MACHINE INTELLIGENCE 1, ASSIGNMENT 9

Exercise H9.1

(a) Let's say $\underline{x}$ is the closest point of the decision boundary to $\underline{x}^\alpha$

$$d(\underline{x}^\alpha, w, b) = \left| (\underline{x}^\alpha - \underline{x}) \cdot \frac{\vec{w}}{\|w\|} \right|$$

$\underline{x}$ satisfies,

$\underline{w}^T x + b = 0$ since $\underline{x}$ is on the decision boundary.

From constraint we have,

$$\min_{\alpha=1,\dots p} \left| w^T \underline{x}^{(\alpha)} + b \right| = 1$$

this means $w^T \underline{x}^\alpha + b \geq 1$ for some p, $y^\alpha = +1$

or $w^T \underline{x}^\alpha + b \leq -1$ for some p, $y^\alpha = -1$

$w^T x^\alpha \geq 1 - b$

$w^T \underline{x}^\alpha \leq -1 - b$

$$d = \left| \frac{1 - b - (-b) + k}{\|w\|} \right| = \frac{|1+k|}{\|w\|}, \quad \begin{matrix} k \geq 0 \\ k \in \mathbb{R} \end{matrix} \quad \text{for } y^\alpha = +1$$

$$d = \left| \frac{-1 - b - k + b}{\|w\|} \right| = \frac{1+k}{\|w\|} \quad \begin{matrix} k \geq 0 \\ k \in \mathbb{R} \end{matrix} \quad \text{for } y^\alpha = -1$$

$$\text{so. } \frac{1+k}{\|w\|} \geq \frac{1}{\|w\|} \Rightarrow d \geq \frac{1}{\|w\|} \quad //$$

b)

$$L = \frac{1}{2}\|w\|^2 + \frac{C}{P}\sum_\alpha \vartheta_\alpha - \sum \alpha_\alpha\left[y^\alpha(w^T \underline{x}^\alpha + b) - 1 + \vartheta_\alpha\right]$$
$$- \sum \beta_\alpha \vartheta_\alpha$$

$$\frac{\partial L}{\partial w} = w - \sum \alpha_\alpha y^\alpha \underline{x}^\alpha = 0$$

$$w = \sum_\alpha \alpha_\alpha y^\alpha \underline{x}^\alpha$$

$$\frac{\partial L}{\partial b} = \sum_\alpha \alpha_\alpha y_\alpha = 0$$

$$\frac{\partial L}{\partial \vartheta_\alpha} = \frac{C}{P} - \alpha_\alpha - \beta_\alpha = 0$$

$$\beta_\alpha = \frac{C}{P} - \alpha_\alpha$$

if we substitute $\beta_\alpha = \frac{C}{P} - \alpha_\alpha$ and $w = \sum_\alpha \alpha_\alpha y^\alpha \underline{x}^\alpha$ into the Lagrangian

$$L = \frac{1}{2} \sum \alpha_\alpha y^\alpha x^\alpha \sum \alpha_\alpha y^\alpha x_\alpha - \sum \alpha_\alpha y^\alpha \underline{x}^\alpha \sum y^\alpha x^\alpha_\alpha$$
$$- \sum \alpha_\alpha y^\alpha b + \sum \alpha_\alpha$$

$$L = \sum \alpha_\alpha - \frac{1}{2}\sum_\alpha \sum_k \alpha_\alpha \alpha_k y^\alpha y^k x^{T\alpha} x^k$$

$\alpha_\alpha = \lambda_\alpha \rightarrow$ Lagrangian multiplier.

β_α is also Lagrangian m.

$$\beta_\alpha \geq 0 \Rightarrow \frac{C}{P} - \alpha_\alpha \geq 0 \Rightarrow 0 \leq \alpha_\alpha \leq \frac{C}{P}$$