LÉVY PROCESSES SIMULATIONS

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This tutorial explores the world of Brownian and Cauchy processes.

Question 1:

Generate a vector T consisting of N = 53 uniformly spaced points ranging from 0 to 1.

NB: This vector represents time expressed in years. Each interval corresponds to a week.

Question 2:

Build a function that takes the previous vector as argument and returns a Wiener process W1 defined as:

$$\Delta W_n \sim Gauss(0; \Delta T_n)$$

Plot M = 5 paths of the Wiener process on the same figure.

Question 3:

Let's define the Brownian process B with drift $\mu = 0.1$ and volatility $\sigma = 0.3$:

$$dB_t = \mu \cdot dt + \sigma \cdot dW_t$$

$$B_0 = 0$$

Build a function that takes the vector T, the drift μ and the volatility σ as arguments and generates a path of the corresponding Brownian process.

Plot M = 5 paths of the Brownian process on the same figure.

Question 4:

Generate M = 1000 paths of the previous Brownian process and plot the histogram of the empirical distribution of the last position B_N along with the theoretical distribution.

Question 5:

Consider two independent Wiener processes W_1 and W_2 and define two correlated Brownian processes B_1 and B_2 as:

$$dB_{1t} = \mu_1 \cdot dt + \sigma_1 \cdot dW_{1t}$$

$$B_{10} = 0$$

$$dB_{2t} = \mu_2 \cdot dt + \sigma_2 \cdot (\rho \cdot dW_{1t} + \sqrt{1-\rho^2} \cdot dW_{2t})$$

$$B_{20} = 0$$

The parameters are: $\mu_1 = 0.1$, $\sigma_1 = 0.3$, $\mu_2 = -0.2$, $\sigma_2 = 0.1$, and $\rho = 0.5$.

1. Plot M = 5 paths in a 2D plot.
2. Generate M = 1000 paths of the previous 2D Brownian process and plot the histogram of the empirical distribution of the last position B_N along with the theoretical distribution.
3. Redo question 5.b. varying the correlation ρ from -1 to 1 with a step of 0.05.
   Plot the trajectory of the empirical and theoretical averages and standard deviations of the last position B_N indexed by the correlation ρ in a 2D plot.

Question 6:

Redo questions 2, 3, and 4 for a Cauchy process:

$$\Delta Z_n \sim Cauchy(0; \Delta T_n)$$

$$dC_t = \mu \cdot dt + \sigma \cdot dZ_t$$

$$C_0 = 0$$

NB: This is a Cauchy process. Brownian and Cauchy processes are examples of Lévy processes.