GAUTENG DEPARTMENT OF EDUCATION

JOHANNESBURG NORTH DISTRICT

2021

GRADE 12

MATHEMATICS

PAPER 1

PRE-TRIAL EXAM

MARKING GUIDELINES

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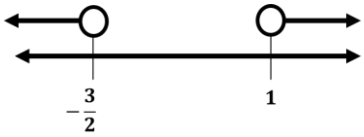
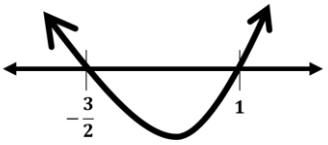
MARKS: **150**

TIME: **3 HOURS**

DATE: **13 AUGUST 2021**

QUESTION	CALCULATION	MARK ALLOCATION
1.1.1	$4x^2 - 25 = 0$ $(2x - 5)(2x + 5) = 0$ $x = \frac{5}{2}\ or\ x = -\frac{5}{2}$ OR $4x^2 = 25$ $x^2 = \frac{25}{4}$ $\sqrt{x^2} = \pm\sqrt{\frac{25}{4}}$ $x = \frac{5}{2}\ or\ x = -\frac{5}{2}$	✓✓ factors ✓ Answers (3) ✓$x^2 = \frac{25}{4}$ ✓$\pm\sqrt{\frac{25}{4}}$ ✓Answer
1.1.2	$3x^2 + 5x - 4 = 0$ $x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-4)}}{2(3)}$ $x = \frac{-5 \pm \sqrt{73}}{6}$ $x = -2{,}26\ or\ x = 0{,}59$ OR $3x^2 + 5x = 4$ $x^2 + \frac{5x}{3} + \frac{25}{36} = \frac{4}{3} + \frac{25}{36}$ $\left(x + \frac{5}{6}\right)^2 = \frac{73}{36}$ $x = -\frac{5}{6} \pm \frac{\sqrt{73}}{6}$ $x = -2{,}26\ or\ x = 0{,}59$	✓ standard form ✓ correct substitution ✓✓ Answers (4) ✓for adding $\frac{25}{36}$ both sides ✓ $x = -\frac{5}{6} \pm \frac{\sqrt{73}}{6}$ ✓✓ Answers
1.1.3	$2^x - 5 \cdot 2^{x+1} = -144$ $2^x(1 - 5.2) = -144$ $2^x(-9) - 144$ $2^x = 16$ $2^x = 2^4$ $\therefore x = 4$	✓Factorise ✓Simplification ✓Answer (3)
1.1.4	$2x^2 + x - 3 > 0$ $(x - 1)(2x + 3) > 0$ $x > 1\ or\ x < -\frac{3}{2}$ or $(-\infty, -\frac{3}{2} \cup (1; \infty)$	✓critical values ✓✓notation

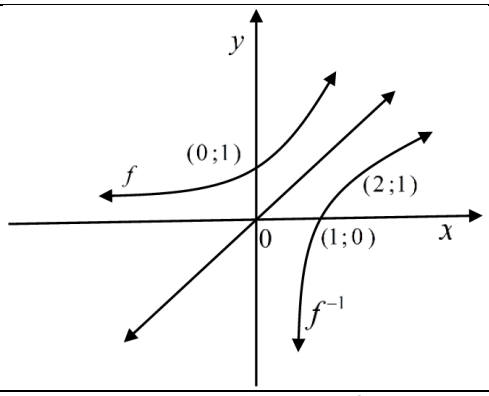
	or  or 	(3)
1.2.1	$4^{x+2} \cdot 8^{y+1} = 2^{1-x}$ $2^{2(x+2)} \cdot 2^{3(y+1)} = 2^{1-x}$ $2x + 4 + 3y + 3 = 1 - x$ $3y = -3x - 6$ $y = -x - 2$	✓Exponential Law ✓Exponential Law ✓Simplify (3)
1.2.2	$y = -x - 2$……(1) $x^2 + y^2 + xy = 7$…….(2) substitute for y in (2) using expression from (1) $x^2 + (-x - 2)^2 + x(-x - 2) = 7$ $x^2 + x^2 + 4x + 4 - x^2 - 2x = 7$ $x^2 + 2x - 3 = 0$ $(x - 1)(x + 3) = 0$ $x = 1 \quad or \quad x = -3$ $\therefore y = -(1) - 2 \; or \; y = -(-3) - 2$ $y = -3 \quad or \; y = 1$	✓Substitution ✓Standard form ✓Factors ✓x-values ✓y-values (5)
1.3	$6x^2 + 2gx - 3x - g = 0$ $6x^2 + 2x(g - 3) - g = 0$ $\Delta = b^2 - 4ac$ $\Delta = (2g - 3)^2 - 4(6)(-g)$ $\Delta = 4g^2 - 12g + 9 + 24g$ $\Delta = g^2 + 12g + 9$ $\Delta = (2g + 3)^2$ $\therefore \Delta = perfect\ square, thus\ rational\ roots.$	✓coefficient of x ✓correct substitution into formula ✓factors ✓conclusion (4)
		[25]

QUESTION	CALCULATION	MARK ALLOCATION
2.1	$37 - x - (x + 5) = x + 13 - (37 - x)$ $37 - x - x - 5 = x + 13 - 37 + x$ $-4x = -56$ $x = 14$	✓use of common difference for an AP ✓Simplification ✓Answer (3)
2.2	T_1 =19, T_2 = 23, T_3 = 27 d = 4 $T_n = 19 + (n - 1)4$ $T_n = 4n + 15$	✓Correct substitution ✓✓Answer (3)
2.3	$S_3 = \frac{a(r^n - 1)}{r - 1}; r \neq 1$ $91 = \frac{a(3^3 - 1)}{3 - 1}$ $91 = \frac{a.26}{2}$ $a = 7$ OR $91 = a + ar + ar^2$ $91 = a + 3a + 9a$ $91 = 13a$ $a = 7$	✓Formula ✓Substitution ✓Answer ✓$a + ar + ar^2$ ✓Simplification ✓Answer (3)
2.4	$S_\infty = \frac{a}{1 - r}; -1 < r < 1$ $\frac{375}{4} = \frac{a}{1 - r}$ $375(1 - r) = 4a$ $a = \frac{375(1 - r)}{4} \ldots\ldots eq\ 1$ $S_2 = \frac{a(r^2 - 1)}{r - 1} \ldots\ldots eq\ 2$ $90 = \frac{a(r - 1)(r + 1)}{r - 1}$ $90 = \frac{375(1 - r)(r + 1)}{4}$ $90 = \frac{-375(r - 1)(r + 1)}{4}$ $90 = \frac{375(r^2 - 1)}{4}$	✓ Equation 1 ✓ Equation 2 ✓ Substitution

	$r^2 = \frac{1}{25}$ $r = \frac{1}{5}\ or\ r = -\frac{1}{5}$ $a = \frac{375\left(1-\frac{1}{5}\right)}{4} = 75\ or\ a = \frac{375\left(1-\left(-\frac{1}{5}\right)\right)}{4} = \frac{225}{2}$	✓ Simplification ✓✓Answers (6)
2.5.1	32699 32896 33091 33284 33475 1st difference 197 195 193 191 2nd difference -2 -2 -2 The 2nd difference is constant, therefore pattern is quadratic.	✓ 1st difference and 2nd difference ✓ conclusion (2)
2.5.2	$T_n = 197 + (n-1)(-2)$ $T_n = 199 - 2n$	✓correct substitution ✓$T_n = 199 - 2n$ (2)
2.5.3	$2a = -2 \quad \therefore\ a = -1$ $3a + b = 197$ $3(-1) + b = 197 \quad \therefore b = 200$ $a + b + c = 32699$ $-1 + 200 + c = 32699 \quad \therefore c = 32\,500$ $T_n = -n^2 + 200n + 32\,500$	✓ a ✓ b ✓ c ✓ T_n (4)
2.5.4	1st derivative: $T_n' = -2n + 200$ $0 = -2n + 200$ $n = 100$ Maximum will be on the 100th day. OR Complete square: $-T_n = n^2 - 200n - 32\,500$ $-T_n = n^2 - 200n + (-100)^2 - 32\,500 - (-100)^2$ $-T_n = (n-100)^2 - 42500$ $T_n = -(n-100)^2 + 42500$ by inspection, maximum will be on the 100th day.	✓✓1st derivate ✓Answer ✓✓complete the square ✓Answer (3)
		[26]

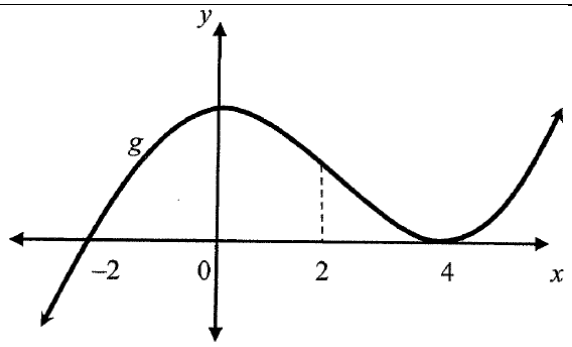
QUESTION	CALCULATION	MARK ALLOCATION
3.1	A(0; 6)	✓Answer (1)
3.2	$x = -\frac{b}{2a} = -\frac{5}{2(-1)} = 2{,}5 \quad \therefore S(5;\ 6)$ OR $y = x + 1$ $6 = x + 1$ $x = 5$ $S(5; 6)$ OR $y = -x^2 + 5x + 6$ $6 = -x^2 + 5x + 6$ $-x^2 + 5x = 0$ $x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0\ or\ x = 5$ $S(5; 6)$ OR $-x^2 + 5x + 6 = x + 1$ $x^2 - 4x - 1 = 0$ $(x - 5)(x + 1) = 0$ $x = 5\ or\ x = -1$ Valid $x\ value: x = 5 \quad \therefore S(5; 6)$	using axis of symmetry: ✓*x*-value ✓*y*-value ✓Equating equation to 6 ✓*x*-value ✓Equating equation to 6 ✓*x*-value ✓Equating equations ✓ valid *x*-value (2)
3.3	$-x^2 + 5x + 6 = 0$ $x^2 - 5x - 6 = 0$ $(x - 6)(x + 1) = 0$ $x = 6\ or\ x = -1$ $B(-1; 0)\ and\ C(6; 0)$	✓ Factors ✓ ✓ Answers (3)
3.4	$f(x) - g(x) = 5$ $-x^2 + 5x + 6 - (x + 1) = 5$ $-x^2 + 5x + 6 - x - 1 = 5$ $x^2 - 4x = 0$ $x(x - 4) = 0$ $x = 0\ or\ x = 4$ $OR = 4\ units$	✓Subtract g from f ✓Equate to 5 ✓Solve for x ✓State OR = 4 units (4)
3.5.1	$x = -\frac{b}{2a} = -\frac{5}{2(-1)} = 2{,}5 \quad or \quad \frac{5}{2}$	✓Formula ✓CA axis of symmetry ✓CA substitution

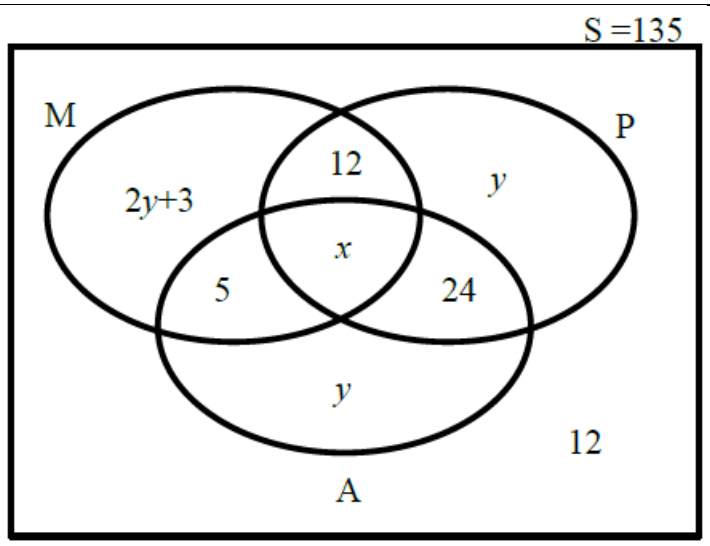
	$f(2,5) = -(2,5)^2 + 5(2,5) + 6 = 12,25 \quad or \quad \frac{49}{4}$ $M(2,5; 12,25)\ or\ M\left(\frac{5}{2}; \frac{49}{4}\right)$ OR $x = \frac{-1+6}{2} = \frac{5}{2}$ $y = -(2,5)^2 + 5(2,5) + 6 = \frac{49}{4}$ $M\left(\frac{5}{2}; \frac{49}{4}\right)$ OR $f'(x) = -2x + 5 = 0 \qquad \therefore x = \frac{5}{2}$ $y = -(\frac{5}{2})^2 + 5\left(\frac{5}{2}\right) + 6 = \frac{49}{4}$	✓CA Answer ✓Midpoint Formula ✓CA axis of symmetry ✓CA substitution ✓CA Answer ✓1st derivative equal to 0 ✓CA axis of symmetry ✓CA substitution ✓CA Answer (4)
3.5.2	$PQ = -x^2 + 4x + 5$ $(PQ)' = -2x + 4 = 0 \qquad \therefore x = 2$ $PQ\ max. = -(2)^2 + 4(2) + 5 = 9\ units$ OR $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$ $PQ\ max. = -(2)^2 + 4(2) + 5 = 9\ units$	✓PQ in terms of *x* ✓1st derivative equal to 0 ✓CA *x*-value ✓CA Answer ✓PQ in terms of *x* ✓substitution ✓CA *x*-value ✓CA Answer (4)
		[19]
QUESTION	**CALCULATION**	**MARK ALLOCATION**
4.1	$y = 2^x$ when *x* = 0 $f(0) = 2^0 = 1$ thus C(0; 1) $g(x) = -(x-1)^2 + q$ $1 = -(0-1)^2 + q$ $q = 2$	✓*y*-intercept ✓Substitute and simplify (2)
4.2	$g(x) = -(x-1)^2 + 2$ D(1; 2)	✓✓Answer (2)
4.3	t = 2	✓ Answer (1)
4.4	$f^{-1}: x = 2^y$ $y = log_2 x$	✓✓Answer (2)

4.5		✓form of the graph ✓*x*-intercept ✓any other coordinate on the graph (3)
4.6	$g(x) = -(x-1)^2 + 2$ $g(x+1) - 2 = -(x+1-1)^2 + 2 - 2$ $h(x) = -x^2$	✓correct substitution into g ✓Answer
4.7	Domain: $x \geq 0 \quad or \quad x \leq 0$	✓Answer
		[13]
QUESTION	**CALCULATION**	**MARK ALLOCATION**
5.1.1	$A = P(1-i)^n$ $79\,866{,}96 = 180\,000(1-0{,}15)^n$ $79\,866{,}96 = 180\,000(0{,}85)^n$ $(0{,}85)^n = \frac{79\,866{,}96}{180\,000}$ $n = \frac{\log\left(\frac{79\,866{,}96}{180\,000}\right)}{\log(0{,}85)}$ $n = 4{,}999 \ldots years$ $n = 5\ years$	✓Substitution ✓use of logs. ✓Answer (3)
5.1.2	$A = P\left(1 + \frac{i}{4}\right)^{n\times 4}$ $A = 49\,000\left(1 + \frac{0{,}1}{4}\right)^{5\times 4}$ $A = R80\,292{,}21$ Yes, the money will be enough to buy the car.	✓values of *i* and *n*. ✓substitution ✓conclusion (consistent with answer) (3)
5.2.1	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $P = \frac{7853{,}15\left[1-\left(1+\frac{0{,}1025}{12}\right)^{-234}\right]}{\frac{0{,}1025}{12}}$ $= R793\,749{,}25$	✓n = 234 ✓i = $\frac{0{,}1025}{12}$ ✓substitution into present value formula ✓Answer

	OR Balance outstanding = $= 800\,000\left(1+\frac{0{,}1025}{12}\right)^{6} - \frac{7853{,}15\left[\left(1+\frac{0{,}1025}{12}\right)^{6}-1\right]}{\frac{0{,}1025}{12}}$ $= 841\,885{,}56 - 48\,136{,}62$ $= R793\,748{,}94$	✓n = 6 in both ✓i = $\frac{0{,}1025}{12}$ ✓A – F ✓Answer (4)
5.2.2	$A = P(1+i)^n$ $A = 793\,749{,}25\left(1+\frac{0{,}1025}{12}\right)^{3}$ $= R814\,263{,}3052$ New instalment: $P = \frac{x[1-(1+i)^{-n}]}{i}$ $814\,263{,}3052 = \frac{x[1-\left(1+\frac{0{,}1025}{12}\right)^{-231}]}{\frac{0{,}1025}{12}}$ $= R8\,089{,}20$	✓ $793\,749{,}25\left(1+\frac{0{,}1025}{12}\right)^{3}$ ✓ n = 231 ✓ substitution for new P ✓substitution for n and i ✓Answer (5)
		[15]
QUESTION	**CALCULATION**	**MARK ALLOCATION**
6.1	$f(x) = -3x^2$ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x) = \lim_{h\to 0}\frac{-3(x+h)^2-3x^2}{h}$ $f'(x) = \lim_{h\to 0}\frac{-3x^2-6xh-3h^2+3x^2}{h}$ $f'(x) = \lim_{h\to 0}\frac{-6xh-3h^2}{h}$ $f'(x) = \lim_{h\to 0}\frac{h(-6x-3h)}{h}$ $f'(x) = -6x$	✓1st principles formula ✓$-6xh-3h^2$ ✓$\frac{h(-6x-3h)}{h}$ ✓Answer (4)
6.2	$y = 7x^4 - 5\sqrt{x} - \frac{3}{x}$ $y = 7x^4 - 5x^{\frac{1}{2}} - 3x^{-1}$ $\frac{dy}{dx} = 28x^3 - \frac{5}{2}x^{-\frac{1}{2}} + 3x^{-2}$	✓ $7x^4 - 5x^{\frac{1}{2}} - 3x^{-1}$ ✓$28x^3$ ✓$-\frac{5}{2}x^{-\frac{1}{2}}$ ✓$+3x^{-2}$ (4)
6.3	$g(x) = ax^3 - 24x + b$ $g'^{(x)} = 3ax^2 - 24$ $0 = 3ax^2 - 24$	✓1st derivative of g

	$0 = 3a(-2)^2 - 24$ $12a = 24$ $a = 2$ $17 = 2(-2)^3 - 24(-2) + b$ $17 = -16 + 48 + b$ $b = -15$	✓equating 1st derivative to 0 and substituting $x = -2$ ✓Answer for a ✓substituting $x = -2$ and y=17 into g ✓Answer for b (5)
		[13]
QUESTION	**CALCULATION**	**MARK ALLOCATION**
7.1.1	$f(x) = -2x^3 + 5x^2 + 4x - 3$ $0 = (x - 3)(-2x^2 - x + 1)$ $x - 3 = 0 \; or \; -2x^2 - x + 1 = 0$ $x = 3 \; or \; 2x^2 + x - 1 = 0$ $(3; 0) \qquad (2x - 1)(x + 1) = 0$ $2x - 1 = 0 \; or \; x + 1 = 0$ $x = \frac{1}{2} \; or \; x = -1$ $\left(\frac{1}{2}; 0\right) \qquad (-1; 0)$	✓$-2x^2 - x + 1$ ✓$(3; 0)$ ✓factors or formula ✓both coordinates $\left(\frac{1}{2}; 0\right) \qquad (-1; 0)$ (4)
7.1.2	$f(x) = -2x^3 + 5x^2 + 4x - 3$ $f'(x) = -6x^2 + 10x + 4$ $3x^2 - 5x - 2 = 0$ $(3x + 1)(x - 2) = 0$ $x = -\frac{1}{3} \; or \; x = 2$	✓$-6x^2 + 10x + 4$ ✓equating 1st derivative to 0 ✓factors or formula ✓Answers (4)
7.1.3	$f'(x) = -6x^2 + 10x + 4$ $f''(x) = -12x + 10$ $0 = -12x + 10$ $12x = 10$ $x = \frac{5}{6}$ $\therefore x < \frac{5}{6}$	✓2nd derivative ✓Answer (2)

7.2.1		✓form ✓both *x*-intercepts ✓*x*-value of both turning points ✓*x*-value of point of inflection (4)
7.2.2	$x < -2 \ or \ \ x > 2; \quad x \neq 4$	✓$x < -2$ ✓$x > 2$ ✓$x \neq 4$ (3)
		[17]
QUESTION	**CALCULATION**	**MARK ALLOCATION**
8.1	$s(t) = \ t^2 + 15t$ $v = s'(t) = (2t + 15)m/s$	✓✓ Answer (2)
8.2	$v = s'(t) = (2t + 15)m/s$ $v = s'(25) = 2(25) + 15 = 65m/s$	✓✓CA Answer (2)
8.3	$v = s'(t) = (2t + 15)m/s$ $a = s''(t) = 2m/s^2$	✓Answer (1)
8.4	$a = s''(t) = 2m/s^2$	✓Answer (1)
8.5	$s(t) = \ t^2 + 15t = 250$ $t^2 + 15t - 250 = 0$ $(t + 25)(t - 10) = 0$ $t = -25 \ or \ t = 10$ $\therefore t = 10s$ $v = s'^{(t)} = 2(10) + 15 = 35m/s$	✓Equating s(t) = 250 ✓Factors ✓t = 10s ✓Answer (4)
		[10]
QUESTION	**CALCULATION**	**MARK ALLOCATION**

9.1		✓12 ✓5; 12; 24 ✓2y+3; y; y ✓ x (4)
9.2	$2y + 3 + 5 + x + 12 = 5 + x + 24 + y$ $y = 16$ $x + y + 5 + 24 = 60$ $x + 16 + 5 + 24 = 60$ $x = 15$	✓✓Equate expressions for Maths and Accounting ✓Answer for *y* ✓Answer for *x* (4)
9.3	$P(M\ or\ P\ and\ A) = \frac{2y + 3 + 5 + 12 + x + 24}{135}$ $= \frac{32 + 3 + 5 + 12 + 12 + 24}{135}$ $\frac{91}{135}$ *or* 0,67	✓$2y + 3 + 5 + 12$ ✓x+24 ✓Answer (3)
		[12]

TOTAL: 150