Solución a las derivadas parciales del ejercicio 6.

$$a)\ f = e^{\frac{x}{y}} + e^{\frac{z}{y}} \begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{1}{y} e^{\frac{x}{y}} \\ \dfrac{\partial f}{\partial y} = \dfrac{-x}{y^2} e^{\frac{x}{y}} + \dfrac{-z}{y^2} e^{\frac{z}{y}} \\ \dfrac{\partial f}{\partial z} = \dfrac{1}{y} e^{\frac{z}{y}} \end{cases}$$

(Creo que el b) es el más difícil de todos, dejadlo para el final mejor)

$$b)\ f = ln\left(\frac{\sqrt{x^2+y^2}-x}{\sqrt{x^2+y^2}+x}\right) = ln\left(\frac{(x^2+y^2)^{\frac{1}{2}}-x}{(x^2+y^2)^{\frac{1}{2}}+x}\right) = ln\left(1 + \frac{-2x}{(x^2+y^2)^{\frac{1}{2}}+x}\right)$$

$$= \begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{1}{\dfrac{(x^2+y^2)^{\frac{1}{2}}-x}{(x^2+y^2)^{\frac{1}{2}}+x}} \left(\dfrac{(-2)\left((x^2+y^2)^{\frac{1}{2}}+x\right) - (-2x)\left(\frac{1}{2}(x^2+y^2)^{\frac{-1}{2}}2x+1\right)}{\left((x^2+y^2)^{\frac{1}{2}}+x\right)^2}\right) = \left(-\dfrac{2}{(x^2+y^2)^{\frac{1}{2}}}\right) \\ \dfrac{\partial f}{\partial y} = \dfrac{1}{\dfrac{(x^2+y^2)^{\frac{1}{2}}-x}{(x^2+y^2)^{\frac{1}{2}}+x}} \left(\dfrac{-(-2x)\left(\frac{1}{2}(x^2+y^2)^{\frac{-1}{2}}2y\right)}{\left((x^2+y^2)^{\frac{1}{2}}+x\right)^2}\right) = \left(\dfrac{2x}{y(x^2+y^2)^{\frac{1}{2}}}\right) \end{cases}$$

$$c)\ f = e^{x^2+y^2+z^2} \begin{cases} \dfrac{\partial f}{\partial x} = 2xe^{x^2+y^2+z^2} \\ \dfrac{\partial f}{\partial y} = 2ye^{x^2+y^2+z^2} \\ \dfrac{\partial f}{\partial z} = 2ze^{x^2+y^2+z^2} \end{cases}$$

$$d)\ f = \sqrt{xy+\frac{x}{y}} = \left(xy+\frac{x}{y}\right)^{1/2} \begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{1}{2}\left(xy+\dfrac{x}{y}\right)^{-1/2}\left(y+\dfrac{1}{y}\right) \\ \dfrac{\partial f}{\partial y} = \dfrac{1}{2}\left(xy+\dfrac{x}{y}\right)^{-1/2}\left(x-\dfrac{x}{y^2}\right) \end{cases}$$

$$e)\ f = x^3+y^3-3axy \begin{cases} \dfrac{\partial f}{\partial x} = 3x^2-3ay \\ \dfrac{\partial f}{\partial y} = 3y^2-3ax \end{cases}$$

$$f)\ f = \frac{x-y}{x+y} \begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{(x+y)-(x-y)}{(x+y)^2} = \dfrac{2y}{(x+y)^2} \\ \dfrac{\partial f}{\partial y} = \dfrac{-(x+y)-(x-y)}{(x+y)^2} = \dfrac{-2x}{(x+y)^2} \end{cases}$$

$$g)\ f = ln\left(x+\sqrt{x^2+y^2}\right)\begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{1+\frac{1}{2}(x^2+y^2)^{\frac{-1}{2}}2x}{x+\sqrt{x^2+y^2}} = \dfrac{1}{\sqrt{x^2+y^2}} \\ \dfrac{\partial f}{\partial y} = \dfrac{\frac{1}{2}(x^2+y^2)^{\frac{-1}{2}}2y}{x+\sqrt{x^2+y^2}} = \dfrac{y}{x\sqrt{x^2+y^2}+(x^2+y^2)} \end{cases}$$

$$h)\ f = x^y \begin{cases} \dfrac{\partial f}{\partial x} = yx^{y-1} \\ \dfrac{\partial f}{\partial y} = x^y \ln(x) \end{cases}$$

$$i)\ f = z^{xy} \begin{cases} \dfrac{\partial f}{\partial x} = z^{xy}\ln(z)\, y \\ \dfrac{\partial f}{\partial y} = z^{xy}\ln(z)\, x \\ \dfrac{\partial f}{\partial z} = xy\, z^{xy-1} \end{cases}$$

$$j)\ f = \frac{x+y}{\sqrt[3]{x^2+y^2}} = \frac{x+y}{(x^2+y^2)^{\frac{1}{3}}}$$

$$\begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{(x^2+y^2)^{1/3}-(x+y)\frac{1}{3}(x^2+y^2)^{-2/3}2x}{(x^2+y^2)^{2/3}} = \dfrac{(x^2+y^2)-(x+y)\frac{2}{3}x}{(x^2+y^2)^{4/3}} \\ \dfrac{\partial f}{\partial y} = \dfrac{(x^2+y^2)^{1/3}-(x+y)\frac{1}{3}(x^2+y^2)^{-2/3}2y}{(x^2+y^2)^{2/3}} = \dfrac{(x^2+y^2)-(x+y)\frac{2}{3}y}{(x^2+y^2)^{4/3}} \end{cases}$$

$$k)\ f = x^2(sen\ y)^2 \begin{cases} \dfrac{\partial f}{\partial x} = 2x(sen\ y)^2 \\ \dfrac{\partial f}{\partial y} = x^2 2(sen\ y)(\cos y) \end{cases}$$

$$l)\ f = \frac{e^{ax}(sen\ x + a\ cosy)}{a^2+b^2}\begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{1}{a^2+b^2}\left(ae^{ax}(sen\ x + a\ cosy) + e^{ax}(\cos x)\right) \\ \dfrac{\partial f}{\partial y} = -\dfrac{1}{a^2+b^2}\left(e^{ax}\ a\ (\text{sen}\, x)\right) \end{cases}$$