Taylor’s Formula

Taylor's Formula (only with 1 variable). Of order N, in point x=a

$$f(x) = f(a) + \sum_{n=1}^{N} \frac{f^{(n}}{n!}(x-a)^n + R_N$$

First order:

$$f(x) = f(a) + f_x(a)(x-a) + R_1(a)$$

Second order:

$$f(x) = f(a) + f_x(a)(x-a) + \frac{1}{2} f_{xx}(a)(x-a)^2 + R_2(a)$$

$f(x) = e^x$, in $x = 0$

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First order:

$$f(x) = f(0) + f'(0)(x-0) = 1 + x + R_1$$

Second order:

$$f(x) = f(0) + f'(0)(x-0) + \frac{1}{2!}f''(0)(x-0)^2 + R_2 =$$
$$= 1 + x + \frac{1}{2}(x)^2 + R_2$$

Third order:

$$f(x) = f(0) + f'(0)(x-0) + \frac{1}{2!}f''(0)(x-0)^2 + \frac{1}{3!}f'''(0)(x-0)^3 + R_3 =$$
$$= 1 + x + \frac{1}{2}(x)^2 + \frac{1}{6}(x)^3 + R_3$$

$f(x) = \ln(x)$, in $x = 1$

$f(x) = \ln(x)$, in $x = 1$

First order:

$$f(x) = f(1) + f'(1)(x-1) = 0 + (x-1) + R_1$$

Second order:

$$f(x) = f(1) + f'(1)(x-1) + \frac{1}{2!}f''(1)(x-1)^2 + R_2 =$$
$$= x - 1 - \frac{1}{2}(x-1)^2 + R_2$$

Third order:

$$f(x) = f(1) + f'(1)(x-1) + \frac{1}{2!}f''(1)(x-1)^2 + \frac{1}{3!}f'''(1)(x-1)^3 + R_3 =$$
$$= x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 + R_3$$

Exercise 1) $f(x) = \sin(x)$, in $x = 0$

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First order:

$$f(x) = f(0) + f'(0)(x - 0) = 0 + (x) + R_1$$

Second order:

$$f(x) = f(0) + f'(0)(x - 0) + \frac{1}{2!}f''(0)(x - 0)^2 + R_2 =$$
$$= x + R_2$$

Third order:

$$f(x) = f(0) + f'(0)(x - 0) + \frac{1}{2!}f''(0)(x - 0)^2 + \frac{1}{3!}f'''(0)(x - 0)^3 + R_3 =$$
$$= x - \frac{x^3}{6} + R_3$$

Implicit Functions

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This is a normal function in which we can obtain a clear relationship between y and x

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$$Implicit\ Functions: \quad \frac{x}{y+y^2} - 1 = 0 \ \rightarrow \ We\ can\ not\ obtain\ y\ as\ a\ function\ of\ x$$

$$¿\frac{dy}{dx}?$$

It exist a relationship between both variables, but it can not be directly obtained

To obtain $\frac{dy}{dx}$, we can use the next formula:

$$\frac{dy}{dx} = -\frac{\frac{\partial f(x,y)}{\partial x}}{\frac{\partial f(x,y)}{\partial y}}$$

An implicit function will exist in (x_0, y_0), if:

1) $F(x_0, y_0) = 0$ $\qquad$ $(\text{The } equation\ exist\ and\ is\ true\ in\ that\ point)$

2) We can obtain the derivatives in the point (x_0, y_0)

3) $\frac{\partial f(x,y)}{\partial y} \neq 0$ $\qquad$ (In the point (x_0, y_0))

Example:

$$F(x, y) = x^2y + xy^2 - 16 = 0, \qquad in\ (2,2)$$

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Derivatives: $\begin{cases} \dfrac{\partial f(x,y)}{\partial x} = 2xy + y^2 = 8 + 4 = 12 \\ \dfrac{\partial f(x,y)}{\partial y} = x^2 + 2xy = 4 + 8 = 12 \end{cases}$

$$\left.\begin{array}{l} 1)\ \textit{Se cumple}\text{: } 4 * 2 + 2 * 4 - 16 = 0 \\ \\ 2)\ \textit{We have calculated the derivatives} \\ \\ 3)\ \dfrac{\partial f(x,y)}{\partial y} = x^2 + 2xy = 12 \neq 0 \end{array}\right\} \text{The implicit function exists}$$

$$F(x,y) = x^2y + xy^2 - 16 = 0$$

Derivative: $\frac{dy}{dx} = -\frac{\frac{\partial f(x,y)}{\partial x}}{\frac{\partial f(x,y)}{\partial y}} = -\frac{2xy+y^2}{x^2+2xy}$

In $(2,2) \rightarrow \frac{dy}{dx} = -1$

Exercise 3)

$$F(x, y) = x\ln(y) + y - e = 0, \qquad in = (0, e)$$

1) Calculate if it exist, dy/dx

$$F(x, y) = x \ln(y) + y - e = 0, \qquad in = (0,e)$$

Derivatives: $\begin{cases} \dfrac{\partial f(x,y)}{\partial x} = \ln(y) = 1 \\ \dfrac{\partial f(x,y)}{\partial y} = \dfrac{x}{y} + 1 = 1 \end{cases}$

$$\left.\begin{array}{c} 1)\ 0 \ln(e) + e - e = 0 \\ \text{2) We have calculated the derivatives} \\ 3)\ \dfrac{\partial f(x,y)}{\partial y} = 1 \neq 0 \end{array}\right\} \quad \text{The implicit function exist}$$

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Derivatives: $\begin{cases} \dfrac{\partial f(x,y)}{\partial x} = \ln(y) = 1 \\ \dfrac{\partial f(x,y)}{\partial y} = \dfrac{x}{y} + 1 = 1 \end{cases}$

$$\frac{dy}{dx}(0,e) = -\frac{\dfrac{\partial f(x,y)}{\partial x}}{\dfrac{\partial f(x,y)}{\partial y}} = -\frac{1}{1} = -1$$

Exercise 4)

$$F(x, y, z) = x^2 z + yz - 4 = 0, \qquad in = (1,1,2)$$

1) Calculate the differencial of z respect to x and y

$$F(x,y,z) = x^2z + yz - 4 = 0, \qquad in = (1,1,2)$$

Derivatives: $\begin{cases} \dfrac{\partial f(x,y,z)}{\partial x} = 2xz = 4 \\ \dfrac{\partial f(x,y,z)}{\partial y} = z = 2 \\ \dfrac{\partial f(x,y,z)}{\partial z} = x^2 + y = 2 \end{cases}$

$$\left.\begin{array}{c} 1) \quad (2) + (2) - 4 = 0 \\ \text{2) We have calculated the derivatives} \\ 3) \ \dfrac{\partial f(x,y,z)}{\partial z} = 2 \neq 0 \end{array}\right\} \text{The implicit function exists}$$

$$F(x,y,z) = x^2z + yz - 4 = 0, \qquad in = (1,1,2)$$

Derivatives: $\begin{cases} \frac{\partial f(x,y,z)}{\partial x} = 2xz = 4 \\ \frac{\partial f(x,y,z)}{\partial y} = z = 2 \\ \frac{\partial f(x,y,z)}{\partial z} = x^2 + y = 2 \end{cases}$

$$\frac{\partial z}{\partial x}(1,1,2) = -\frac{\frac{\partial f(x,y,z)}{\partial x}}{\frac{\partial f(x,y,z)}{\partial z}} = -\frac{4}{2} = -2$$

$$\frac{\partial z}{\partial y}(1,1,2) = -\frac{\frac{\partial f(x,y,z)}{\partial y}}{\frac{\partial f(x,y,z)}{\partial z}} = -\frac{2}{2} = -1$$

$$F(x, y, z) = x^2z + yz - 4 = 0, \qquad in = (1,1,2)$$

$$\frac{\partial z}{\partial x}(1,1,2) = -\frac{\dfrac{\partial f(x, y, z)}{\partial x}}{\dfrac{\partial f(x, y, z)}{\partial z}} = -\frac{4}{2} = -2$$

$$\frac{\partial z}{\partial y}(1,1,2) = -\frac{\dfrac{\partial f(x, y, z)}{\partial y}}{\dfrac{\partial f(x, y, z)}{\partial z}} = -\frac{2}{2} = -1$$

Differencial equation of the relationship z=g(x,y) in (1,1,2):

$$dz = -2dx - dy$$