2)

$$z = \frac{Ln(x)}{y}, \qquad x = t^2, \qquad y = t^2 - u$$

$$u = s + 2t^2$$

$Differential\ equation\ of\ z\ respect\ to\ s\ and\ t.$ *In the point (s,t)=(1,1)*

$$z = \frac{Ln(x)}{y}, \qquad x = t^2, \qquad y = t^2 - u$$

$$u = s + 2t^2$$

$$z \to \begin{cases} x \to \{t \\ y \to \begin{cases} t \\ u \to \begin{cases} s \\ t \end{cases} \end{cases} \end{cases}$$

$$z = \frac{Ln(x)}{y}, \qquad x = t^2, \qquad y = t^2 - u$$

$$u = s + 2t^2$$

$$z \to \begin{cases} x \to \{t \\ \\ y \to \begin{cases} t \\ u \to \begin{cases} s \\ t \end{cases} \end{cases} \end{cases}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}\frac{\partial u}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}\frac{\partial u}{\partial t}$$

$$z = \frac{Ln(x)}{y}, \qquad x = t^2, \qquad y = t^2 - u$$

$$u = s + 2t^2$$

$$z \to \begin{cases} x \to \{t \\ y \to \begin{cases} t \\ u \to \begin{cases} s \\ t \end{cases} \end{cases} \end{cases}$$

$$\frac{\partial z}{\partial s} = \left(-\frac{Ln(x)}{y^2}\right)(-1)(1)$$

$$\frac{\partial z}{\partial t} = \left(\frac{1}{xy}\right)(2t) + \left(-\frac{Ln(x)}{y^2}\right)(2t) + \left(-\frac{Ln(x)}{y^2}\right)(-1)(4t)$$

$$z = \frac{Ln(x)}{y}, \qquad x = t^2, \qquad y = t^2 - u$$

$$u = s + 2t^2$$

(s,t)=(1,1) $\begin{cases} x = 1 \\ u = 3 \rightarrow y = -2 \end{cases}$

$$\frac{\partial z}{\partial s} = \left(-\frac{Ln(1)}{4}\right)(-1)(1) = 0$$

$$\frac{\partial z}{\partial t} = \left(\frac{1}{-2}\right)(2) + \left(-\frac{Ln(1)}{4}\right)(2) + \left(-\frac{Ln(1)}{4}\right)(-1)(4) = -1$$

$$dz = \frac{\partial z}{\partial s} ds + \frac{\partial z}{\partial t} dt$$

$$dz = (0)ds + (-1)dt = -dt$$

3)

$$F(x, y) = x\ln(y) + y - e = 0, \qquad In\ the\ point = (0, e)$$

1) Analyze if the implicit function, $y = f(x)$, exists in that point

2) Calculte dy/dx

$$F(x,y) = x\ln(y) + y - e = 0, \qquad \textit{In the point} = (0,e)$$

Derivatives: $\begin{cases} \dfrac{\partial f(x,y)}{\partial x} = \ln(y) = 1 \\ \dfrac{\partial f(x,y)}{\partial y} = \dfrac{x}{y} + 1 = 1 \end{cases}$

$$\left.\begin{array}{l} 1)\ 0\ln(e) + e - e = 0 \\ \quad 2)\ \text{It is continuous} \\ \quad 3)\ \dfrac{\partial f(x,y)}{\partial y} = 1 \neq 0 \end{array}\right\} \quad \text{The implicit function exists}$$

$F(x, y) = x\ln(y) + y - e = 0,\qquad In\ the\ point = (0,e)$

Derivatives: $\begin{cases} \dfrac{\partial f(x,y)}{\partial x} = \ln(y) = 1 \\ \dfrac{\partial f(x,y)}{\partial y} = \dfrac{x}{y} + 1 = 1 \end{cases}$

$$\frac{dy}{dx}(0,e) = -\frac{\frac{\partial f(x,y)}{\partial x}}{\frac{\partial f(x,y)}{\partial y}} = -\frac{1}{1} = -1$$

If x increases, then y will decrease

4)

$$F(x, y, z) = x^2 z + yz - 4 = 0, \qquad In\ the\ point = (1,1,2)$$

1) Analyze if the implicit function, $z = g(x, y)$, exists in that point

2) Calculate $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

$F(x, y, z) = x^2 z + yz - 4 = 0,$ $\quad In\ the\ point$ = (1,1,2)

Derivatives: $\begin{cases} \dfrac{\partial f(x,y,z)}{\partial x} = 2xz = 4 \\ \dfrac{\partial f(x,y,z)}{\partial y} = z = 2 \\ \dfrac{\partial f(x,y,z)}{\partial z} = x^2 + y = 2 \end{cases}$

$\left.\begin{array}{l} 1)\ (2) + (2) - 4 = 0 \\ \text{2) It is continuous} \\ \text{3) } \dfrac{\partial f(x,y,z)}{\partial z} = 2 \neq 0 \end{array}\right\}$ The implicit function exists

$F(x,y,z) = x^2 z + yz - 4 = 0,$ $\qquad$ *In the point* = (1,1,2)

Derivatives: $\begin{cases} \frac{\partial f(x,y,z)}{\partial x} = 2xz = 4 \\ \frac{\partial f(x,y,z)}{\partial y} = z = 2 \\ \frac{\partial f(x,y,z)}{\partial z} = x^2 + y = 2 \end{cases}$

$$\frac{\partial z}{\partial x}(1,1,2) = -\frac{\frac{\partial f(x,y,z)}{\partial x}}{\frac{\partial f(x,y,z)}{\partial z}} = -\frac{4}{2} = -2$$

$$\frac{\partial z}{\partial y}(1,1,2) = -\frac{\frac{\partial f(x,y,z)}{\partial y}}{\frac{\partial f(x,y,z)}{\partial z}} = -\frac{2}{2} = -1$$

$F(x, y, z) = x^2 z + yz - 4 = 0,$ $\quad$ $In\ the\ point$= (1,1,2)

$$\frac{\partial z}{\partial x}(1,1,2) = -\frac{\frac{\partial f(x,y,z)}{\partial x}}{\frac{\partial f(x,y,z)}{\partial z}} = -\frac{4}{2} = -2$$

$$\frac{\partial z}{\partial y}(1,1,2) = -\frac{\frac{\partial f(x,y,z)}{\partial y}}{\frac{\partial f(x,y,z)}{\partial z}} = -\frac{2}{2} = -1$$

Differential equation of z=g(x,y) in (1,1,2):

$$dz = -2dx - dy$$

$$f(x,y)=\frac{\sqrt{x^4+y^4}}{x}\to m=$$

$$f(x,y)=3x^4+4x^2y^2+5y^4\to m=$$

$$f(x,y)=\frac{x^2y^2}{x^3+y^3}\to m=$$

$$f(x,y)=\frac{2xy}{x^2+y^2}\to m=$$

$$f(x,y)=\frac{\sqrt{x^4+y^4}}{x}\rightarrow m=1$$

$$f(x,y)=3x^4+4x^2y^2+5y^4\rightarrow m=4$$

$$f(x,y)=\frac{x^2y^2}{x^3+y^3}\rightarrow m=1$$

$$f(x,y)=\frac{2xy}{x^2+y^2}\rightarrow m=0$$

5) Grado de homogeneidad de las siguientes ecuaciones:

$$f(x,y,z) = \ln\left(\frac{x-2y}{y+3z}\right) \to m =$$

$$f(x,y,z) = e^{3x+y} + \sqrt[3]{xz} \to m =$$

$$f(x,y,z) = e^{\sqrt{\frac{x^2}{yz}}} \to m =$$

$$f(x,y,z) = x\,Ln\left(\frac{y}{z}\right) + \sqrt{2yz} \to m =$$

Grado de homogeneidad de las siguientes ecuaciones:

$$f(x,y,z) = ln\frac{x-2y}{y+3z} \rightarrow m = 0$$

$$f(x,y,z) = e^{3x+y} + \sqrt[3]{xz} \rightarrow m = \text{no tiene}$$

$$f(x,y,z) = e^{\sqrt{\frac{x^2}{yz}}} \rightarrow m = 0$$

$$f(x,y,z) = x\ Ln\left(\frac{y}{z}\right) + \sqrt{2yz} \rightarrow m = 1$$

49.- Let f be a differentiable, homogeneous function of degree 2 with $\frac{\partial f}{\partial x}(3,2) = 3$ and $\frac{\partial f}{\partial y}(3,2) = 4$. Compute $f(3,2)$.

49.- Let f be a differentiable, homogeneous function of degree 2 with $\frac{\partial f}{\partial x}(3,2)=3$ and $\frac{\partial f}{\partial y}(3,2)=4$. Compute $f(3,2)$.

Euler's Theorem

$$x\frac{\partial f(\bar{x})}{\partial x}+y\frac{\partial f(\bar{x})}{\partial y}=m\,f(\bar{x})$$

$$3\cdot 3+2\cdot 4=2\cdot f(3,2)$$

$$17=2\,f(3,2)\rightarrow f(3,2)=8.5$$

50.- Let $f(x,y,z)$ be a differentiable, homogeneous function of degree 3 such that the components of its gradient vector at $(1,2,3)$ are $(5,2,2)$. What is the value of the function at the point $(1,2,3)$?

50.- Let $f(x,y,z)$ be a differentiable, homogeneous function of degree 3 such that the components of its gradient vector at (1,2,3) are (5,2,2). What is the value of the function at the point (1,2,3)?

Euler's Theorem

$$x\frac{\partial f(\bar{x})}{\partial x}+y\frac{\partial f(\bar{x})}{\partial y}+z\frac{\partial f(\bar{x})}{\partial z}=m\,f(\bar{x})$$

$$1\cdot 5+2\cdot 2+3\cdot 2=3\cdot\ f(1,2,3)\rightarrow f(1,2,3)=5$$