Exercise H8.1 : Vapnik - Chervonenkis dimension

$$\bar{C}_{(N+1,N)} = 2\sum_{k=0}^{N}\binom{N}{k}$$

$$= 2\sum_{k=0}^{N}\binom{N}{k}1^{k}1^{N-k}$$

$$= 2 \cdot 2^{N} = 2^{N+1}$$

$$\underbrace{\hat{C}_{(N+1,N)}}_{2^{N+1}} + \acute{C}_{(N+1,N-1)} = \bar{C}_{(N+2,N)}$$

$$\acute{C}_{(N+1,N-1)} = 2 \cdot \sum_{k=0}^{N-1}\binom{N}{k}$$

$$= 2 \cdot \Big(\underbrace{\sum_{k=0}^{N}\binom{N}{k}}_{2^{N}} - 1 \Big)$$

$$= 2^{N+1} - 2$$

$$\acute{C}_{(N+2,N)} = 2^{N+1} + 2^{N+1} - 2 = 2^{N+2} - 2 < 2^{N+2}$$

We have $\check{C}_{(N+1,N)} = 2^{N+1}$ and $\acute{C}_{(N+2,N)} < 2^{N+2}$

so Vapnik - Chervonenkis dimension $d_{VC} = N+1$.