Part 2: Calculus

Real-Valued Function: A mapping that represents a relationship between a set of variables (inputs) and a unique value.

$$y = f(X)$$

Domain: The set o all the possible values of X.

Image or range: The set of all the possible value of y.

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Logarithmic function, $Ln\left(F(X)\right)$: $\mathbb{R} - \{F(X) \leq 0\}$

Some functions don't exist for specific points: There exist some discontinuity

Limits: Help us analyzing the discontinuity and continuity of the functions

Removable discontinuity in $x = x_0$:

- The function does not exist in x_0
- The limit of the function exists, is the same from left and right

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Infinity discontinuity in $x = x_0$:

- When the limit of the function form left or right tends to infinity

Derivative:

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

If it exist we denoted as:

$$f_x = f'(x_0) = \frac{df(x_0)}{dx}$$

This derivative will be also the slope of the tangent line of the function at that point

Limits:

- Polynomial: We look only the temr with the highest degree

  - $\lim_{x \to \infty} 2x^3 - 2x =$

  - $\lim_{x \to -\infty} 3x^3 - 4x^2 =$

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- $\lim_{x \to -\infty} 3x^3 - 4x^2 = \lim_{x \to -\infty} 3x^3 = -\infty$

$$\frac{f(x)}{g(x)} = \frac{0}{0} \; or \; \frac{\infty}{\infty}$$

$$\downarrow$$

$$L'H\hat{o}pital$$

$$\downarrow$$

$$\frac{f'(x)}{g'(x)}$$

$$L'H\hat{o}pital$$

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

$$L'H\hat{o}pital$$

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{0}{0}$$

$$\downarrow$$

$$\lim_{x \to 2} \frac{1}{2x} = \frac{1}{4}$$

$$\lim_{x \to \infty} \frac{e^x}{x^2 - 4}$$

$$\lim_{x\to\infty} \frac{e^x}{x^2 - 4} = \frac{\infty}{\infty}$$

$$\downarrow$$

$$\lim_{x\to\infty} \frac{e^x}{2x} = \frac{\infty}{\infty}$$

$$\downarrow$$

$$\lim_{x\to\infty} \frac{e^x}{2} = \infty$$

Another indeterminations

- $\infty - \infty$: We transform it into $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x \to 1} \left(\frac{2}{x^2 - 1} - \frac{1}{x - 1} \right)$$

$$\lim_{x \to 1}\left(\frac{2}{x^2 - 1} - \frac{1}{x - 1}\right) = \lim_{x \to 1}\left(\frac{2}{(x-1)(x+1)} - \frac{x+1}{(x-1)(x+1)}\right) =$$

$$\lim_{x \to 1}\left(\frac{1 - x}{(x-1)(x+1)}\right) = -\frac{1}{2}$$

Another indeterminations:

- $\lim_{x\to a} f(x)^{g(x)}$ = $1^{\infty} \to e^{\lim_{x\to a} g(x)[f(x)-1]}$

$$\lim_{x\to 0}(e^{x}+x)^{\frac{1}{x}} = \lim_{x\to 0}(1)^{\infty}$$

$$\lim_{x\to 0}(e^{x}+x)^{\frac{1}{x}} = e^{\lim\limits_{x\to 0}\frac{1}{x}(e^{x}+x-1)} = e^{\frac{0}{0}}$$

$$\downarrow$$

$$e^{\lim\limits_{x\to 0}\frac{e^{x}+x-1}{x}} = (l'Hopital) = e^{\lim\limits_{x\to 0}\frac{e^{x}+1}{1}} = e^{2}$$