education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

NASIONALE
SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE VRAESTEL 1

SEPTEMBER 2021

PUNTE: 150
TYD: 3 ure

Die vraestel bestaan uit 7 bladsye en een inligtingsblad

Blaai om asseblief

education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 1

SEPTEMBER 2021

MARKS: 150
TIME: 3 hours

This question paper consists of 7 pages and a one-page information sheet.

Please turn over

VRAAG 1

1.1 Los op vir x:

1.1.1 $x(2-x)=0$ (2)

1.1.2 $(2x+3)(3-x) = 2$ (Laat jou antwoord korrek tot TWEE desimale plekke) (4)

1.1.3 $27^x \cdot 9^{x-2} = 1$ (3)

1.1.4 $\sqrt{5-2x} = \frac{x}{2}+4$ (5)

1.1.5 $x(x+4) \geq -3$ (4)

1.2 Los op vir x en y

$$(2x-y)(x+2y-3)=0$$
$$x-y=1$$

(5)

[23]

VRAAG 2

2.1 Die eerste drie terme van 'n kwadratiese ry is: $1\ ;\ -5\ ;-13\ ;\ldots$

2.1.1 Bepaal die algemene term van die ry. (4)

2.1.2 Watter term in die ry is -643? (3)

2.2 In 'n gegewe rekenkundige ry is die eerste term 2, die laaste term 29 en die som van al die terme 155. Bereken die gemene verskil. (4)

2.3 Bereken: $\sum_{p=3}^{14}(15-4p)$ (3)

2.4 Die eerste term van 'n meetkundige ry is 9 en die verhouding van die som van die eerste 8 terme tot die som van die eerste 4 terme is $97 : 81$.
Bereken die eerste drie terme van die ry, as dit gegee word dat al die terme positief is. (5)

2.5 Beskou die meetkundige reeks : $2(p-5)+(p-5)^2+\frac{1}{2}(p-5)^3+\ldots\ldots \quad p\neq 5$

2.5.1 Vir watter waardes van p sal die reeks konvergeer? (3)

2.5.2 Bereken die som tot oneindigheid as $p=4$. (3)

[25]

Blaai om asseblief

QUESTION 1

1.1 Solve for x:

1.1.1 $x(2-x)=0$ (2)

1.1.2 $(2x+3)(3-x) = 2$ (Leave your answer correct to TWO decimal places) (4)

1.1.3 $27^x \cdot 9^{x-2} = 1$ (3)

1.1.4 $\sqrt{5-2x} = \frac{x}{2}+4$ (5)

1.1.5 $x(x+4) \geq -3$ (4)

1.2 Solve for x and y.

$$(2x-y)(x+2y-3)=0$$
$$x-y=1$$

(5)

[23]

QUESTION 2

2.1 The first three terms of a quadratic sequence are: $1\ ;\ -5\ ;-13\ ;\ldots$

2.1.1 Determine the general term of the sequence. (4)

2.1.2 Which term has a value of -643? (3)

2.2 In a given arithmetic series, the first term is 2, the last term is 29 and the sum of all the terms is 155. Calculate the common difference. (4)

2.3 Calculate: $\sum_{p=3}^{14}(15-4p)$ (3)

2.4 The first term of a geometric series is 9 and the ratio of the sum of the first eight terms to the sum of the first four terms is $97 : 81$.
Calculate the first three terms of the sequence if it is given that all the terms are positive. (5)

2.5 Consider the geometric series: $2(p-5)+(p-5)^2+\frac{1}{2}(p-5)^3+\ldots\ldots\ ,\ p\neq 5$

2.5.1 For which value of p will the series be convergent? (3)

2.5.2 Calculate the sum to infinity if $p=4$. (3)

[25]

Please turn over

QUESTION 3

Given: $g(x)=\frac{-6}{x-3}+1$

3.1 Determine the:

3.1.1 equations of the asymptotes of g. (2)

3.1.2 y-intercept of g. (1)

3.1.3 x- intercept of g. (2)

3.1.4 Sketch the graph of g, showing all intercepts with the axis and asymptotes. (3)

3.2 For which values of x is $g(x)>0$? (2)

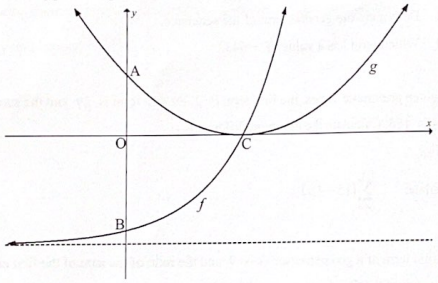
3.3 If $h(x)=x+c$ is the axis of symmetry of g, determine the value of c. (2)

3.4 Describe in words the transformation of g to $h(x)=\frac{6}{x+3}+1$ (2)

[14]

QUESTION 4

4.1 In the diagram, $f(x)=2^x-8$ and $g(x)=ax^2+bx+c$ are drawn. A $(0\,;4{,}5)$ is the y - intercept of g. C is the x – intercept of both f and g. B is the y-intercept of f. C is the turning point of g.



4.1.1 Determine the coordinates of B and C. (3)

4.1.2 Calculate the values of a and b. (4)

4.1.3 Write down the range of $y=-f(x)$. (2)

4.1.4 If $h(x)=f(2x)+8$, determine the equation of $h^{-1}(x)$ in the form $y=\ldots$. (3)

4.1.5 Determine the values of x for which

(a) $f(x)-g(x)\le 0$ (2)

(b) $x\,.\,g'(x)>0$ (2)

VRAAG 3

Gegee: $g(x)=\frac{-6}{x-3}+1$

3.1 Bepaal die:

3.1.1 vergelykings van die asimtote van g. (2)

3.1.2 y-afsnit van g. (1)

3.1.3 x- afsnit van g. (2)

3.1.4 Skets die grafiek van g, toon alle afsnitte met die asse en asimtote. (3)

3.2 Vir watter waardes van x is $g(x)>0$? (2)

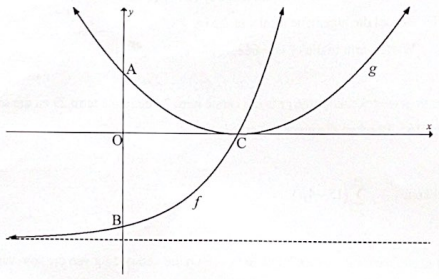
3.3 As $h(x)=x+c$ 'n simmetrie-as van g is, bepaal die waarde van c. (2)

3.4 Beskryf die transformasie van g na $h(x)=\frac{6}{x+3}+1$ in woorde. (2)

[14]

VRAAG 4

4.1 In die onderstaande diagram is, $f(x)=2^x-8$ en $g(x)=ax^2+bx+c$ geskets. A $(0\,;4{,}5)$ is die y-afsnit van g. C is die x-afsnit van beide f en g. B is die y-afsnit van f. C is die draaipunt van g.



4.1.1 Bepaal die koördinate van B en C. (3)

4.1.2 Bereken die waardes van a en b. (4)

4.1.3 Gee die waardeversameling van $y=-f(x)$. (2)

4.1.4 As $h(x)=f(2x)+8$, bepaal die vergelyking van $h^{-1}(x)$ in die vorm $y=\ldots$. (3)

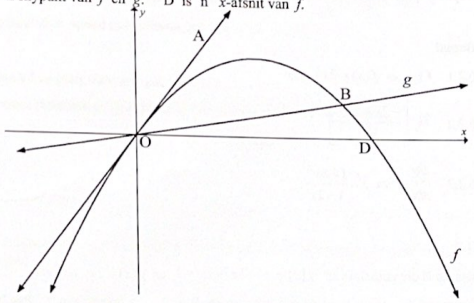
4.1.5 Bepaal die waardes van x waar:

(a) $f(x)-g(x)\le 0$ (2)

(b) $x\,.\,g'(x)>0$ (2)

4.2 In die onderstaande diagram is $f(x)=4x-x^2$ en $g(x)=\frac{1}{2}x$ geskets.

'n Raaklyn aan f, wat deur die oorsprong gaan, is geskets. A is 'n punt op die raaklyn. B is 'n snypunt van f en g. D is 'n x-afsnit van f.



Bereken $A\hat{O}B$. (5)
[21]

VRAAG 5

5.1 Die Murray familie het twee kinders. Vir sy seun, Peter, besluit die pa om 'n eenmalige bedrag met geboorte te belê, maar vir sy dogter, Cally besluit hy om maandeliks R150 te belê van die einde van die maand van haar geboorte en dan aan die einde van elke maand daarna.

5.1.1 Die bedrag wat vir Peter belê is, is R18 000 teen 'n rentekoers van 12% p.j. maandeliks saamgestel. Hoe oud sal Peter wees teen die tyd wat sy aanvanklike belegging aangegroei het tot R 150 000? Laat jou antwoord tot die naaste jaar. (4)

5.1.2 Cally is gebore op 12 Maart. Die bank bied dieselfde rentekoers aan as vir Peter se belegging. Wat sal Cally se belegging werd wees aan die einde van die maand waarin sy 18? (4)

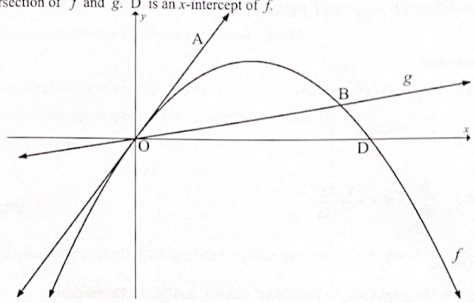
5.2 Tom gaan 'n lening aan met die volgende voorwaardes: Hy sal die lening terug betaal in 15 halfjaarlikse paaimente van R2 500, beginnende 6 maande van nou af. Die rentekoers is 16% p.j. kwartaalliks saamgestel.

5.2.1 Toon aan dat die halfjaarlikse rentekoers 16,32% p.j. is. (3)

5.2.2 Bereken die waarde van die lening. (4)
[15]

4.2 In the diagram below, $f(x)=4x-x^2$ and $g(x)=\frac{1}{2}x$ are sketched.

A tangent to f is drawn at the origin. A is a point on this tangent to f. B is a point of intersection of f and g. D is an x-intercept of f.



Calculate $A\hat{O}B$. (5)
[21]

QUESTION 5

5.1 The Murray family has two children. For the son, Peter, the father invested a lump sum at his birth but for the daughter, Cally, he decided to make monthly payments of R150 from the end of the month she was born and at the end of each month thereafter.

5.1.1 For Peter an amount of R18 000 was invested at a bank which offered an interest rate of 12% p.a. compounded monthly. What was Peter's age when the initial amount grew to R 150 000? Leave your answer to the nearest year. (4)

5.1.2 Cally was born on the 12 March. The bank offered the same interest rate as for Peter's investment. What would Cally's investment be worth at the end of the month in which she turned 18? (4)

5.2 Tom takes out a loan on the following conditions: He will repay the loan by means of 15 semi-annual payments of R2 500, starting 6 months from now. Interest is charged at 16% p.a. compounded quarterly.

5.2.1 Show that the semi-annual interest rate is 16,32% p.a.. (3)

5.2.2 Calculate the value of the loan. (4)
[15]

QUESTION 6

6.1 Given: $f(x)=3x^2-2$

Determine $f'(x)$ from FIRST PRINCIPLES. (5)

6.2 Determine

6.2.1 $f'(x)$ if $f(x)=3x^2-5x$ (2)

6.2.2 $D_x\left[\frac{-2x+\sqrt{x}}{x^2}\right]$ (4)

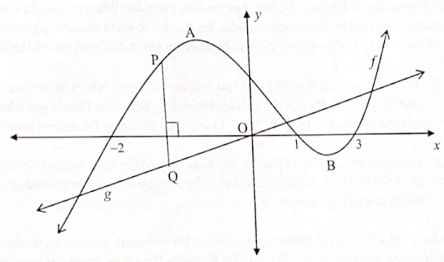
6.2.3 $\frac{dy}{dx}$, if $y=\frac{1-8x^3}{1-2x}$. (3)

[14]

QUESTION 7

In the diagram, the graphs of $f(x)=x^3+bx^2+cx+d$ and $g(x)=2x$ are drawn.

The graph of f passes through the x-axis at $x=-2$, $x=1$ and $x=3$. A and B are the turning points of f. P is a point on f and Q is a point on g such that PQ is perpendicular to the x-axis. $x_p<0$.



7.1 Show that $f(x)=x^3-2x^2-5x+6$ (2)

7.2 Calculate the x-coordinate of B. (3)

7.3 A tangent to f has a gradient of -1. Explain why the point of contact of the tangent and the graph of f lies between A and B. (1)

7.4 For which values of x will f be concave up? (2)

7.5 Determine the maximum length of the line PQ. (5)

[13]

VRAAG 6

6.1 Gegee: $f(x)=3x^2-2$

Bepaal $f'(x)$ vanuit EERSTE BEGINSELS. (5)

6.2 Bepaal

6.2.1 $f'(x)$ as $f(x)=3x^2-5x$ (2)

6.2.2 $D_x\left[\frac{-2x+\sqrt{x}}{x^2}\right]$ (4)

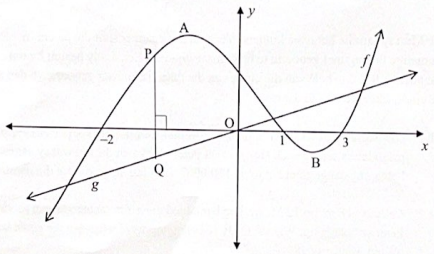
6.2.3 $\frac{dy}{dx}$, as $y=\frac{1-8x^3}{1-2x}$. (3)

[14]

VRAAG 7

In die diagram is die grafieke van $f(x)=x^3+bx^2+cx+d$ en $g(x)=2x$ geskets.

Die x-afsnitte van f is $x=-2$, $x=1$ en $x=3$. A en B is die draaipunte van f. P is 'n punt op f en Q is 'n punt op g sodat PQ loodreg op die x-as is. $x_p<0$.



7.1 Toon aan dat $f(x)=x^3-2x^2-5x+6$. (2)

7.2 Bereken die x-koördinaat van B. (3)

7.3 'n Raaklyn aan f het 'n gradiënt van -1. Verduidelik waarom die raakpunt van die raaklyn en die grafiek van f tussen die punte A en B sal lê. (1)

7.4 Vir watter waardes van x sal f konkaaf op wees? (2)

7.5 Bepaal die maksimum lengte van die lyn PQ. (5)

[13]

VRAAG 8

Die wins van 'n taxi is afhanklik van die spoed waarteen die voertuig gery word.

Die wins (P) in rande per uur is bereken met die formule $P = -\frac{3}{80}x^2 + 6x - 180$,

waar x die gemiddelde spoed in kilometer per uur is en $x \geq 30$.

Bepaal:

8.1 Die spoed as geen wins gemaak word nie. (3)

8.2 Die mees ekonomiese spoed met die ooreenstemmende wins per uur. (4)

8.3 Die spoed waarteen die eienaar 'n verlies sal toon. (3)

[10]

VRAAG 9

1.1 236 Studente word gevra watter selfoon verskaffer hulle gebruik. Die uitslag word in die tabel vertoon.

Selfoon verskaffer	Manlik	Vroulik	Totaal
Vodacom	48	52	100
MTN	40	34	74
Cell C	32	30	62
Totaal	120	116	236

9.1.1 In die gebeurtenisse Manlike studente en Vodacom onderling uitsluitend? Motiveer jou antwoord. (2)

9.1.2 Wat is die waarskynlikheid dat 'n student wat willekeurig gekies word, manlike is en Vodacom as verskaffer sal gebruik? (1)

9.1.3 Bewys dat manlike studente en Vodacom nie onafhanklik van mekaar is nie. (3)

9.2 Beskou die woord STAPLED.

As die letters lukraak gekies word en al die letters word gebruik, bepaal die waarskynlikeid dat die nuwe word met 'n klinker sal begin en eindig. (3)

9.3 Die getalle 0, 1, 2, 3, 4, 5 en 6 word gebruik om 'n drie syfer kode te maak.

9.3.1 Hoeveel unieke kodes kan gevorm word as die getalle herhaal mag word? (2)

9.3.2 Hoveel unieke kodes kan gevorm word as die getalle nie herhaal mag word nie? (2)

9.3.3 As die getalle herhaal mag word, hoeveel unieke drie syfer kodes kan gevorm word wat groter as 300 en deelbaar deur 5 is? (2)

[15]

TOTAAL: 150

QUESTION 8

The profit made on a taxi is dependent on the average speed at which it is being driven.

The profit (P) in rands per hour is calculated from the formula $P = -\frac{3}{80}x^2 + 6x - 180$,

where x is the average speed in kilometres per hour and $x \geq 30$.

Determine:

8.1 The speed at which no profit is yielded. (3)

8.2 The most economical speed and the corresponding profit per hour. (4)

8.3 The speed at which the owner will show a loss. (3)

[10]

QUESTION 9

9.1 236 Students were asked which cellphone provider they use. The results are shown in the table.

Cellphone provider	Male	Female	Total
Vodacom	48	52	100
MTN	40	34	74
Cell C	32	30	62
Total	120	116	236

9.1.1 Are the events Male students and Vodacom mutually exclusive? Justify your answer. (2)

9.1.2 What is the probability that a student chosen at random will be Male and use Vodacom as a service provider? (1)

9.1.3 Prove that the Male students and Vodacom are not independent. (3)

9.2 Consider the word STAPLED.

If the letters are randomly selected, and all letters are used, what is the probability that the new word would start and end with a vowel. (3)

9.3 The numbers 0, 1, 2, 3, 4, 5 and 6 are used to form a three digit code.

9.3.1 How many unique codes can be formed if the numbers may be repeated? (2)

9.3.2 How many unique codes can be formed if the numbers may not be repeated? (2)

9.3.3 In the case where the digits can be repeated, how many 3 digit codes formed are numbers that are greater than 300 and exactly divisible by 5? (2)

[15]

TOTAL: 150