Solución de las derivadas del ejercicio 5.

a) $f(x) = e^5 = constante \quad \rightarrow \quad f'(x) = 0$

b) $f(x) = \frac{e^x}{x} \quad \rightarrow \quad f'(x) = \frac{e^x(x)-e^x}{x^2} = e^x\left(\frac{x-1}{x}\right)$

c) $f(x) = \frac{1}{x} = x^{-1} \quad \rightarrow \quad f'(x) = -1x^{-2} = -\frac{1}{x^2}$

d) $f(x) = e^{\frac{-x}{2}} \quad \rightarrow \quad f'(x) = e^{\frac{-x}{2}}\left(\frac{-1}{2}\right)$

e) $f(x) = \frac{1+x}{1-x} \quad \rightarrow \quad f'(x) = \frac{(1-x)-(-(1+x))}{(1-x)^2} = \frac{2}{(1-x)^2}$

f) $f(x) = \sqrt{2x-1} = (2x-1)^{\frac{1}{2}} \quad \rightarrow \quad f'(x) = \frac{1}{2}(2x-1)^{\frac{-1}{2}}(2) = \frac{1}{\sqrt{2x-1}}$

g) $f(x) = \ln(x) + \sqrt{x^2+1} \quad \rightarrow$

$$f'(x) = \frac{1}{x} + \frac{1}{2}(x^2+1)^{\frac{-1}{2}}2x = \frac{1}{x} + \frac{x}{\sqrt{x^2+1}}$$

h) $f(x) = \frac{\pi}{x} + \ln(2) \quad \rightarrow \quad f'(x) = -\frac{\pi}{x^2}$

i) $f(x) = \frac{2}{2x-1} - \frac{1}{x} \quad \rightarrow \quad f'(x) = \frac{-4}{(2x-1)^2} + \frac{1}{x^2}$

j) $f(x) = \frac{1+\sqrt{x}}{1-\sqrt{x}} = \frac{1+x^{\frac{1}{2}}}{1-x^{\frac{1}{2}}} \quad \rightarrow$

$$f'(x) = \frac{\frac{1}{2}x^{-\frac{1}{2}}\left(1-x^{\frac{1}{2}}\right) - (-\frac{1}{2}x^{-\frac{1}{2}}(1+x^{1/2}))}{(1-x^{1/2})^2} = \frac{x^{-\frac{1}{2}}}{(1-x^{1/2})^2}$$

$$= \frac{1}{\sqrt{x}\left(1-\sqrt{x}\right)^2}$$

k) $f(x) = \ln(x)\log(x) - \ln(a)\, log_a(x) \;\rightarrow\; f'(x) =$
$\frac{1}{x}\log(x) + \ln(x)\frac{1}{x\ln(10)} - \frac{\ln(a)}{x\ln(a)} = \frac{1}{x}\log(x) + \frac{1}{x}\log(x) - \frac{1}{x} =$
$\frac{1}{x}(2\log(x) - 1)$

(Para este hay que saberse bien las propiedades de los logaritmos)

l) $f(x) = \frac{3}{56(2x-1)^7} - \frac{1}{24(2x-1)^6} - \frac{1}{40(2x-1)^5} \quad \rightarrow$

$$f'(x) = \frac{3(-7)2}{56(2x-1)^8} - \frac{1(-6)(2)}{24(2x-1)^7} - \frac{1(-5)(2)}{40(2x-1)^6} =$$
$$= \frac{-3}{4(2x-1)^8} + \frac{2}{4(2x-1)^7} + \frac{1}{4(2x-1)^6} =$$
$$= \frac{-3}{4(2x-1)^8} + \frac{2(2x-1)}{4(2x-1)^8} + \frac{(2x-1)^2}{4(2x-1)^8} =$$
$$= \left(\frac{1}{4(2x-1)^8}\right)(-3 + 2(2x-1) + (2x-1)^2) =$$
$$= \left(\frac{1}{4(2x-1)^8}\right)(4x^2 - 4) = \left(\frac{x^2-1}{(2x-1)^8}\right)$$

m) $f(x) = \sqrt[3]{2e^x - 2^x + 1} + (\ln(x))^5 \quad \rightarrow$

$$f'(x) = \frac{1}{3\left(\sqrt[3]{2e^x - 2^x + 1}\right)^2}(2e^x - 2^x\ln(2)) + \frac{5(\ln(x))^4}{x}$$

n) $f(x) = x^2 10^{2x} \quad \rightarrow$

$$f'(x) = 2x\,(10^{2x}) + x^2(10^{2x})2\ln(10) =$$
$$= 2x(10^{2x})(1 + xln(10)$$

o) $f(x) = (\ln(x))^2 - \ln(\ln(x)) \quad \rightarrow \quad f'(x) = \frac{2\ln(x)}{x} - \frac{1}{\ln(x)x}$

p) $f(x) = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left(x + \sqrt{x^2 - a^2}\right) \rightarrow$

$$f'(x) = \frac{1}{2}\sqrt{x^2 - a^2} + \frac{x}{2}\frac{2x}{2\sqrt{x^2 - a^2}} - \frac{a^2}{2}\frac{1 + \frac{2x}{2\sqrt{x^2 - a^2}}}{x + \sqrt{x^2 - a^2}} =$$

$$= \sqrt{x^2 - a^2}$$

q) $f(x) = x^4(a - 2x^3)^2 \quad \rightarrow$

$$f'(x) = 4x^3(a - 2x^3)^2 + x^4 2(a - 2x^3)(-6x^2) =$$

$$= 4x^3(a - 2x^3)[(a - 5x^3)]$$

r) $f(x) = \sqrt{(x + a)(x + b)(x + c)} \quad \rightarrow$

$$f'(x) = \frac{[(x + b)(x + c) + (x + a)(x + c) + (x + b)(x + b)]}{2\sqrt{(x + a)(x + b)(x + c)}} =$$

$$= \sqrt{(x + a)(x + b)(x + c)}\left[\frac{1}{2(x + a)} + \frac{1}{2(x + b)} + \frac{1}{2(x + c)}\right]$$

s) $f(x) = \sqrt[3]{x + \sqrt{x}} \quad \rightarrow \quad f'(x) = \frac{1}{3\left(\sqrt[3]{x + \sqrt{x}}\right)^2}\left(1 + \frac{1}{2\sqrt{x}}\right)$

t) $f(x) = (2x + 1)(3x + 2)\sqrt[3]{3x + 2} \quad \rightarrow$

$$f'(x) = 2(3x + 2)^{\frac{4}{3}} + (2x + 1)\frac{4}{3}(3x + 2)^{\frac{1}{3}}3 =$$

$$= 2(3x + 2)^{\frac{4}{3}} + (8x + 4)(3x + 2)^{\frac{1}{3}}$$

u) $f(x) = \frac{4}{3}\sqrt[4]{\frac{x-1}{x+2}} \quad \rightarrow$

$$f'(x) = \frac{4}{3}\frac{1}{4\sqrt[4]{\left(\frac{x - 1}{x + 2}\right)^3}}\frac{x + 2 - (x - 1)}{(x + 2)^2} = \frac{1}{(x + 2)^2}\frac{1}{\sqrt[4]{\left(\frac{x - 1}{x + 2}\right)^3}}$$

v) $f(x) = \ln(sen(x)) \rightarrow f'(x) = \frac{1}{sen(x)}\cos(x)$

w) $f(x) = sen(x^4) \rightarrow f'(x) = \cos(x^4)\,4x^3$

x) $f(x) = (\cos(x))^4 \rightarrow f'(x) = -4(\cos(x))^3 sen(x)$

y) $f(x) = \left(sen(x)\right)^2 \cos(x^6) \rightarrow$

$f'(x) = 2sen(x)\cos(x)\cos(x^6) - \left(sen(x)\right)^2 sen(x^6)6(x^5)$