Matrices: Exercises

Exercise

$$\mathrm{X} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

Verify:

$$X^2 - 7X + 10I_2 = 0_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathrm{X} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

Verify:

$$X^2 - 7X + 10I_2 =$$

$$\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} - 7\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} + 10\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathrm{X} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

Verify:

$$X^2 - 7X + 10I_2 =$$

$$\begin{pmatrix} 18 & 14 \\ 7 & 11 \end{pmatrix} - \begin{pmatrix} 28 & 14 \\ 7 & 21 \end{pmatrix} + \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

$$\mathrm{X} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

Verify:

$$X^2 - 7X + 10I_2 =$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0_2$$

Exercise:

Obtain the matrix A of size 2 that verify the next equation:

$$A\begin{pmatrix}0\\2\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$$

Knowing that A is a symmetric matrix and that $|A| = 1$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$\left.\begin{array}{r} 2a_{12} = 2 \\ 2a_{22} = 4 \\ a_{11}a_{22} - a_{12}a_{12} = 1 \end{array}\right\} \begin{cases} a_{11} = 1 \\ a_{12} = 1 \\ a_{22} = 2 \end{cases}$$

1 a) Exercise

$$\begin{vmatrix} 3 & 2 & 5 \\ 2 & 1 & 4 \\ 3 & 1 & 6 \end{vmatrix} =$$

1 a) Exercise

Order 3

$$\begin{vmatrix} 3 & 2 & 5 \\ 2 & 1 & 4 \\ 3 & 1 & 6 \end{vmatrix} =$$

$$= 3 \cdot 1 \cdot 6 + 5 \cdot 2 \cdot 1 + 3 \cdot 2 \cdot 4$$
$$-5 \cdot 1 \cdot 3 - 3 \cdot 1 \cdot 4 - 2 \cdot 2 \cdot 6 =$$
$$= 52 - 51 = 1$$

1 a) Exercise

$$\begin{vmatrix} 3 & 2 & 5 \\ 2 & 1 & 4 \\ 3 & 1 & 6 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 5 \\ 2 & 1 & 4 \\ 0 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 7 & 5 \\ 2 & 5 & 4 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$(+)1 \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = (15 - 14) = 1$$

1 f) Exercise

$$A = \begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

$$|A|?$$

$$|A| = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 \end{vmatrix}$$

Row 2 + Row 1

Row 3 + Row 1

Row 4 + Row 1

$$|A| = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & -2 \end{vmatrix}$$

Row 4 – Row 3

$$|A| = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & -4 \end{vmatrix}$$

Row 4 – Row 2

$$|A| = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & -4 \end{vmatrix} = - \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -4 \end{vmatrix}$$

We interchange Row 2 and Row 3

$$|A| = -\begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -4 \end{vmatrix} = -(-1)(2)(2)(-4) = -16$$

1 i) Exercise

$$A = \begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix}$$

$$|A|?$$

$$|A| = \begin{vmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & -2 & 0 & 2 \end{vmatrix}$$

Row 3 – Row 2

Row 4 – Row 2

$$|A| = \begin{vmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & -2 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 1 & 1 \\ 1 & 4 & 1 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

Column 2 + Column 4

$$|A| = \begin{vmatrix} 3 & 2 & 1 & 1 \\ 1 & 4 & 1 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 3 & 2 & 1 \\ 1 & 4 & 1 \\ 0 & -2 & 2 \end{vmatrix}$$

$$|A| = 2\begin{vmatrix} 3 & 2 & 1 \\ 1 & 4 & 1 \\ 0 & -2 & 2 \end{vmatrix} = 2\begin{vmatrix} 3 & 3 & 1 \\ 1 & 5 & 1 \\ 0 & 0 & 2 \end{vmatrix}$$

Column 2 + Column 3

$$|A| = 2\begin{vmatrix} 3 & 3 & 1 \\ 1 & 5 & 1 \\ 0 & 0 & 2 \end{vmatrix} = (2)(2)\begin{vmatrix} 3 & 3 \\ 1 & 5 \end{vmatrix} = 4(15 - 3) = 48$$

1 q) Exercise

$$A = \begin{pmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{pmatrix}$$

$$|A|?$$

$$|A| = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = \begin{vmatrix} x & -x & 0 \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$$

Row 1 – Row 2

$$|A| = \begin{vmatrix} x & -x & 0 \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = \begin{vmatrix} x & 0 & 0 \\ a & (x+a+b) & c \\ a & b+a & x+c \end{vmatrix}$$

Column 2 + Columns 1

$$|A| = \begin{vmatrix} x & 0 & 0 \\ a & (x+a+b) & c \\ a & b+a & x+c \end{vmatrix} = \begin{vmatrix} x & 0 & 0 \\ 0 & x & -x \\ a & b+a & x+c \end{vmatrix}$$

Row 2 – Row 3

$$|A| = \begin{vmatrix} x & 0 & 0 \\ 0 & x & -x \\ a & b+a & x+c \end{vmatrix} = \begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ a & b+a & (x+a+b+c) \end{vmatrix}$$

Column 3 + Columns 2

$$|A| = \begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ a & b+a & x+a+b+c \end{vmatrix} = x \cdot x \cdot (x+a+b+c)$$

Exercise: Determine for which values of m the determinant of A is 0

$$A = \begin{pmatrix} -m & 5 \\ 2 & 3 - m \end{pmatrix}$$

$$|A| = \begin{vmatrix} -m & 5 \\ 2 & 3-m \end{vmatrix} = m^2 - 3m - 10$$

$$m^2 - 3m - 10 = 0 \rightarrow m = \frac{3 \pm \sqrt{9+40}}{2} \begin{cases} m = 5 \\ m = -2 \end{cases}$$

$$|A| = 0 \begin{cases} m = 5 \\ or \\ m = -2 \end{cases}$$