Solución, se divide en dos partes, y_h y y_p

$$\underbrace{y^{(n} + a_1 y^{(n-1} + \cdots + a_{n-1} y' + a_n y}_{y_h} = \underbrace{f(x)}_{y_p}$$

$$y = y_h + y_p$$

$$f(x) = a + bx + cx^2 + \cdots \rightarrow y_p = A + Bx + Cx^2 + \cdots$$

Manteniendo el grado del polinomio.

Si 0 aparecía “m” veces como raíz antes:

$$y_p = x^m(A + Bx + Cx^2 + \cdots)$$

$$f(x) = ae^{kx} \rightarrow y_p = Ae^{kx}$$

Si k aparecía “m” veces como raíz antes:

$$y_p = x^m(Ae^{kx})$$

$$f(x) = (a + bx)e^{kx} \rightarrow y_p = (A + Bx)e^{kx}$$

Si k aparecía “m” veces como raíz antes:

$$y_p = x^m = (A + Bx)e^{kx}$$

$$f(x) = e^{kx}(a\ sen\ \delta x + b\cos\delta x) \rightarrow y_p = e^{kx}(A\ sen\ \delta x + B\cos\delta x)$$

Si $\mathrm{k} \pm \delta i$ aparecía “m” veces como raíz antes:

$$y_p = x^m e^{kx}(A\ sen\ \delta x + B\cos\delta x)$$

1)

$$y''' - 25y' = 75x^2$$

$$\downarrow$$

$$\begin{cases} r_1 = 0 \\ r_2 = 5 \\ r_3 = -5 \end{cases}$$

$$y_p = x(A + Bx + Cx^2)$$

$$y''' - 25y' = 75x^2$$

$$y_p = x(A + Bx + Cx^2),$$

$$y' = A + 2Bx + 3Cx^2$$

$$y'' = 2B + 6Cx$$

$$y''' = 6C$$

$$6C - 25(A + 2Bx + 3Cx^2) = 75x^2 \begin{cases} C = -1 \\ A = \dfrac{-6}{25} \\ B = 0 \end{cases}$$

$$y''' - 25y' = 75x^2$$

$$y_p = x\left(\frac{-6}{25} - x^2\right)$$

$$y = C_1 e^{0x} + C_2 e^{5x} + C_3 e^{-5x} + x\left(\frac{-6}{25} - x^2\right)$$

2)

$$y^{(4} + 2y''' + 2y'' = e^{-x}$$

$$\downarrow$$

$$\begin{cases} r_1 = 0 \ (dos\ veces) \\ r_2 = -1 \pm 1i \end{cases}$$

$$y_p = Ae^{-x}$$

$$y_p = Ae^{-x},$$
$$y' = -Ae^{-x}$$
$$y'' = Ae^{-x}$$
$$y''' = -Ae^{-x}$$
$$y^{(4} = Ae^{-x}$$

$$Ae^{-x} - 2Ae^{-x} + 2Ae^{-x} = e^{-x}\{A = 1$$

$$y^{(4} + 2y''' + 2y'' = e^{-x}$$

$$y_p = e^{-x}$$

$$y = C_1 e^{0x} + x C_2 e^{0x} + e^{-x}(C_3\ sen\ x + C_4 \cos x) + e^{-x}$$

3)

$$y''' - 4y' = 2sen\, x$$

$$y''' - 4y' = 2sen\, x$$

$$\downarrow$$

$$\begin{cases} r_1 = 0 \\ r_2 = 2 \\ r_3 = -2 \end{cases}$$

$$y_p = A\, sen\, x + B \cos x$$

$$y_p = A\,sen\,x + B\cos x\,,$$
$$y' = A\cos x - B\,sen\,x$$
$$y'' = -A\,sen\,x - B\cos x$$
$$y''' = -A\,cos\,x + B\,\mathrm{sen}\,x$$

$$-A\,cos\,x + B\,\mathrm{sen}\,x - 4(A\cos x - B\,sen\,x) = 2sen\,x$$
$$\begin{cases} A = 0 \\ B = \frac{2}{5} \end{cases}$$

$$y''' - 4y' = 2sen\, x$$

$$y_p = \frac{2}{5}\cos x$$

$$y = C_1 e^{0x} + C_2 e^{2x} + C_3 e^{-2x} + \frac{2}{5}\cos x$$

4)

$$y'' - 2y' + y = (2 + x)e^{-x}$$

$$y'' - 2y' + y = (2 + x)e^{-x}$$

$$\downarrow$$

$$\begin{cases} r_1 = 1 \\ r_2 = 1 \end{cases}$$

$$y_p = (A + Bx)e^{-x}$$

$$y_p = (A + Bx)e^{-x}$$
$$y' = (-A + B - Bx)e^{-x}$$
$$y'' = (A - 2B + Bx)e^{-x}$$

$$(4A - 4B + 4Bx)e^{-x} = (2 + x)e^{-x}$$
$$\begin{cases} A = \dfrac{3}{4} \\ B = \dfrac{1}{4} \end{cases}$$

$$y'' - 2y'' + y = (2 + x)e^{-x}$$

$$y_p = (\frac{3}{4} + \frac{1}{4}x)e^{-x}$$

$$y = C_1 e^{x} + C_2 x e^{x} + (\frac{3}{4} + \frac{1}{4}x)e^{-x}$$

5)

$$y'' - y = e^x$$

$$y'' - y = e^x$$

$$\downarrow$$

$$\begin{cases} r_1 = 1 \\ r_2 = -1 \end{cases}$$

$$y_p = xAe^x$$

$$y_p = xAe^x$$

$$y' = (x+1)Ae^x$$
$$y'' = (x+2)Ae^x$$

$$y'' - y = e^x$$

$$= (x+2)Ae^x - xAe^x = 2Ae^x = e^x$$

$$\{2A = 1 \rightarrow \left\{A = \frac{1}{2}\right.$$

$$y'' - y = e^{x}$$

$$y_p = x\frac{1}{2}e^{x}$$

$$y = C_1 e^{x} + C_2 e^{-x} + x\frac{1}{2}e^{x}$$

6)

$$y' + y = e^{x}$$

$$y' + y = e^x$$

$$\downarrow$$

$$\{ r_1 = -1$$

$$y_p = Ae^x$$

$$y_p = Ae^x$$

$$y' = Ae^x$$

$$y' + y = e^x$$

$$2Ae^x = e^x$$

$$A = 1/2$$

$$y' + y = e^{x}$$

$$y_p = \frac{1}{2}e^{x}$$

$$y = C_1 e^{-x} + \frac{1}{2}e^{x}$$

7)

$$y'' - 2y' = -6x^2$$

$$y'' - 2y' = -6x^2$$

$$\downarrow$$

$$\begin{cases} r_1 = 0 \\ r_2 = 2 \end{cases}$$

$$y_p = x[A + Bx + Cx^2]$$

$$y_p = Ax + Bx^2 + Cx^3$$

$$y' = A + 2Bx + 3Cx^2$$

$$y'' = 2B + 6Cx$$

$$y'' - 2y' = (2B + 6Cx) - 2(A + 2Bx + 3Cx^2) = -6x^2$$

$$(2B - 2A) + (6Cx - 4Bx) - 6Cx^2 = -6x^2$$

$$y_p = Ax + Bx^2 + Cx^3$$

$$(2B - 2A) + (6Cx - 4Bx) - 6Cx^2 = -6x^2$$

$$\left.\begin{array}{r} 2B - 2A = 0 \\ 6C - 4B = 0 \\ -6C = -6 \end{array}\right\} \left\{\begin{array}{c} C = 1 \\ B = 3/2 \\ A = 3/2 \end{array}\right.$$

$$y_p = \frac{3}{2}x + \frac{3}{2}x^2 + x^3$$

$$y'' - 2y' = -6x^2$$

$$y_p = \frac{3}{2}x + \frac{3}{2}x^2 + x^3$$

$$y = C_1 e^{0x} + C_2 e^{2x} + \frac{3}{2}x + \frac{3}{2}x^2 + x^3$$

8)

$$y^{(4} + y''' = 3 - x$$

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$$\downarrow$$

$$\begin{cases} r_1 = 0, \qquad tres\ veces \\ \qquad r_2 = -1 \end{cases}$$

$$y_p = x^3[A + Bx]$$

$$y_p = Ax^3 + Bx^4$$

$$y' = 3Ax^2 + 4Bx^3$$

$$y'' = 6Ax + 12Bx^2$$

$$y''' = 6A + 24Bx$$

$$y^{(4} = 24B$$

$$y^{(4} + y''' = 3 - x$$

$$24B + (6A + 24Bx) = 3 - x$$

$$\left.\begin{matrix} 24B + 6A = 3 \\ 24Bx = -x \end{matrix}\right\} \begin{cases} B = \dfrac{-1}{24} \\ A = \dfrac{2}{3} \end{cases}$$

$$y_p = \frac{2}{3}x^3 - \frac{1}{24}x^4$$

$$y^{(4} + y''' = 3 - x$$

$$y = C_1 e^{0x} + x C_2 e^{0x} + x^2 C_3 e^{0x} + C_4 e^{-x} + \frac{2}{3} x^3 - \frac{1}{24} x^4$$