

Machine Intelligence ex 04 solution

November 15, 2017

```
In [167]: import numpy as np
          from numpy import genfromtxt
          import pandas as pd

          %matplotlib inline
          import matplotlib.pyplot as plt

In [168]: """
          defining our variables: i did it in functions, since our variables will change later
          (in the end it was almost unnecessary, since i could have done it like g = np.dot(r
          I'm sorry if this got quite confusing in the b and c,.

          The most important thing to know by my solution:

          g = costfunction()

          the functions are self-explanatory
          """

          data = str.split("""-1 -0.1
          0.3 0.5
          2 0.5""", "\n")

          data
          inputs = [float(str.split(data[i], " ")[0]) for i in range(3)]
          targets = [float(str.split(data[i], " ")[1]) for i in range(3)]
          inputs

          def returnX():
              # X = np.matrix('1 inputs[0]; 3 inputs[1];3 inputs[2]')
              X= np.matrix([[1, -1],[1, 0.3],[1, 2] ]).T;
              return X;

          def returny ():
              # X = np.matrix('1 inputs[0]; 3 inputs[1];3 inputs[2]')
```

```

y = (np.matrix([[targets[0],targets[1],targets[2]]])).T;
return y;

def returnb():
    # X = np.matrix('1 inputs[0]; 3 inputs[1];3 inputs[2]')
    b = -(np.dot(returnX(),returny()))
    return b;

def returnH ():
    # X = np.matrix('1 inputs[0]; 3 inputs[1];3 inputs[2]')
    H = np.dot(returnX(),(returnX()).T)
    return H;
def costfunction ():
    return np.dot(returnH(),w) + returnb();

```

In [169]: #A

```

i =1;

w= np.matrix([-0.45,0.2]).T;
first_iteration = w -0.1*costfunction();
"""
Xm=np.array([[1, -1],[1, 0.3],[1, 2] ]).transpose()
t=np.array([[ -0.1, 0.5, 0.5 ]])
H=np.dot(Xm, Xm.transpose())
b= - (np.dot(Xm, t.transpose()))
g= lambda w: np.dot(H, w) + b
#w = np.random.uniform(0.0, 1.0, [2, 1])
n = 0.2

err = lambda w: 0.5*np.dot((np.dot(Xm.transpose(), w)-t.transpose()).transpose(),
(np.dot(Xm.transpose(), w)-t.transpose()))[0][0]
"""

full_df_weights = pd.DataFrame({'iteration':[i], 'w1':[-0.45], 'w2':[0.2]})
for i in range(1, 1000):
    #the (0.1 is my learningcurve)
    #Et = err(w)
    w = w -0.1*costfunction();
    # print (w);
    weight_updates = pd.DataFrame({'iteration':[i], 'w1':[w[0]], 'w2':[w[1]]},index=[i])
    full_df_weights = pd.concat([full_df_weights,weight_updates])
    i=i+1

print(full_df_weights)

print('left: the link between w0 and w1 | Right: the evolution of the weights during
print('We see that FULL convergence takes many iterations')

```

```

plt.figure(figsize=(40,20))

plt.subplot(1, 2, 1)
scatter1 = plt.scatter(full_df_weights.w1, full_df_weights.w2)
plt.xlabel('w0', fontsize=40)
plt.ylabel('w1', fontsize=40)
plt.suptitle('left: the link between w0 and w1 | Right: the evolution of the weights',
             fontsize=40)
plt.tick_params(axis='both', which='major', labelsize=30)

#plot2 = plt.plot(full_df_weights.w1, full_df_weights.w2,Et)
plt.subplot(1, 2, 2)
plt.tick_params(axis='both', which='major', labelsize=30)
plt.scatter(full_df_weights.iteration[0:50],full_df_weights.w1[0:50],color='green', )
plt.scatter(full_df_weights.iteration[0:50],full_df_weights.w2[0:50], color='blue',

plt.xlabel('iterations', fontsize=40)
plt.ylabel('weights', fontsize=40)

plt.legend(fontsize=50)
#plt.legend(loc='best')

plt.show()
plt.gcf().clear()

```

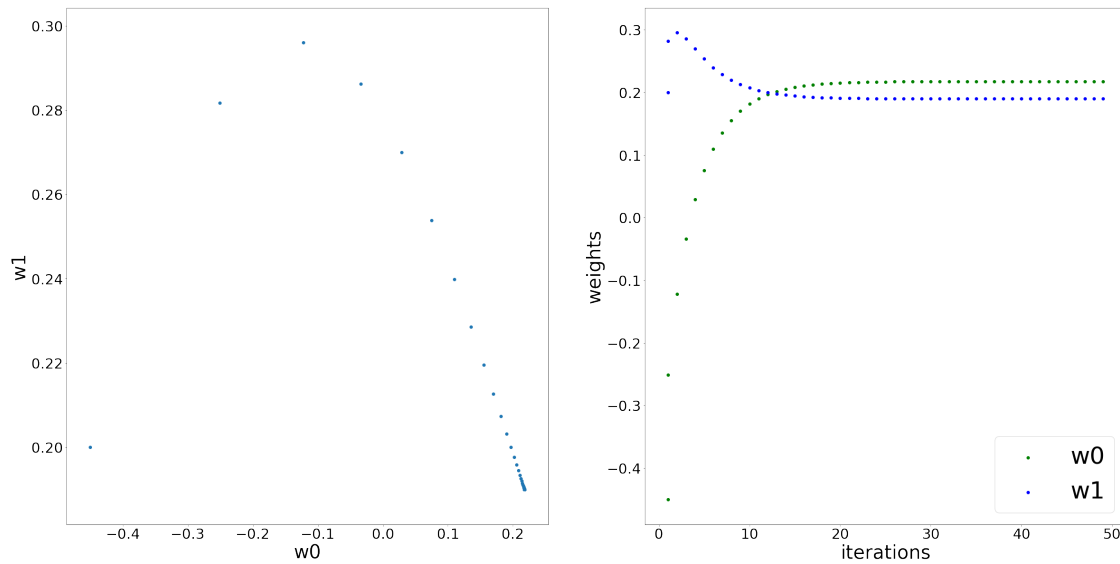
	iteration	w1	w2
0	1	-0.45	0.2
1	1	[[[[-0.251]]]]	[[[[[0.2817]]]]]
2	2	[[[[-0.122321]]]]	[[[[[0.2959447]]]]]
3	3	[[[[-0.03409751]]]]	[[[[[0.28621058]]]]]
4	4	[[[[[0.02892437]]]]]	[[[[[0.26996207]]]]]
5	5	[[[[[0.07515199]]]]]	[[[[[0.25379121]]]]]
6	6	[[[[[0.10961353]]]]]	[[[[[0.23984173]]]]]
7	7	[[[[[0.13555005]]]]]	[[[[[0.22851253]]]]]
8	8	[[[[[0.15517841]]]]]	[[[[[0.21957814]]]]]
9	9	[[[[[0.17007973]]]]]	[[[[[0.21263968]]]]]
10	10	[[[[[0.18141265]]]]]	[[[[[0.20729572]]]]]
11	11	[[[[[0.19004041]]]]]	[[[[[0.20319855]]]]]
12	12	[[[[[0.19661248]]]]]	[[[[[0.20006524]]]]]
13	13	[[[[[0.20162025]]]]]	[[[[[0.19767241]]]]]
14	14	[[[[[0.20543676]]]]]	[[[[[0.19584652]]]]]
15	15	[[[[[0.20834569]]]]]	[[[[[0.19445386]]]]]
16	16	[[[[[0.21056298]]]]]	[[[[[0.19339191]]]]]
17	17	[[[[[0.21225314]]]]]	[[[[[0.19258224]]]]]
18	18	[[[[[0.21354151]]]]]	[[[[[0.19196497]]]]]

19	19	[[[[[0.21452361]]]]]	[[[[[0.19149441]]]]]
20	20	[[[[[0.21527225]]]]]	[[[[[0.19113568]]]]]
21	21	[[[[[0.21584294]]]]]	[[[[[0.19086223]]]]]
22	22	[[[[[0.21627797]]]]]	[[[[[0.19065377]]]]]
23	23	[[[[[0.21660959]]]]]	[[[[[0.19049487]]]]]
24	24	[[[[[0.21686238]]]]]	[[[[[0.19037373]]]]]
25	25	[[[[[0.21705508]]]]]	[[[[[0.19028139]]]]]
26	26	[[[[[0.21720197]]]]]	[[[[[0.190211]]]]]
27	27	[[[[[0.21731395]]]]]	[[[[[0.19015735]]]]]
28	28	[[[[[0.21739931]]]]]	[[[[[0.19011644]]]]]
29	29	[[[[[0.21746438]]]]]	[[[[[0.19008526]]]]]
..
970	970	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
971	971	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
972	972	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
973	973	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
974	974	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
975	975	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
976	976	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
977	977	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
978	978	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
979	979	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
980	980	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
981	981	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
982	982	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
983	983	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
984	984	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
985	985	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
986	986	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
987	987	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
988	988	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
989	989	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
990	990	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
991	991	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
992	992	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
993	993	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
994	994	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
995	995	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
996	996	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
997	997	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
998	998	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
999	999	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]

[1000 rows x 3 columns]

left: the link between w_0 and w_1 | Right: the evolution of the weights during the iterations.
We see that FULL convergence takes many iterations

left: the link between w0 and w1 | Right: the evolution of the weights during the iterations.



<matplotlib.figure.Figure at 0x278ce5a3f60>

```
In [170]: #B
plt.gcf().clear()
i =0;
w= np.matrix([-0.45,0.2]).T;

full_df_weights_line = pd.DataFrame({'iteration':[i], 'w1':[-0.45], 'w2':[0.2]})
learning_rate = (np.dot(costfunction().T, costfunction())
                 /
                 np.dot(costfunction().T, np.dot(returnH(), costfunction()))))

ln = np.squeeze(np.asarray(learning_rate))
"""
print(learning_rate)
print(costfunction())

test = w = w -np.dot(ln, costfunction(), out=None)

print(test)

print (test)

test = w = w -learning_rate*costfunction()
```

```

print(test)
"""

full_df_weights_line = pd.DataFrame({'iteration':[i], 'w1':[-0.45], 'w2':[0.2]})
for i in range(1, 1000):

    w = w -ln*costfunction()
    weight_updates = pd.DataFrame({'iteration':[i], 'w1':[w[0]], 'w2':[w[1]]}, index=[i])
    full_df_weights_line = pd.concat([full_df_weights_line, weight_updates])
    i=i+1
    learning_rate = (np.dot(costfunction().T, costfunction())
                    /
                    np.dot(costfunction().T, np.dot(returnH(), costfunction()))))
    ln = np.squeeze(np.asarray(learning_rate))

print(full_df_weights_line)

print('left: the link between w0 and w1 | Right: the evolution of the weights during')
print('We see that FULL convergence happened AFTER 22 iterations.')

plt.figure(figsize=(40,20))

plt.subplot(1, 2, 1)
scatter1 = plt.scatter(full_df_weights_line.w1, full_df_weights_line.w2, s=60)
plt.xlabel('w0', fontsize=40)
plt.ylabel('w1', fontsize=40)
plt.suptitle('left: the link between w0 and w1 | Right: the evolution of the weights',
             fontsize=40)
plt.tick_params(axis='both', which='major', labelsize=30)

plt.subplot(1, 2, 2)
plt.tick_params(axis='both', which='major', labelsize=30)
plt.scatter(full_df_weights_line.iteration[0:100], full_df_weights_line.w1[0:100], color='red')
plt.scatter(full_df_weights_line.iteration[0:100], full_df_weights_line.w2[0:100], color='blue')

plt.xlabel('iterations', fontsize=40)
plt.ylabel('weights', fontsize=40)

plt.legend(fontsize=50)

```

```
plt.show()
plt.gcf().clear()
```

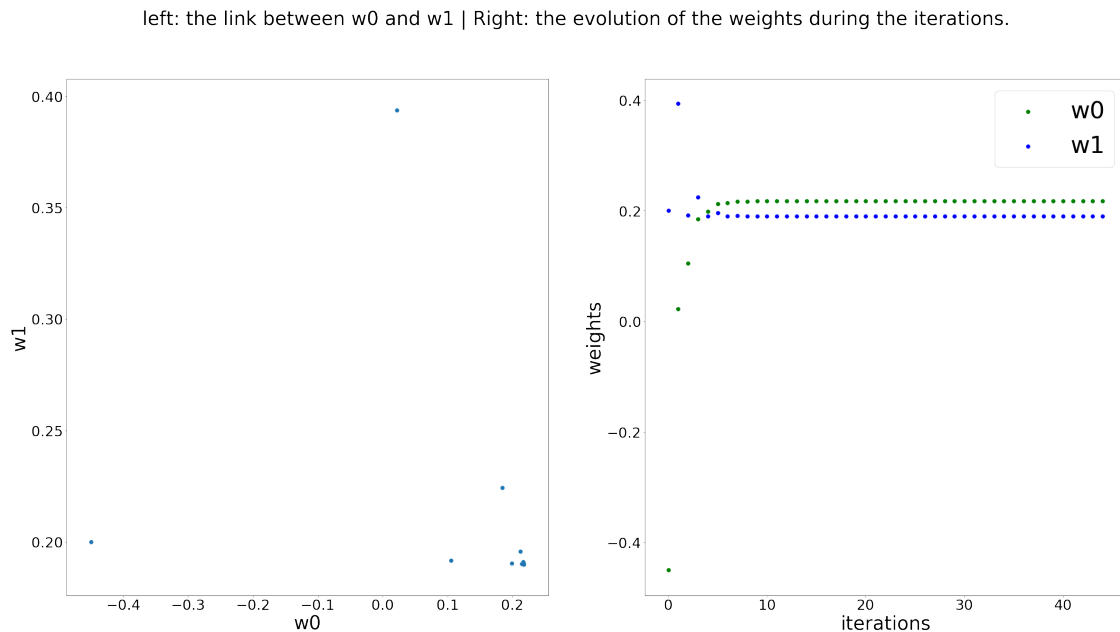
	iteration	w1	w2
0	0	-0.45	0.2
1	1	[[[[[0.02213089]]]]]	[[[[[0.39383464]]]]]
2	2	[[[[[0.10512875]]]]]	[[[[[0.19167337]]]]]
3	3	[[[[[0.18471207]]]]]	[[[[[0.22434653]]]]]
4	4	[[[[[0.19870236]]]]]	[[[[[0.19026982]]]]]
5	5	[[[[[0.21211708]]]]]	[[[[[0.19577727]]]]]
6	6	[[[[[0.21447531]]]]]	[[[[[0.19003324]]]]]
7	7	[[[[[0.21673652]]]]]	[[[[[0.19096158]]]]]
8	8	[[[[[0.21713403]]]]]	[[[[[0.18999336]]]]]
9	9	[[[[[0.21751519]]]]]	[[[[[0.19014984]]]]]
10	10	[[[[[0.21758219]]]]]	[[[[[0.18998664]]]]]
11	11	[[[[[0.21764644]]]]]	[[[[[0.19001301]]]]]
12	12	[[[[[0.21765773]]]]]	[[[[[0.1899855]]]]]
13	13	[[[[[0.21766856]]]]]	[[[[[0.18998995]]]]]
14	14	[[[[[0.21767047]]]]]	[[[[[0.18998531]]]]]
15	15	[[[[[0.21767229]]]]]	[[[[[0.18998606]]]]]
16	16	[[[[[0.21767261]]]]]	[[[[[0.18998528]]]]]
17	17	[[[[[0.21767292]]]]]	[[[[[0.18998541]]]]]
18	18	[[[[[0.21767298]]]]]	[[[[[0.18998527]]]]]
19	19	[[[[[0.21767303]]]]]	[[[[[0.18998529]]]]]
20	20	[[[[[0.21767304]]]]]	[[[[[0.18998527]]]]]
21	21	[[[[[0.21767304]]]]]	[[[[[0.18998528]]]]]
22	22	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
23	23	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
24	24	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
25	25	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
26	26	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
27	27	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
28	28	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
29	29	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
..
970	970	[[[[[nan]]]]]	[[[[[nan]]]]]
971	971	[[[[[nan]]]]]	[[[[[nan]]]]]
972	972	[[[[[nan]]]]]	[[[[[nan]]]]]
973	973	[[[[[nan]]]]]	[[[[[nan]]]]]
974	974	[[[[[nan]]]]]	[[[[[nan]]]]]
975	975	[[[[[nan]]]]]	[[[[[nan]]]]]
976	976	[[[[[nan]]]]]	[[[[[nan]]]]]
977	977	[[[[[nan]]]]]	[[[[[nan]]]]]
978	978	[[[[[nan]]]]]	[[[[[nan]]]]]
979	979	[[[[[nan]]]]]	[[[[[nan]]]]]
980	980	[[[[[nan]]]]]	[[[[[nan]]]]]
981	981	[[[[[nan]]]]]	[[[[[nan]]]]]
982	982	[[[[[nan]]]]]	[[[[[nan]]]]]

983	983	[[[[[nan]]]]]	[[[[[nan]]]]]
984	984	[[[[[nan]]]]]	[[[[[nan]]]]]
985	985	[[[[[nan]]]]]	[[[[[nan]]]]]
986	986	[[[[[nan]]]]]	[[[[[nan]]]]]
987	987	[[[[[nan]]]]]	[[[[[nan]]]]]
988	988	[[[[[nan]]]]]	[[[[[nan]]]]]
989	989	[[[[[nan]]]]]	[[[[[nan]]]]]
990	990	[[[[[nan]]]]]	[[[[[nan]]]]]
991	991	[[[[[nan]]]]]	[[[[[nan]]]]]
992	992	[[[[[nan]]]]]	[[[[[nan]]]]]
993	993	[[[[[nan]]]]]	[[[[[nan]]]]]
994	994	[[[[[nan]]]]]	[[[[[nan]]]]]
995	995	[[[[[nan]]]]]	[[[[[nan]]]]]
996	996	[[[[[nan]]]]]	[[[[[nan]]]]]
997	997	[[[[[nan]]]]]	[[[[[nan]]]]]
998	998	[[[[[nan]]]]]	[[[[[nan]]]]]
999	999	[[[[[nan]]]]]	[[[[[nan]]]]]

[1000 rows x 3 columns]

left: the link between w0 and w1 | Right: the evolution of the weights during the iterations.
We see that FULL convergence happened AFTER 22 iterations.

<matplotlib.figure.Figure at 0x278cd6e3a20>



<matplotlib.figure.Figure at 0x278cdbee550>


```
In [171]: #C
```

```
i =0;
w= np.matrix([-0.45,0.2]).T;

full_df_weights_conj = pd.DataFrame({'iteration':[i], 'w1':[-0.45], 'w2':[0.2]})

np.seterr(divide='ignore', invalid='ignore')

w= np.matrix([-0.45,0.2]).T;
d = -(costfunction())

learning_rate_conjugate = -(np.dot(d.T,costfunction())
                             /
                             np.dot(d.T,np.dot(returnH(),d)))
lnc = np.squeeze(np.asarray(learning_rate_conjugate))

full_df_weights_conj = pd.DataFrame({'iteration':[i], 'w1':[-0.45], 'w2':[0.2]})
beta_upper_half = np.dot(costfunction().T,costfunction())

for i in range(1, 100):
    d = np.squeeze(np.asarray(d))
    oldgradient = costfunction()
    beta_lower_half = np.dot(costfunction().T,costfunction())
    dneu = np.matrix([d[0],d[1]]).T

    #print(w,dneu,lnc)
    if (beta_upper_half == 0):

        weight_updates_conj = pd.DataFrame({'iteration':[i], 'w1':[w[0]], 'w2':[w[1]]})
        full_df_weights_conj = pd.concat([full_df_weights_conj,weight_updates_conj])
    else:

        w = w +dneu*lnc

    new_gradient =costfunction()
    beta_upper_half = np.dot(costfunction().T,costfunction())
    weight_updates_conj = pd.DataFrame({'iteration':[i], 'w1':[w[0]], 'w2':[w[1]]},index=[i])
    full_df_weights_conj = pd.concat([full_df_weights_conj,weight_updates_conj])
    i=i+1

    beta = -(beta_upper_half/beta_lower_half)

    d = costfunction() + dneu*beta
```

```

d = np.squeeze(np.asarray(d))
dneu = np.matrix([d[0],d[1]]).T
learning_rate_conjugate = -(np.dot(dneu.T,costfunction())/np.dot(dneu.T,np.dot(r
lnc = np.squeeze(np.asarray(learning_rate_conjugate))

print(full_df_weights_conj)
print('left: the link between w0 and w1 | Right: the evolution of the weights during
print('We see that convergence happened AFTER 2(!) iterations.')

plt.figure(figsize=(40,20))

plt.subplot(1, 2, 1)
scatter1 = plt.scatter(full_df_weights_conj.w1, full_df_weights_conj.w2,s=60)
plt.xlabel('w0', fontsize=40)
plt.ylabel('w1', fontsize=40)
plt.suptitle('left: the link between w0 and w1 | Right: the evolution of the weights
            fontsize=40)
plt.tick_params(axis='both', which='major', labelsize=30)

plt.subplot(1, 2, 2)
plt.tick_params(axis='both', which='major', labelsize=30)
plt.scatter(full_df_weights_conj.iteration[0:100],full_df_weights_conj.w1[0:100],col
plt.scatter(full_df_weights_conj.iteration[0:100],full_df_weights_conj.w2[0:100], c

plt.xlabel('iterations', fontsize=40)
plt.ylabel('weights', fontsize=40)

plt.legend(fontsize=50)

plt.show()
plt.gcf().clear()

```

	iteration	w1	w2
0	0	-0.45	0.2
1	1	[[[[[0.02213089]]]]]	[[[[[0.39383464]]]]]
2	2	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
3	3	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
3	3	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
4	4	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
4	4	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
5	5	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]
5	5	[[[[[0.21767305]]]]]	[[[[[0.18998527]]]]]

[illegible]

```

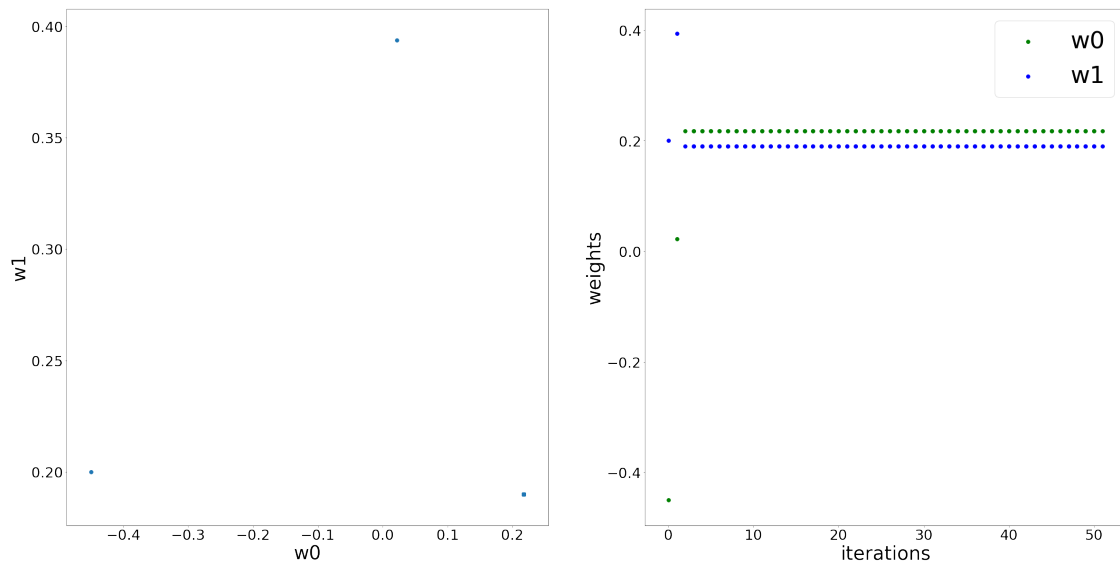
98      98  [[[[[ 0.21767305]]]]]  [[[[[ 0.18998527]]]]]
98      98  [[[[[ 0.21767305]]]]]  [[[[[ 0.18998527]]]]]
99      99  [[[[[ 0.21767305]]]]]  [[[[[ 0.18998527]]]]]
99      99  [[[[[ 0.21767305]]]]]  [[[[[ 0.18998527]]]]]

```

[197 rows x 3 columns]

left: the link between w0 and w1 | Right: the evolution of the weights during the iterations.
We see that convergence happened AFTER 2(!) iterations.

left: the link between w0 and w1 | Right: the evolution of the weights during the iterations.



<matplotlib.figure.Figure at 0x278ce5bcc88>