integer number instead of an integer will result in an error in the future
    return reshape(newshape, order=order)

Fold 0 best kernel width 10.000000 best regularizer 0.010000 rsquare 0.960664 rsquare linear 0.929993
Fold 1 best kernel width 10.000000 best regularizer 0.010000 rsquare 0.955961 rsquare linear 0.910612
Fold 2 best kernel width 10.000000 best regularizer 0.010000 rsquare 0.958977 rsquare linear 0.917495
Fold 3 best kernel width 10.000000 best regularizer 0.010000 rsquare 0.956318 rsquare linear 0.928879
Fold 4 best kernel width 10.000000 best regularizer 0.010000 rsquare 0.956620 rsquare linear 0.911763
```



Task5 :

It can be said that when considering computational complexity, performance of the `crossvalidate_krr` function used in `test_handpositions` function would be very bad if we use the whole dataset. Many nested for loops are used in the function.