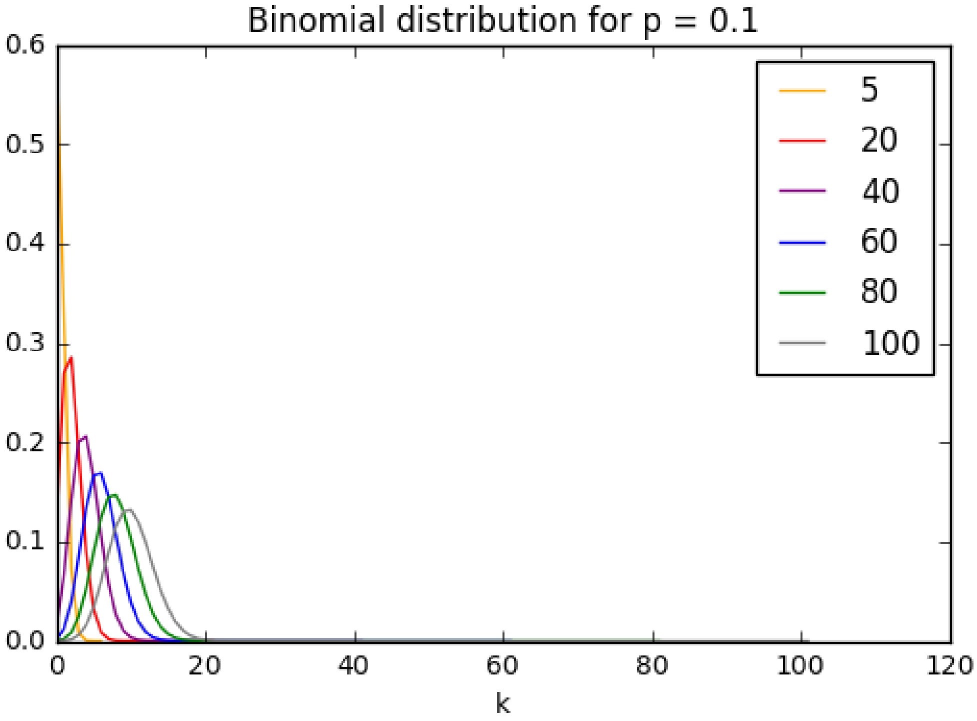
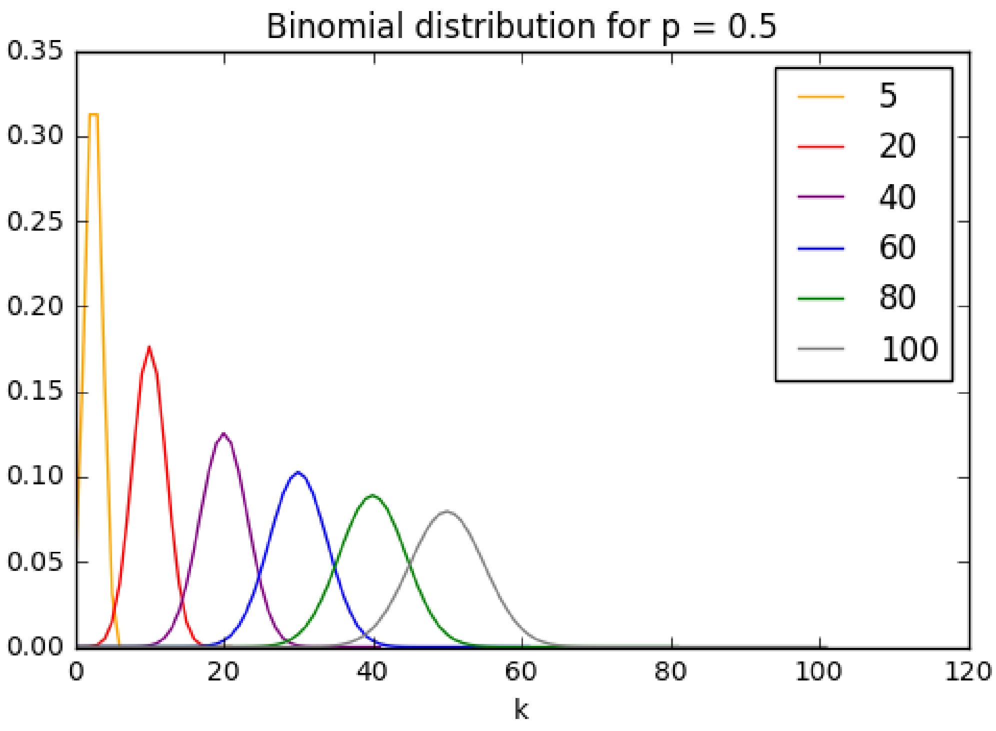
MI8_ex3

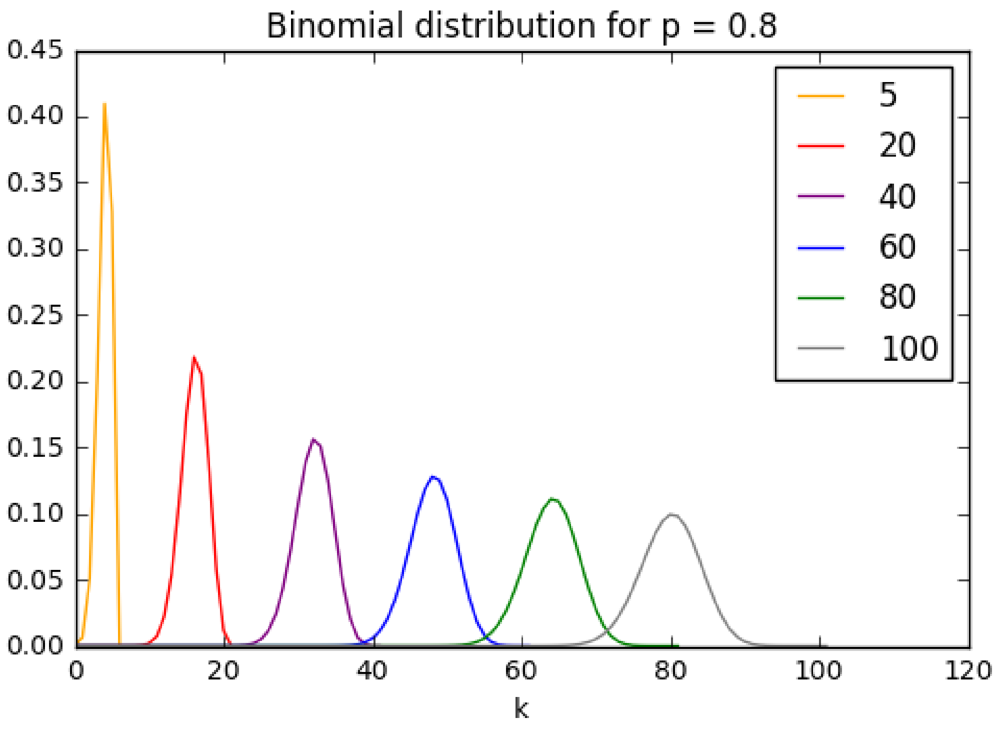
January 10, 2018

```
In [126]: import numpy as np
          import math
          import matplotlib.pyplot as plt

          def fact(n):
              if n == 1 or n == 0:
                  return 1
              else :
                  return n * fact(n-1)

          cpn = np.zeros((200,200))
          cpn[:,0] = 1
          for i in range(1,200) :
              for j in range(1,i+1) :
                  cpn[i][j] = cpn[i-1][j-1] + cpn[i-1][j]

          def binomial(k,n,p):
              return cpn[n][k] * p**k * (1-p)**(n-k)

          def normal(x,m,s):
              return np.exp(-(x-m)**2 / (2 * s**2)) / (s * (2*math.pi)**0.5)

          def poisson(k,l):
              return l**k * np.exp(-l) / fact(k)

In [92]: def binomial_fct(n,p) :
             res = []
             for k in range(n+2):
                 res.append(binomial(k,n,p))
             return res

         def plot_binomial(p) :
             colors = ['orange','red','purple','blue','green','grey']
             i = 0
             nlist = (5,20,40,60,80,100)
             for n in nlist :
                 plt.plot(range(n+1),binomial_fct(n,p),color=colors[i])
                 plt.xlabel('k')
```

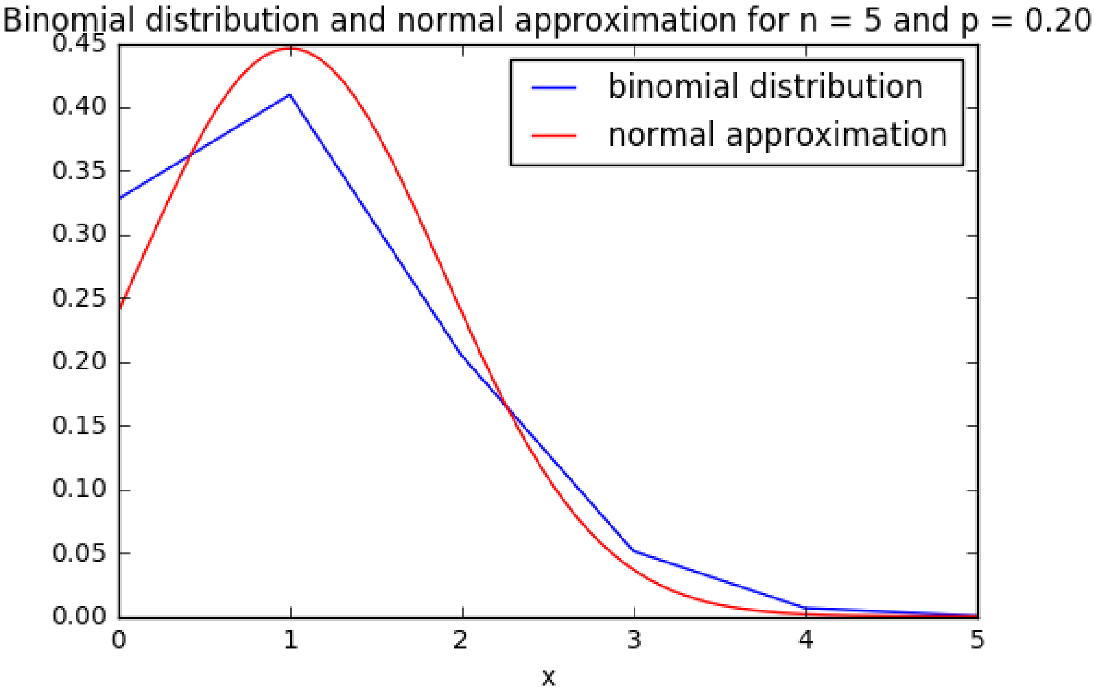
```
        plt.title('Binomial distribution for p = %.1f' %p)
        i += 1
    plt.legend(labels = nlist)
    plt.show()

## (b)

plot_binomial(0.1)
plot_binomial(0.5)
plot_binomial(0.8)
```

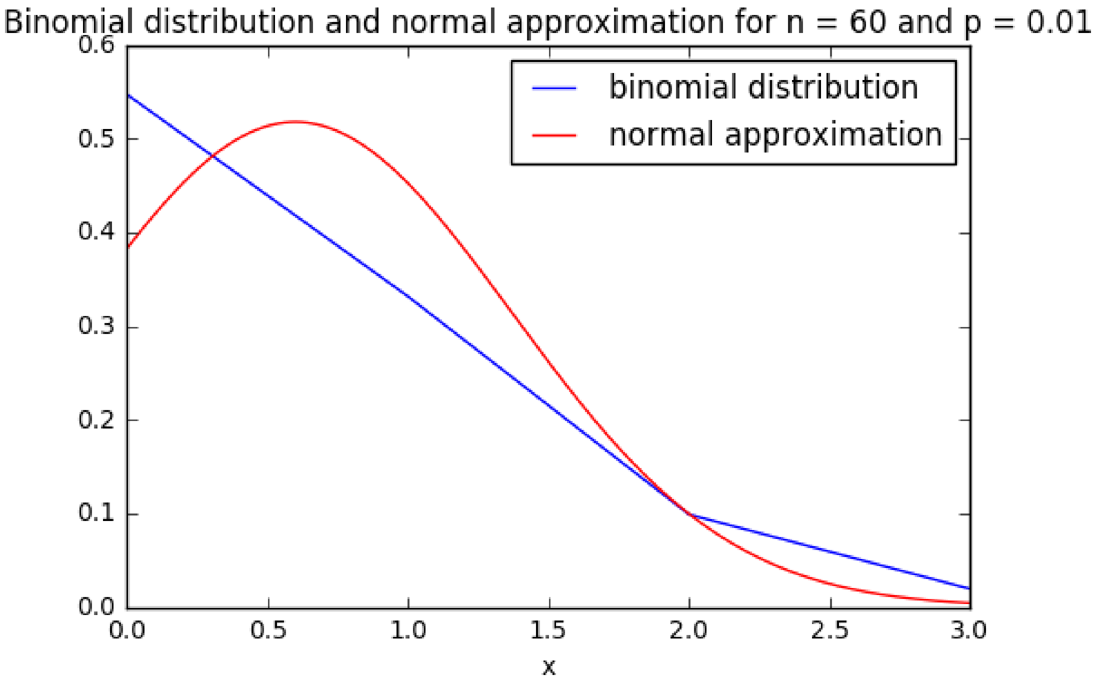


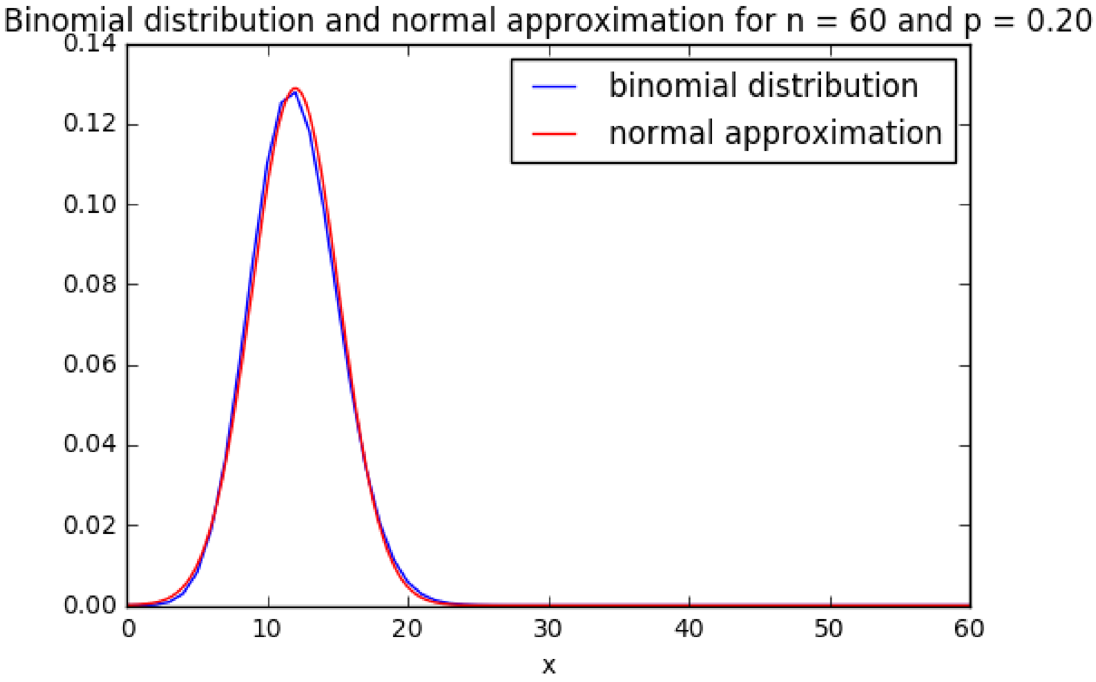

Binomial distribution for p = 0.5
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
20
40
60
80
100
120
k
5
20
40
60
80
100


Binomial distribution for p = 0.8
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
20
40
60
80
100
120
k
5
20
40
60
80
100

```
In [120]: def normal_approx(x,n,p):
              return normal(x,n*p,(n*p*(1-p))**0.5)

          def plot_binomial_normal(n,p) :
              x = np.linspace(0,n+2,100*(n+2)+1)
              plt.plot(range(n+2),binomial_fct(n,p),color='blue')
              plt.plot(x,normal_approx(x,n,p),color='red')
              plt.xlabel('x')
              plt.title('Binomial distribution and normal approximation for n = %i and p = %.2
              plt.xlim(0,5*n*p)
              plt.legend(labels = ('binomial distribution','normal approximation'))
              plt.show()

          ## (b)

          plot_binomial_normal(5,0.2)
          plot_binomial_normal(60,0.01)
          plot_binomial_normal(60,0.2)
```

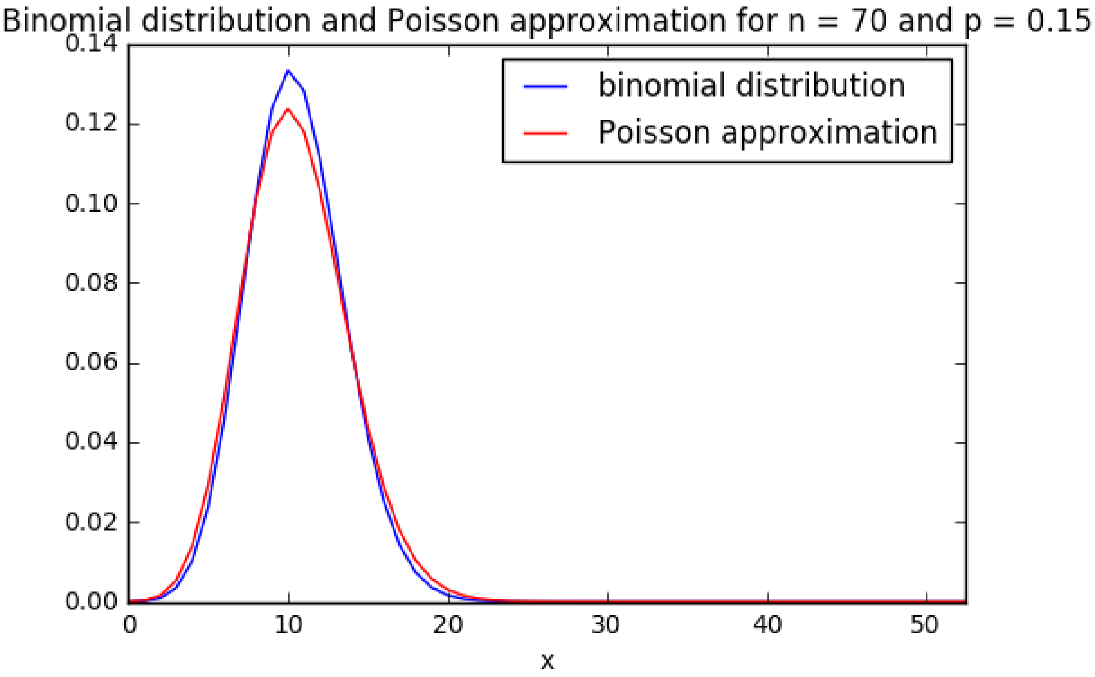


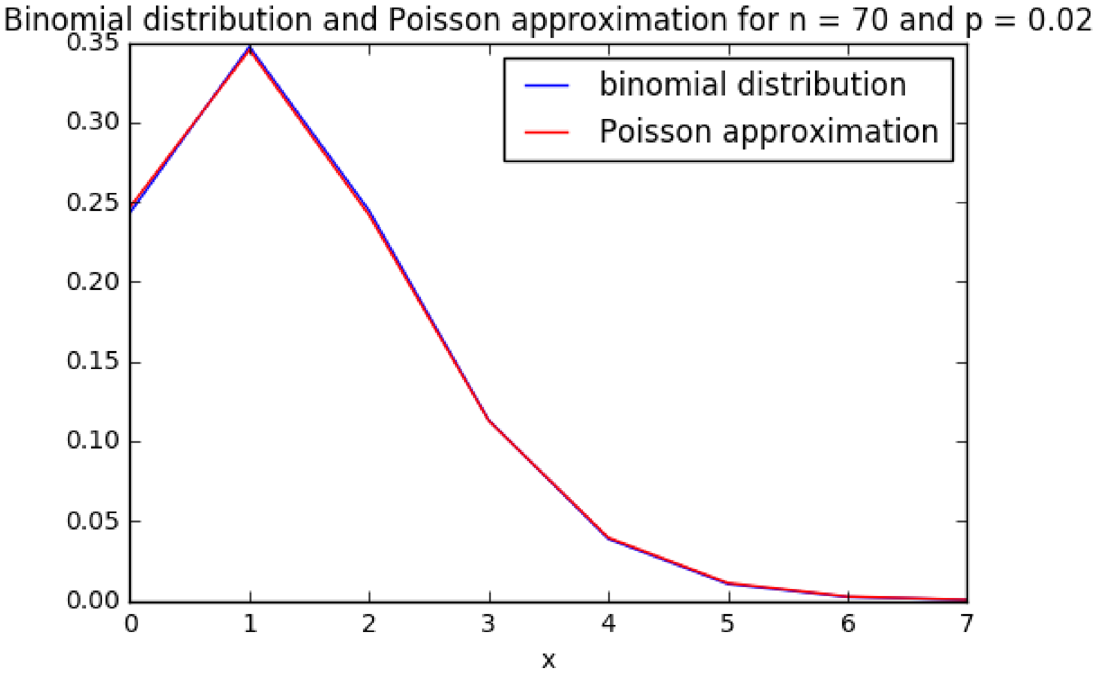




The binomial distribution is a good approximation of the binomial distribution if n is large enough (it seems good for n = 60). It is widely used because it is easier ot use and much faster to compute Above are 2 examples of bad normal approximation because of too small n or p.

```
In [134]: def poisson_approx(k,n,p):
              return poisson(k,n*p)

          def poisson_approx_fct(n,p):
              res = []
              for k in range(n+2):
                  res.append(poisson_approx(k,n,p))
              return res

          def plot_binomial_poisson(n,p) :
              plt.plot(range(n+2),binomial_fct(n,p),color='blue')
              plt.plot(range(n+2),poisson_approx_fct(n,p),color='red')
              plt.xlabel('x')
              plt.title('Binomial distribution and Poisson approximation for n = %i and p = %.
              plt.xlim(0,5*n*p)
              plt.legend(labels = ('binomial distribution','Poisson approximation'))
              plt.show()

          ## (c)

          plot_binomial_poisson(70,0.15)
          plot_binomial_poisson(70,0.02)
```





The first graph shows a bad approximation because of a too high p and the second graph shows a good approximation with Poisson law. This approximation is good for a large number n (at least 20) and a small p (at most 0.05).