Chain Rule

$$z = f(y) \rightarrow If\ y\ changes, z\ changes\text{:}\ \ \frac{\partial z}{\partial y} = \frac{\partial f(y)}{\partial y}$$

$$y = g(x) \rightarrow If\ x\ changes, y\ changes\text{:}\ \ \frac{\partial y}{\partial x} = \frac{\partial g(x)}{\partial x}$$

$$\downarrow$$

$$If\ x\ changes, y\ changes$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}\frac{\partial y}{\partial x}$$

$$z = 2y^2 \quad \rightarrow \quad \frac{\partial z}{\partial y} =$$

$$y = 3x + 2 \rightarrow \frac{\partial y}{\partial x} =$$

$$\frac{\partial z}{\partial x} =$$

$$z = 2y^2 \quad \rightarrow \quad \frac{\partial z}{\partial y} = 4y$$

$$y = 3x + 2 \rightarrow \frac{\partial y}{\partial x} = 3$$

$$\frac{\partial z}{\partial x} = (4y)(3) = 12y$$

Chain Rule

If z is a function of several variables that depend on other variables, we can apply the chain rule

$$z \begin{cases} x_1 \to t \\ x_2 \to t \\ \quad \dots \\ x_n \to t \end{cases}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x_1}\frac{\partial x_1}{\partial t} + \frac{\partial z}{\partial x_2}\frac{\partial x_2}{\partial t} + \cdots + \frac{\partial z}{\partial x_n}\frac{\partial x_n}{\partial t}$$

For complex relationships we can first draw all the relations .

$Example$:

$$u = xyz \begin{cases} x = \ln(t) \\ y = t^2 \\ z = \dfrac{1}{t} \end{cases}, \qquad ¿\frac{du}{dt} \text{ in t} = 1?$$

$$Example:$$

$$u = xyz \begin{cases} x = \ln(t) \\ y = t^2 \\ z = \dfrac{1}{t} \end{cases} \rightarrow u \begin{cases} x \rightarrow t \\ y \rightarrow t \\ z \rightarrow t \end{cases}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t}$$

$$\frac{du}{dt} = yz\left(\frac{1}{t}\right) + xz(2t) + xy\left(\frac{-1}{t^2}\right)$$

$$\text{In } \ t = 1 \begin{cases} x = 0 \\ y = 1 \\ z = 1 \end{cases} \rightarrow \frac{du}{dt} = 1$$

Exercise

$$z = 3x + y, \qquad x = t^2 + 1, \qquad y = e^t$$

$$¿\frac{dz}{dt} \ in \ t = 0?$$

$$z = 3x + y, \qquad x = t^2 + 1, \qquad y = e^t$$

$$z \begin{cases} x \to t \\ y \to t \end{cases}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{dz}{dt} = 3 * 2t + 1 * e^t = 6t + e^t$$

$$z = 3x + y, \qquad x = t^2 + 1, \qquad y = e^t$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{dz}{dt} = 3 * 2t + 1 * e^t = 6t + e^t$$

$$\text{In t} = 0 \ \rightarrow \frac{dz}{dt} = 0 + 1 = 1$$

Homogeneous Function of degree m

$$f(t\bar{x}) = t^m f(\bar{x})$$

If we multiply all the variables inside the function by t, the value of the function is multiplied by t^m.

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$$f(x, y) = x^2 y \rightarrow f(tx, ty) =$$

Homogeneous Function of degree m

$$f(t\bar{x}) = t^m f(\bar{x})$$

If we multiply all the variables inside the function by t, the value of the function is multiplied by t^m.

$$f(x,y) = x^2 y \rightarrow f(tx,ty) = (tx)^2(ty) = t^3 x^2 y = t^3 f(x,y)$$

The function is an homogeneous function of degree 3

Homogeneous Function of degree m

$$f(t\bar{x}) = t^m f(\bar{x})$$

If we multiply all the variables inside the function by t, the value of the function is multiplied by t^m.

$$f(x,y) = \frac{x}{y} \to f(tx,ty) =$$

Homogeneous Function of degree m

$$f(t\bar{x}) = t^m f(\bar{x})$$

If we multiply all the variables inside the function by t, the value of the function is multiplied by t^m.

$$f(x,y) = \frac{x}{y} \rightarrow f(tx,ty) = \frac{tx}{ty} = \frac{x}{y} = t^0 f(x,y)$$

The function is an homogeneous function of degree 0

Homogeneous Function of degree m

$$f(t\bar{x}) = t^m f(\bar{x})$$

If we multiply all the variables inside the function by t, the value of the function is multiplied by t^m.

$$f(x, y) = \frac{x}{y + y^2} \rightarrow f(tx, ty) =$$

Homogeneous Function of degree m

$$f(t\bar{x}) = t^m f(\bar{x})$$

If we multiply all the variables inside the function by t, the value of the function is multiplied by t^m.

$$f(x,y) = \frac{x}{y + y^2} \rightarrow f(tx, ty) = \frac{tx}{ty + (ty)^2} = \frac{x}{y + ty^2}$$

It is not an homogeneous function (we can not extract the t)

Euler's Theorem

$$x_1 \frac{\partial f(\bar{x})}{\partial x_1} + x_2 \frac{\partial f(\bar{x})}{\partial x_2} + \cdots + x_n \frac{\partial f(\bar{x})}{\partial x_n} = m\, f(\bar{x})$$

Euler's Theorem

$$x_1 \frac{\partial f(\bar{x})}{\partial x_1} + x_2 \frac{\partial f(\bar{x})}{\partial x_2} + \cdots + x_n \frac{\partial f(\bar{x})}{\partial x_n} = m\, f(\bar{x})$$

$$f(x, y) = x^2 y \ \rightarrow$$

$$f(x, y) = \frac{x}{y} \rightarrow$$

Euler's Theorem

$$x_1 \frac{\partial f(\bar{x})}{\partial x_1} + x_2 \frac{\partial f(\bar{x})}{\partial x_2} + \cdots + x_n \frac{\partial f(\bar{x})}{\partial x_n} = m\, f(\bar{x})$$

$$f(x,y) = x^2 y \;\rightarrow\; x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = x(2xy) + y(x^2) = 3\, x^2 y = 3\, f(x,y)$$

$$f(x,y) = \frac{x}{y} \rightarrow$$

Euler's Theorem

$$x_1 \frac{\partial f(\bar{x})}{\partial x_1} + x_2 \frac{\partial f(\bar{x})}{\partial x_2} + \cdots + x_n \frac{\partial f(\bar{x})}{\partial x_n} = m\, f(\bar{x})$$

$$f(x,y) = x^2 y \;\rightarrow\; x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x(2xy) + y(x^2) = 3\,x^2 y = 3\,f(x,y)$$

$$f(x,y) = \frac{x}{y} \rightarrow\; x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x\left(\frac{1}{y}\right) + y\left(\frac{-x}{y^2}\right) = 0 = 0\,f(x,y)$$

Properties

$$\left.\begin{matrix} f(x)\ of\ degree\ m_1 \\ g(x)\ of\ degree\ m_1 \end{matrix}\right\} f(x)+g(x)\ of\ degree\ (m_1)$$

$$\left.\begin{matrix} f(x)\ of\ degree\ m_1 \\ g(x)\ of\ degree\ m_2 \end{matrix}\right\} f(x)g(x)\ of\ degree\ (m_1+m_2)$$

$$\left.\begin{matrix} f(x)\ of\ degree\ m_1 \\ g(x)\ of\ degree\ m_2 \end{matrix}\right\} \frac{f(x)}{g(x)}\ of\ degree(m_1-m_2)$$

$$\left.\begin{matrix} f(x)\ of\ degree\ m_1 \\ g(x)\ of\ degree\ m_2 \end{matrix}\right\} g\circ f=g\big(f(x)\big)\ of\ degree\ (m_1m_2)$$

Properties

$$f(x)\ of\ degree\ m \rightarrow \frac{\partial f}{\partial x}\ of\ degree\ (m-1)$$

$$f(x,y) = \frac{\sqrt{x^4 + y^4}}{x} \rightarrow m =$$

$$f(x,y) = 3x^4 + 4x^2y^2 + 5y^4 \rightarrow m =$$

$$f(x,y) = \frac{x^2y^2}{x^3 + y^3} \rightarrow m =$$

$$f(x,y) = \frac{2xy}{x^2 + y^2} \rightarrow m =$$

$$f(x,y) = \frac{\sqrt{x^4 + y^4}}{x} \rightarrow m = 1$$

$$f(x,y) = 3x^4 + 4x^2y^2 + 5y^4 \rightarrow m = 4$$

$$f(x,y) = \frac{x^2y^2}{x^3 + y^3} \rightarrow m = 1$$

$$f(x,y) = \frac{2xy}{x^2 + y^2} \rightarrow m = 0$$