10.- Determinar los óptimos locales de las siguientes funciones sujetas a las restricciones que se indican:

f) $f(x,y) = x + y$

s.a: $x^2 + y^2 = 1$

$$f(x,y,z) = x + y$$

s.a. $\{x^2 + y^2 - 1 = 0$

$$L = x + y + \lambda \; (x^2 + y^2 - 1)$$

$$L = x + y + \lambda \quad (x^2 + y^2 - 1)$$

Puntos críticos:
$$\begin{cases} \dfrac{\partial L}{\partial x} = 1 + 2x\lambda = 0 \\ \dfrac{\partial L}{\partial y} = 1 + 2y\lambda = 0 \\ \dfrac{\partial L}{\partial \lambda} = x^2 + y^2 - 1 = 0 \end{cases}$$

$$L = x + y + \lambda \quad (x^2 + y^2 - 1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= 1 + 2x\lambda = 0 \\ \frac{\partial L}{\partial y} &= 1 + 2y\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= x^2 + y^2 - 1 = 0 \end{aligned}\right\} \begin{cases} \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \lambda = \frac{-1}{\sqrt{2}} \\ \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right), \lambda = \frac{1}{\sqrt{2}} \end{cases}$$

$$L = x + y + \lambda \ \ (x^2 + y^2 - 1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= 1 + 2x\lambda = 0 \\ \frac{\partial L}{\partial y} &= 1 + 2y\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= x^2 + y^2 - 1 = 0 \end{aligned}\right\} \left\{ HL = \begin{pmatrix} 2\lambda & 0 \\ 0 & 2\lambda \end{pmatrix} \right.$$

1º punto. $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right), \lambda = \frac{-1}{\sqrt{2}} \rightarrow HL = \begin{pmatrix} \frac{-2}{\sqrt{2}} & 0 \\ 0 & \frac{-2}{\sqrt{2}} \end{pmatrix}$

Definida Negativa -> Máximo Local

2º punto. $\left(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right), \lambda = \frac{1}{\sqrt{2}} \rightarrow HL = \begin{pmatrix} \frac{2}{\sqrt{2}} & 0 \\ 0 & \frac{2}{\sqrt{2}} \end{pmatrix}$

Definida Positiva -> Mínimo Local

(No hace falta usar la restricción ya que sale directamente signo definido)

10.- Determinar los óptimos locales de las siguientes funciones sujetas a las restricciones que se indican:

i) $f(x,y) = x\,y$

s.a: $x^2 + y^2 = 1$

$$f(x, y, z) = xy$$

s.a. $\{x^2 + y^2 - 1 = 0$

$$L = xy + \lambda \quad (x^2 + y^2 - 1)$$

$$L = xy + \lambda \quad (x^2 + y^2 - 1)$$

Puntos críticos:
$$\begin{cases} \dfrac{\partial L}{\partial x} = y + 2x\lambda = 0 \\ \dfrac{\partial L}{\partial y} = x + 2y\lambda = 0 \\ \dfrac{\partial L}{\partial \lambda} = x^2 + y^2 - 1 = 0 \end{cases}$$

$$L = xy + \lambda \quad (x^2 + y^2 - 1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= y + 2x\lambda = 0 \\ \frac{\partial L}{\partial y} &= x + 2y\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= x^2 + y^2 - 1 = 0 \end{aligned}\right\} \begin{cases} \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \lambda = \frac{-1}{2} \\ \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right), \lambda = \frac{1}{2} \\ \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \lambda = \frac{1}{2} \\ \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right), \lambda = \frac{-1}{2} \end{cases}$$

$$L = xy + \lambda \quad (x^2 + y^2 - 1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= y + 2x\lambda = 0 \\ \frac{\partial L}{\partial y} &= x + 2y\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= x^2 + y^2 - 1 = 0 \end{aligned}\right\} \left\{ HL = \begin{pmatrix} 2\lambda & 1 \\ 1 & 2\lambda \end{pmatrix} \right.$$

1º punto. $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ $\lambda=\frac{-1}{2}\rightarrow HL=\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}$ *S.D.N*

Sin tener en cuenta la restricción:

$$Q(h_1,h_2)=-h_1^2+2h_1h_2-h_2^2$$

Restringimos para el caso: $Jg\begin{pmatrix}h_1\\ h_2\end{pmatrix}=0$

$$(2x \quad 2y)\begin{pmatrix}h_1\\ h_2\end{pmatrix}=0\rightarrow\left(\frac{2}{\sqrt{2}} \quad \frac{2}{\sqrt{2}}\right)\begin{pmatrix}h_1\\ h_2\end{pmatrix}=0\rightarrow h_2=-h_1$$

1º punto. $h_2 = -h_1$

$$Q(h_1, h_2) = -h_1^2 + 2h_1h_2 - h_2^2$$

Restringida:

$$Q(h_1) = -h_1^2 - 2h_1^2 - h_1^2 = -4h_1^2 < 0$$

Definida Negativa -> Máximo local condicionado

2º punto. $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ $\lambda = \frac{1}{2} \rightarrow HL = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ *S.D.P*

Sin tener en cuenta la restricción:

$$Q(h_1, h_2) = h_1^2 + 2h_1h_2 + h_2^2$$

Restringimos para el caso: Jg $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0$

$(2x \quad 2y) \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0 \rightarrow \left(\frac{2}{\sqrt{2}} \quad \frac{-2}{\sqrt{2}}\right) \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0 \rightarrow h_2 = h_1$

2º punto. $h_2 = h_1$

$$Q(h_1, h_2) = h_1^2 + 2h_1h_2 + h_2^2$$

Restringida:

$$Q(h_1) = h_1^2 + 2h_1^2 + h_1^2 = 4h_1^2 > 0$$

Definida Positiva -> Mínimo local condicionado

1º y 4º puntos son iguales. Máximos locales condicionados.

2º y 3º puntos son iguales. Mínimos locales condicionados.

14.- Sea $f(x,y,z) = x + 2y + 2z$:

a) Calcular, si existen, los óptimos locales de *f* en $S = \left\{ (x,y,z) \in \mathbb{R}^3 \middle| x + 4z^2 = 5 \right\}$.

b) Calcular, si existen, los óptimos locales de *f* en $T = \left\{ (x,y,z) \in \mathbb{R}^3 \middle| x^2 + y^2 + 4z^2 = 6 \right\}$.

$$f(x, y, z) = x + 2y + 2z$$

s.a. $\{x + 4z^2 - 5 = 0$

$$L = x + 2y + 2z + \lambda(x + 4z^2 - 5)$$

$$L = x + 2y + 2z + \lambda(x + 4z^2 - 5)$$

$$\text{Puntos críticos:} \begin{cases} \dfrac{\partial L}{\partial x} = 1 + \lambda = 0 \\ \dfrac{\partial L}{\partial y} = 2 = 0 \quad \rightarrow Imposible \\ \dfrac{\partial L}{\partial z} = 2 + \lambda 8z = 0 \\ \dfrac{\partial L}{\partial \lambda} = x + 4z^2 - 5 = 0 \end{cases}$$

No hay óptimos locales con esa restricción

$$f(x, y, z) = x + 2y + 2z$$

s.a. $\{x^2 + y^2 + 4z^2 - 6 = 0$

$$L = x + 2y + 2z + \lambda(x^2 + y^2 + 4z^2 - 6)$$

$$L = x + 2y + 2z + \lambda(x^2 + y^2 + 4z^2 - 6)$$

Puntos críticos:
$$\begin{cases} \dfrac{\partial L}{\partial x} = 1 + \lambda 2x = 0 \\ \dfrac{\partial L}{\partial y} = 2 + \lambda 2y = 0 \\ \dfrac{\partial L}{\partial z} = 2 + \lambda 8z = 0 \\ \dfrac{\partial L}{\partial \lambda} = x^2 + y^2 + 4z^2 - 6 = 0 \end{cases}$$

$$L = x + 2y + 2z + \lambda(x^2 + y^2 + 4z^2 - 6)$$

$$\left.\begin{aligned}
\frac{\partial L}{\partial x} &= 1 + \lambda 2x = 0 \\
\frac{\partial L}{\partial y} &= 2 + \lambda 2y = 0 \\
\frac{\partial L}{\partial z} &= 2 + \lambda 8z = 0 \\
\frac{\partial L}{\partial \lambda} &= x^2 + y^2 + 4z^2 - 6 = 0
\end{aligned}\right\} \left\{\begin{aligned}
&\left(1,2,\frac{1}{2}\right), \lambda = \frac{-1}{2} \\
&\left(-1,-2,\frac{-1}{2}\right), \lambda = \frac{1}{2}
\end{aligned}\right.$$

$$L = x + 2y + 2z + \lambda(x^2 + y^2 + 4z^2 - 6)$$

$$\left.\begin{array}{r}\dfrac{\partial L}{\partial x} = 1 + \lambda 2x = 0\\ \dfrac{\partial L}{\partial y} = 2 + \lambda 2y = 0\\ \dfrac{\partial L}{\partial z} = 2 + \lambda 8z = 0\\ \dfrac{\partial L}{\partial \lambda} = x^2 + y^2 + 4z^2 - 6 = 0\end{array}\right\} \left\{ HL = \begin{pmatrix} 2\lambda & 0 & 0 \\ 0 & 2\lambda & 0 \\ 0 & 0 & 8\lambda \end{pmatrix} \right.$$

1º punto $\left(1,2,\frac{1}{2}\right), \lambda = \frac{-1}{2} \rightarrow HL = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix}$

Definida negativa -> Máximo local condicionado

2º punto $\left(-1,-2,\frac{-1}{2}\right), \lambda = \frac{1}{2} \rightarrow HL = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

Definida positiva -> Mínimo local condicionado

15.- Sea $f(x,y)=\dfrac{y-x}{y}$. Estudiar la existencia de máximos y mínimos relativos condicionados a la restricción $x-y^2=1$.

$$f(x,y,z) = \frac{y-x}{y}$$

s.a. $\{x - y^2 - 1 = 0$

$$L = \frac{y-x}{y} + \lambda(x - y^2 - 1) = 1 - \frac{x}{y} + \lambda(x - y^2 - 1)$$

$$L = 1 - \frac{x}{y} + \lambda(x - y^2 - 1)$$

Puntos críticos: $$\begin{cases} \dfrac{\partial L}{\partial x} = -\dfrac{1}{y} + \lambda = 0 \\ \dfrac{\partial L}{\partial y} = \dfrac{x}{y^2} - \lambda 2y = 0 \\ \dfrac{\partial L}{\partial \lambda} = x - y^2 - 1 = 0 \end{cases}$$

$$L = 1 - \frac{x}{y} + \lambda(x - y^2 - 1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= -\frac{1}{y} + \lambda = 0 \\ \frac{\partial L}{\partial y} &= \frac{x}{y^2} - \lambda 2y = 0 \\ \frac{\partial L}{\partial \lambda} &= x - y^2 - 1 = 0 \end{aligned}\right\} \begin{cases} (2,1), \lambda = 1 \\ (2,-1), \lambda = -1 \end{cases}$$

$$L = 1 - \frac{x}{y} + \lambda(x - y^2 - 1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= -\frac{1}{y} + \lambda = 0 \\ \frac{\partial L}{\partial y} &= \frac{x}{y^2} - \lambda 2y = 0 \\ \frac{\partial L}{\partial \lambda} &= x - y^2 - 1 = 0 \end{aligned}\right\} HL = \begin{pmatrix} 0 & \dfrac{1}{y^2} \\ \dfrac{1}{y^2} & \dfrac{-2x}{y^3} - 2\lambda \end{pmatrix}$$

1º punto: $(2{,}1), \lambda = 1 \rightarrow HL = \begin{pmatrix} 0 & 1 \\ 1 & -6 \end{pmatrix}$

$$Q(h_1, h_2) = 2(h_1 h_2) - 6h_2^2 \qquad Indefinida$$

$$Jg\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} 1 & -2y \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0 \rightarrow h_1 = 2h_2$$

$$Q(h_2) = 4h_2^2 - 6h_2^2 = -2h_2^2 < 0$$

$$Definida\ Negativa \rightarrow Máximo\ local\ condicionado$$

2º punto: $(2,-1), \lambda = -1 \rightarrow HL = \begin{pmatrix} 0 & 1 \\ 1 & 6 \end{pmatrix}$

$$Q(h_1, h_2) = 2(h_1 h_2) + 6h_2^2 \quad Indefinida$$

$$Jg\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = (1 \quad -2y)\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = (1 \quad 2)\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = 0 \rightarrow h_1 = -2h_2$$

$$Q(h_2) = -4h_2^2 + 6h_2^2 = 2h_2^2 > 0$$

$$Definida\ Positiva \rightarrow Mínimo\ local\ condicionado$$