$$f(x, y) = 3 - 2x^2 + y^3$$

1) Partial derivatives
2) Hessian
3) Differential in the point (1,2)
4) Directional derivative. Direction = $\left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$, point=$(1,2)$
5) Tangent plane in the point (1,2)

$$3 - 2x^2 + y^3 \begin{cases} \dfrac{\partial f}{\partial x} = -4x \begin{cases} \dfrac{\partial^2 f}{\partial x^2} = -4 \\ \dfrac{\partial^2 f}{\partial x \partial y} = 0 \end{cases} \\ \dfrac{\partial f}{\partial y} = 3y^2 \begin{cases} \dfrac{\partial^2 f}{\partial y \partial x} = 0 \\ \dfrac{\partial^2 f}{\partial y^2} = 6y \end{cases} \end{cases}$$

Hessian Matrix of $3 - 2x^2 + y^3$

$$Hf(\bar{x}) = \begin{bmatrix} -4 & 0 \\ 0 & 6y \end{bmatrix}$$

Differential:

$$3 - 2x^2 + y^3 \begin{cases} \dfrac{\partial f}{\partial x} = -4x \\ \dfrac{\partial f}{\partial y} = 3y^2 \end{cases}$$

$$df(\bar{x}) = \frac{\partial f(\bar{x})}{\partial x} dx + \frac{\partial f(\bar{x})}{\partial y} dy = (-4x)dx + (3y^2)dy$$

in the point (1,2) $\rightarrow df(1,2) = -4dx + 12dy$

Directional derivative:

$$3-2x^2+y^3 \begin{cases} \dfrac{\partial f}{\partial x} = -4x \\ \dfrac{\partial f}{\partial y} = 3y^2 \end{cases}$$

In the direction: $\left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$

$$D_{\vec{v}} f(\bar{x}) = (-4x)\left(\frac{3}{\sqrt{10}}\right) + (3y^2)\left(\frac{1}{\sqrt{10}}\right)$$

in the point (1,2) $\rightarrow D_{\vec{v}} f(1,2) = (-4)\left(\frac{3}{\sqrt{10}}\right) + (12)\left(\frac{1}{\sqrt{10}}\right) = 0$

Tangent plane of $3 - 2x^2 + y^3$:

$$z = f(x_0, y_0) + (-4x)(x - x_0) + (3y^2)(y - y_0)$$

Tangent plane of f in the point $(1,2)$:

$$\mathrm{z} = 9 - 4(x - 1) + 12(y - 1)$$

$$f(x, y) = \ln(xy)$$

1) Partial derivatives
2) Hessian
3) Differential in the point (3,3)
4) Directional derivative. Direction $=\left(\frac{2}{\sqrt{8}}, \frac{2}{\sqrt{8}}\right)$, point$=(4,2)$
5) Tangent plane in the point (1,1)

$$\ln(xy)\begin{cases}\dfrac{\partial f}{\partial x}=\dfrac{y}{xy}=\dfrac{1}{x} & \begin{cases}\dfrac{\partial^2 f}{\partial x^2}=\dfrac{-1}{x^2}\\ \dfrac{\partial^2 f}{\partial x\partial y}=0\end{cases}\\ \dfrac{\partial f}{\partial y}=\dfrac{x}{xy}=\dfrac{1}{y} & \begin{cases}\dfrac{\partial^2 f}{\partial y\partial x}=0\\ \dfrac{\partial^2 f}{\partial y^2}=\dfrac{-1}{y^2}\end{cases}\end{cases}$$

Hessian Matrix of $\ln(xy)$

$$Hf(\bar{x}) = \begin{bmatrix} \frac{-1}{x^2} & 0 \\ 0 & \frac{-1}{y^2} \end{bmatrix}$$

Directional derivative:

$$\ln(xy)\begin{cases}\dfrac{\partial f}{\partial x}=\dfrac{1}{x}\\[2mm]\dfrac{\partial f}{\partial y}=\dfrac{1}{y}\end{cases}$$

$$df(\bar{x})=\frac{\partial f(\bar{x})}{\partial x}dx+\frac{\partial f(\bar{x})}{\partial y}dy=\left(\frac{1}{x}\right)dx+\left(\frac{1}{y}\right)dy$$

in the point (3,3) $\rightarrow df(3,3)=\frac{1}{3}dx+\frac{1}{3}dy$

Directional derivative:

$$\ln(xy)\begin{cases}\dfrac{\partial f}{\partial x}=\dfrac{1}{x}\\[2ex] \dfrac{\partial f}{\partial y}=\dfrac{1}{y}\end{cases}$$

In the direction: $\left(\frac{2}{\sqrt{8}},\frac{2}{\sqrt{8}}\right)$

$$D_{\vec{v}}f(\bar{x})=\left(\frac{1}{x}\right)\left(\frac{2}{\sqrt{8}}\right)+\left(\frac{1}{y}\right)\left(\frac{2}{\sqrt{8}}\right)$$

in the point (4,2) $\rightarrow D_{\vec{v}}f(4,2)=\left(\frac{1}{4}\right)\left(\frac{2}{\sqrt{8}}\right)+\left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{8}}\right)=\left(\frac{3}{4}\right)\left(\frac{2}{\sqrt{8}}\right)$

Tangent plane of $\ln(xy)$:

$$z = \ln(xy) + \left(\frac{1}{x}\right)(x - x_0) + \left(\frac{1}{y}\right)(y - y_0)$$

Tangent plane of f in the point $(1,1)$:

$$z = 0 + (x - 1) + (y - 1)$$

$$f(x,y) = x^4 + 2xy - y^2$$

1) Partial derivatives
2) Hessian
3) Differential in the point (2,3)
4) Directional derivative. Direction = $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$, point=$(1,1)$
5) Tangent plane in the point (1,-1)

$$
x^4 + 2xy - y^2 \begin{cases} \dfrac{\partial f}{\partial x} = 4x^3 + 2y & \begin{cases} \dfrac{\partial^2 f}{\partial x^2} = 12x^2 \\ \dfrac{\partial^2 f}{\partial x \partial y} = 2 \end{cases} \\ \dfrac{\partial f}{\partial y} = 2x - 2y & \begin{cases} \dfrac{\partial^2 f}{\partial y \partial x} = 2 \\ \dfrac{\partial^2 f}{\partial y^2} = -2 \end{cases} \end{cases}
$$

Matriz Hessian $x^4+2xy-y^2$

$$Hf(\bar{x}) = \begin{bmatrix} 12x^2 & 2 \\ 2 & -2 \end{bmatrix}$$

Differential:

$$x^4+2xy-y^2\begin{cases}\dfrac{\partial f}{\partial x}=4x^3+2y\\[2ex]\dfrac{\partial f}{\partial y}=2x-2y\end{cases}$$

$$df(\bar{x})=(4x^3+2y)dx+(2x-2y)dy$$

in the point (2,3) $\rightarrow df(2,3)=38dx-2dy$

Directional derivative:

$$x^4+2xy-y^2 \begin{cases} \dfrac{\partial f}{\partial x} = 4x^3+2y \\ \dfrac{\partial f}{\partial y} = 2x-2y \end{cases}$$

In the direction: $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$

$$D_{\vec{v}} f(\bar{x}) = (4x^3+2y)\left(\frac{1}{\sqrt{10}}\right) + (2x-2y)\left(\frac{3}{\sqrt{10}}\right)$$

in the point (1,1) $\rightarrow D_{\vec{v}} f(1,1) = (6)\left(\frac{1}{\sqrt{10}}\right) + (2-2)\left(\frac{3}{\sqrt{10}}\right) = \frac{6}{\sqrt{10}}$

Tangent plane of $x^4+2xy-y^2$ in the point $(1,-1)$

$$z=(1-2-1)+(4-2)(x-1)+(2+2)(y+1)$$

$$z=-2+2(x-1)+4(y+1)$$

$$f(x,y) = \frac{x^2}{1+y}$$

0) Level set of value 4

1) Partial derivatives

2) Hessian

3) Differential in the point (1,1)

4) Directional derivative. Direction $=\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$, point$=(2,1)$

5) Tangent plane in the point (-1,1)

Level set of value 4: Set of points in which the function is equal to 4:

$$f(x,y) = \frac{x^2}{1+y} = 4 \rightarrow x^2 = 4(1+y) = 4 + 4y$$

$$\downarrow$$

$$y = \frac{x^2 - 4}{4} = \frac{x^2}{4} - 1$$

$$\frac{x^2}{1+y}\begin{cases}\dfrac{\partial f}{\partial x}=\dfrac{2x}{1+y} & \begin{cases}\dfrac{\partial^2 f}{\partial x^2}=\dfrac{2}{1+y}\\[2ex]\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{-2x}{(1+y)^2}\end{cases}\\[6ex]\dfrac{\partial f}{\partial y}=\dfrac{-x^2}{(1+y)^2} & \begin{cases}\dfrac{\partial^2 f}{\partial y\partial x}=\dfrac{-2x}{(1+y)^2}\\[2ex]\dfrac{\partial^2 f}{\partial y^2}=\dfrac{2x^2}{(1+y)^3}\end{cases}\end{cases}$$

Matriz Hessian $\frac{x^2}{1+y}$

$$Hf(\bar{x}) = \begin{bmatrix} \dfrac{2}{1+y} & \dfrac{-2x}{(1+y)^2} \\ \dfrac{-2x}{(1+y)^2} & \dfrac{2x^2}{(1+y)^3} \end{bmatrix}$$

Differential:

$$\frac{x^2}{1+y}\begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{2x}{1+y} \\ \dfrac{\partial f}{\partial y} = \dfrac{-x^2}{(1+y)^2} \end{cases}$$

$$df(\bar{x}) = \left(\frac{2x}{1+y}\right)dx + \left(\frac{-x^2}{(1+y)^2}\right)dy$$

in the point (1,1) $\rightarrow df(1{,}1) = dx - \frac{1}{4}dy$

Directional derivative:

$$\frac{x^2}{1+y}\begin{cases} \dfrac{\partial f}{\partial x} = \dfrac{2x}{1+y} \\ \dfrac{\partial f}{\partial y} = \dfrac{-x^2}{(1+y)^2} \end{cases}$$

In the direction: $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$D_{\vec{v}} f(\bar{x}) = \left(\frac{2x}{1+y}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{-x^2}{(1+y)^2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

in the point (2,1) $\rightarrow D_{\vec{v}} f(2,1) = (2)\left(\frac{1}{\sqrt{2}}\right) + (-1)\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

Tangent plane of $\frac{x^2}{1+y}$ in the point $(-1, 1)$

$$z = \frac{(-1)^2}{1+1} + \left(\frac{-2}{1+1}\right)(x+1) + \left(\frac{-(-1)^2}{(1+1)^2}\right)(y+1)$$

$$z = f(x, y) = \frac{1}{2} - (x+1) - \frac{1}{4}(y+1)$$