57.- Let $f(x,y) = e^{x^2+y^2}$.

a) Compute the contour curves of $f(x, y)$ and represent them graphically.

b) Compute $\nabla f(2,1)$.

c) Compute the directional derivative of f at the point $(2,1)$ following the direction $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

d) Taking into account $\begin{cases} x = 2 + \ln t^2 \\ y = e^{t^3-1} \end{cases}$, compute $\frac{df}{dt}(1)$.

a) Level set: If the exercise doesn't give us the value of the level set, we can choose a couple of them

$$
\begin{aligned}
&level\ \ 1 &&\rightarrow e^{x^2+y^2} = 1 \rightarrow \ \ x^2 + y^2 = 0 \\
&level\ e &&\rightarrow e^{x^2+y^2} = e \rightarrow \ \ x^2 + y^2 = 1
\end{aligned}
$$

Gradient in the point (2,1)

$$f = e^{x^2+y^2} \begin{cases} \dfrac{\partial f}{\partial x} = 2x\, e^{x^2+y^2} = 4e^5 \\[2ex] \dfrac{\partial f}{\partial y} = 2y\, e^{x^2+y^2} = 2e^5 \end{cases}$$

c) Directional Derivative:

$$f = e^{x^2+y^2} \begin{cases} \dfrac{\partial f}{\partial x} = 2x\, e^{x^2+y^2} = 4e^5 \\ \\ \dfrac{\partial f}{\partial y} = 2y\, e^{x^2+y^2} = 2e^5 \end{cases}$$

Direction: $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$D_{\vec{v}} f(2,1) = (4e^5)\left(\frac{1}{\sqrt{2}}\right) + (2e^5)\left(\frac{1}{\sqrt{2}}\right) = \frac{6e^5}{\sqrt{2}}$$

d) Chain Rule

$$f = e^{x^2+y^2}, \qquad x = 2 + \ln(t^2) + 1,$$
$$y = e^{t^3-1}$$

$$z\begin{cases} x \to t \\ y \to t \end{cases}, \qquad t = 1 \to \begin{cases} x = 2 \\ y = 1 \end{cases}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{df}{dt} = (4e^5)\left(\frac{2}{t}\right) + (2e^5)\left(3t^2 e^{t^3-1}\right) = (14e^5)$$

58.- Let the function $f(x,y) = ln(1 + x^2 + 2y^2)$.

a) Compute the domain of *f*.

b) Compute the gradient vector of *f* at (x,y).

c) Is the function *f* diferentiable?

d) Compute the directional derivative of *f* respect to the direction of $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ at the point (1,1).

e) Does the function *f* verify the conditions of Schwarz's Theorem?

f) Using the result of e), compute the Hessian matrix of *f* at (x, y).

a) Domain.

$$f = \ln(1 + x^2 + 2y^2) \rightarrow \quad 1 + x^2 + 2y^2 > 0$$

It will always be positive, so x and y can take any real value:

Domain: $\{x, y \in R\}$

b) Gradient.

$$f = \ln(1 + x^2 + 2y^2) \begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{2x}{1 + x^2 + 2y^2} \\ \\ \dfrac{\partial f}{\partial y} = \dfrac{4y}{1 + x^2 + 2y^2} \end{cases}$$

c) We have previously calculated the derivatives, it is differentiable

d) Directional derivative en (1,1):

$$f = \ln(1 + x^2 + 2y^2) \begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{2x}{1 + x^2 + 2y^2} = \dfrac{2}{4} = \dfrac{1}{2} \\[2ex] \dfrac{\partial f}{\partial y} = \dfrac{4y}{1 + x^2 + 2y^2} = \dfrac{4}{4} = 1 \end{cases}$$

Direction: $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

$$D_{\vec{v}} f(2,1) = \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + (1)\left(\frac{-1}{\sqrt{2}}\right) = \frac{-1}{2\sqrt{2}}$$

E y f) Schwarz theorem: Hessian Matrix will be symmetric.

$$f = \ln(1 + x^2 + 2y^2) \begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{2x}{1 + x^2 + 2y^2} \\ \\ \dfrac{\partial f}{\partial y} = \dfrac{4y}{1 + x^2 + 2y^2} \end{cases}$$

$$Hf(\bar{x}) = \begin{bmatrix} \dfrac{2[1 + x^2 + 2y^2] - 4x^2}{(1 + x^2 + 2y^2)^2} & \dfrac{-8xy}{(1 + x^2 + 2y^2)^2} \\ \dfrac{-8xy}{(1 + x^2 + 2y^2)^2} & \dfrac{4[1 + x^2 + 2y^2] - 16y^2}{(1 + x^2 + 2y^2)^2} \end{bmatrix}$$

66.- The cost function in terms of the number of hours worked, x, is of the form $C(x) = \begin{cases} ax^2 + b & \text{if } x \in [0,100] \\ c\sqrt{x} & \text{if } x \in (100,200] \end{cases}$. Determine a, b, c knowing that $C(x)$ is continuous, that the slope of the straight tangent at $x = 50$ is 8 and that 121 hours worked imply a cost equal to 990.

1) It is continious if it does not have discontinuity points.

We may have a problema at x=100, because we are changing the function.

At (x=100) the value should be the same

$$ax^2 + b = c\sqrt{x}, \qquad \text{at } x = 100$$

$$10000a + b = 10c$$

2) At x=121, cost = 990.

$$c\sqrt{x}, = 990 \quad \text{at } x = 121$$

11c=990

$$c = 90$$

2) Slope of tangent function = derivative at the point (x = 50).

$$\frac{df}{dx}(50) = 2ax = 100a$$

The exericse says it is 8:

$$\boxed{100a = 8 \rightarrow a = \frac{8}{100}}$$

Now we can obtain:

$$10000\frac{8}{100} + b = 10(90)$$

$$800 + b = 900$$

$$\boxed{b = 100}$$

Merry Christmas!

Happy New year!

And don’t forget about the surveys: encuestas.unizar.es