education

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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

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MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x-1)(x+5)=0$ (2)

1.1.2 $7x^2+5x-9=0$ (Leave your answer correct to TWO decimal places.) (3)

1.1.3 $x^2-16\geq 0$ (3)

1.1.4 $3^{2x}+2.3^x=3$ (5)

1.2 Solve simultaneously for x and y:

$x-2y=1$ and $4x^2-3xy=5+4y$ (6)

1.3 1.3.1 Solve for x in the following equation: $(x+5)(x-6)=26$ (2)

1.3.2 Hence, or otherwise, determine the values of x if
$f(x)=(x+5)(x-6)$ and $0<f(x)<26$ (3)

[24]

QUESTION 2

2.1 Given the quadratic pattern: 12; 21; 34; …

2.1.1 Write down the next TWO terms of the pattern. (2)

2.1.2 Determine the formulae for the nth term of the pattern. (4)

2.1.3 Calculate the value of the 60th term of the pattern. (2)

2.2 Consider the following finite arithmetic series: 4 + 7 + 10 + … + 172

2.2.1 Calculate how many terms are in the series. (2)

2.2.2 Determine the sum of the series. (2)

[12]

QUESTION 3

3.1 Consider the following: $\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^{k-1}$

3.1.1 Write down the first three terms of the series. (2)

3.1.2 Calculate the sum of the series. (3)

3.2 Consider the series: $\cos\theta + \sin 2\theta + 4\sin^2\theta.\cos\theta + ...$ where θ is an acute angle.

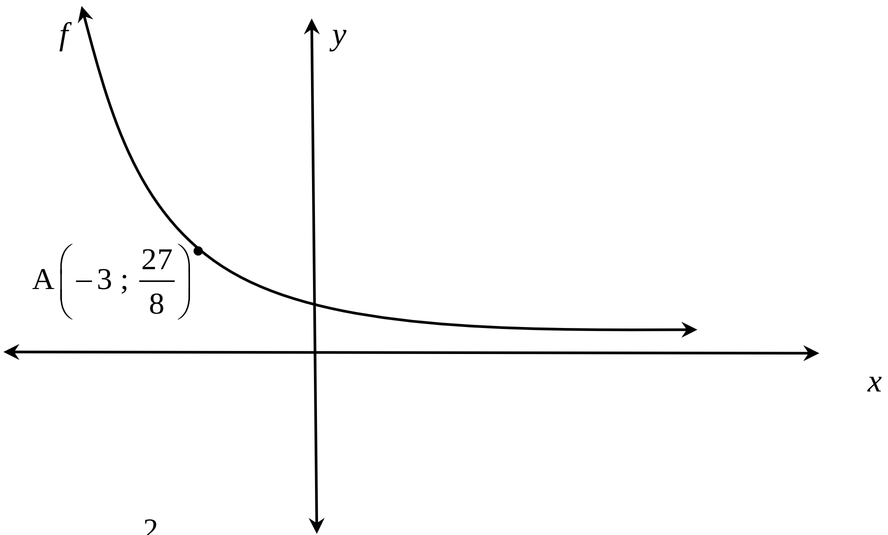
3.2.1 Prove that it is a geometric series. (4)

3.2.2 Calculate for which values of θ it will be a converging series. (3)

[12]

QUESTION 4

The graph of $f(x) = p^x;\ p > 0$ is sketched below. The point $A\left(-3\ ;\ \frac{27}{8}\right)$ is a point on f.



4.1 Show that $p = \frac{2}{3}$. (2)

4.2 Write down the equation of f^{-1} in the form $y = ...$ (2)

4.3 Sketch the graph of f^{-1}. Indicate on your graph the intercept(s) with the axes and one other point. (3)

4.4 Graph k is obtained by reflecting f about the y-axis. Determine the equation of k. Write your answer having a positive exponent. (2)

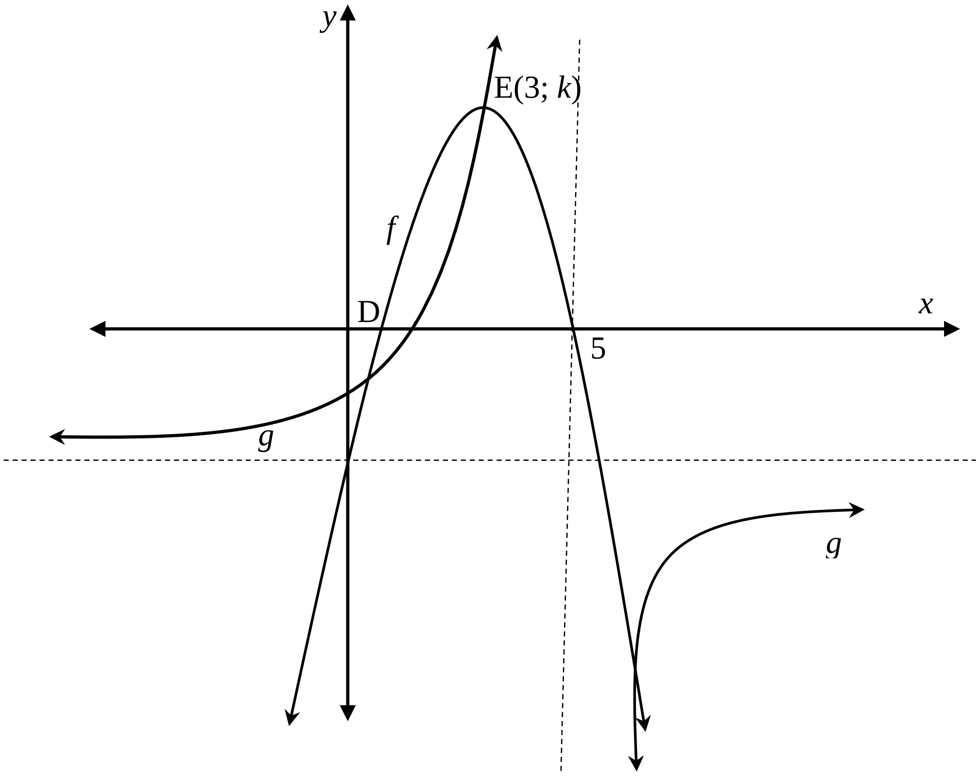
4.5 For which values of x will $\log_{\frac{3}{2}} x \le 0$ (3)

[12]

QUESTION 5

The graphs of $f(x) = ax^2 + bx - 10$ and $g(x) = \dfrac{m}{x+p} + q$ are sketched below.

D and (5 ; 0) are x-intercepts of f and E(3 ; k) is the turning point of f. The vertical asymptote of g intersects f in the point (5 ; 0). The horizontal asymptote of g intersects f at the y-intercept of f.



5.1 Write down the coordinates of the y-intercept of f. (1)

5.2 Determine the coordinates of D, an x-intercept of f. (2)

5.3 Show that the equation of f is $f(x) = -2x^2 + 12x - 10$ (4)

5.4 Calculate the value of k, the y-coordinate of the turning point of f. (2)

5.5 Determine the equation of g in the form $g(x) = \dfrac{m}{x+p} + q$. (3)

5.6 Calculate the y-intercept of g. (2)

5.7 If h is the decreasing axis of symmetry of g, determine the equation of h in the form $y = mx + c$. (2)

5.8 Calculate the x-intercept of g. (3)

5.9 For which values of x will $f(x).g(x) \leq 0$ (2)

5.10 For which value(s) of t will $g(x) + t = 0$ have ONE negative root? (2)

[23]

QUESTION 6

Kevin inherited R800 000 and invested the full amount at a bank. The bank paid him interest at the rate of 8% per annum, compounded monthly.

6.1 If Kevin withdrew R120 000 from the account one year after making the initial investment, how much was in the account two years after making the initial investment? (4)

6.2 Two years after making the initial investment, Kevin decided to enrol at a university. He decided to use the money from his investment to pay for his studies.

6.2.1 Convert the nominal interest rate from monthly to half-yearly. (4)

6.2.2 Kevin will need to withdraw R95 000 every six months to pay for his studies. The first withdrawal is at the beginning of the first semester. Calculate how many complete semesters this investment can finance his studies. (5)

[13]

QUESTION 7

7.1 Given: $f(x) = 3x^2 + 11$

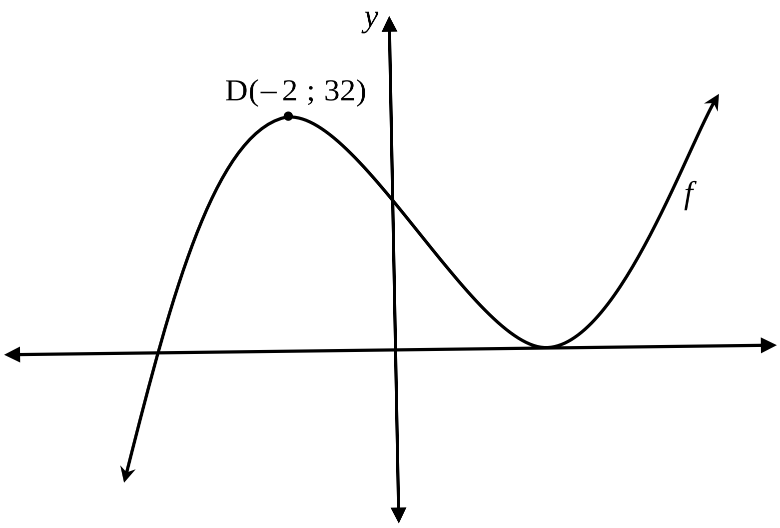
Determine $f^{/}(x)$ from first principles. (5)

7.2 Determine $\frac{dy}{dx}$ if: $y = \frac{2x^4 - \pi\sqrt{x} + 8}{\sqrt{x}}$ (4)

[9]

QUESTION 8

The sketch below represents the curve of $f(x) = ax^3 + bx + c$. The graph f has a turning point at $\text{D}(-2\ ;\ 32)$. The gradient of the tangent to f at $x = 3$ is 15.



8.1 Show that $a = 1$, $b = -12$ and $c = 16$. (6)

8.2 Calculate the:

8.2.1 x coordinate of the point of inflection. (2)

8.2.2 values of x for which f is concave down. (1)

8.3 Write down the coordinates of D', the image of D when f is reflected about the x-axis. (2)

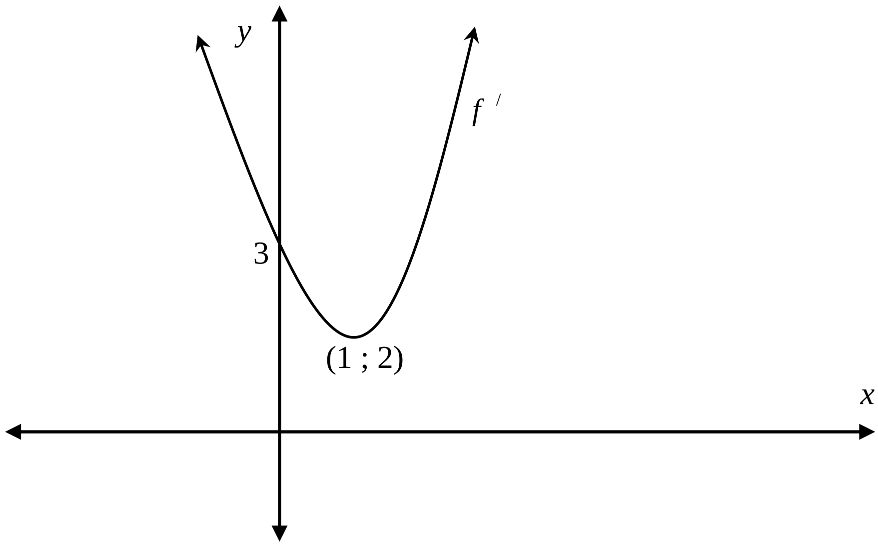
8.4 Is f a function? Give a reason for your answer. (2)

[13]

QUESTION 9

The sketch below shows the graph of

$y = f^{/}(x)$. The turning point of $f^{/}$ is (1 ; 2) and the y-intercept is (0 ; 3).



9.1 For which value(s) of x is f increasing? (1)

9.2 What is the gradient of the tangent to f at $x = 1$? (1)

9.3 Sketch the graph of f if it is given that $f(0) = -2$ and $f\left(\frac{1}{2}\right) = 0$.

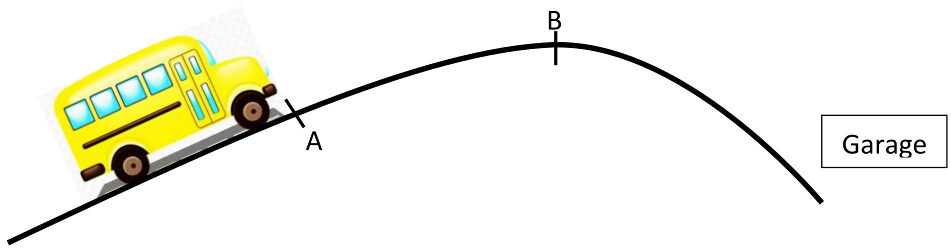
Label all x-intercepts, y-intercept, stationary points and point of inflection, if any. (4)

[6]

QUESTION 10

A school bus is travelling up hill on a mountainous gravel road. Due to the bad state of the road, the diesel tank cracked and diesel started leaking quickly. The bus is at point A, 2 km from the top of the hill (point B), at the time the tank cracked. The position of the bus, with respect to A is given by:

$S(t) = t - \dfrac{t^2}{6}$, where t is in minutes and S is in kilometres.



10.1 How far is the bus from point A after 1 minute? (2)

10.2 If the bus reaches the top of the hill (point B), the bus driver will be able to roll (free wheel) downhill to the garage. Will he reach the top of the hill? Show all your calculations. (4)

10.3 At what speed will the bus be travelling at exactly 1 minute after the diesel tank cracked and diesel start leaking? (2)

10.4 Calculate the acceleration of the bus as it was travelling up hill after the diesel tank cracked. (1)

[9]

QUESTION 11

A certain school wants to paint their computer room. A survey was conducted among the boys and girls who take CAT and IT as a subject to establish their preference of colour.

The results are shown in the table below:

	Blue	**Green**	**Total**
Boys	19	31	50
Girls	41	35	76
Total	60	66	126

11.1 If a learner from this group is selected at random, what is the probability that he/she will choose green? (2)

11.2 Are the events being a boy and choosing blue as a preferred colour independent? Use calculations, correct to TWO decimal places, to motivate your answer. (4)

[6]

QUESTION 12

An athletic association decided to host an athletics event. The following 12 male athletes were selected according to their personal best times for the season, on the 100 m sprint:

North West:	A and B athlete
Gauteng:	A, B and C athlete
Limpopo:	A and B athlete
Mpumalanga:	A, B and C athlete
Free State:	A and B athlete

No athlete had the same personal best times. There are 12 lanes next to each other in this stadium.

12.1 If the 12 athletes are placed according to their personal best times from fastest to slowest in lane 1 to 12, independent of the province they represent, in how many ways can the athletes be placed? (1)

12.2 The organizers of the event decide NOT to place the athletes according to their best times, but to place each athletes' name in a container and select an athlete for each lane randomly. The A athlete from Mpumalanga and the A athlete from Gauteng do not want to run next to each other. How many ways can the 12 athletes be placed next to each other so that the A athlete from Mpumalanga and the A athlete from Gauteng will not run next to each other? (3)

12.3 In how many ways can the athletes be placed randomly next to each other if the two C athletes must run in the two outside lanes (lane 1 or 12) and the A and B athletes must alternate (in lane 2 to 11)? (3)

12.4 At the end of the event, a fun-run is held where only 6 athletes are allowed to run. If the 6 athletes are chosen randomly, what is the probability that there will be ONLY ONE athlete form Free State competing in the fun-run? (4)

[11]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$