Seismology

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Homework 2

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Problem 1

Period T is to angular frequency ω as wavelength is to:

(a) wavenumber k,

(b) velocity c,

(c) frequency f ,

(d) time t,

(e) none of the above

Solution

$\omega = \frac{2\pi}{T}$ and $k = \frac{2\pi}{\lambda}$ so the answer is (a), the wavenumber.

Problem 2

Figure 3.6 in Shearer plots a harmonic plane wave at $t = 0$, traveling in the x direction at 5 km/s.

(a) Write down an equation for this wave that describes displacement $\mathbf{u}$ as a funciton of x and t

(b)

(c)

(d) What is the maximum strain for this wave?

Solution (a)

We are looking to write the wave in the form $\mathbf{u}(x,t) = A\sin(\omega t - kx)$. We choose sin because the wave is 0 at $t = 0, x = 0$.

From the figure we can see that $A = 0.04$, $\lambda = 8000$m, and $v = 5000$m/s
Now we can calculate the period $T = \frac{\lambda}{v} = \frac{8000}{5000} = 1.6$s, the angular velocity $\omega = \frac{2\pi}{T} = \frac{5}{4}\pi = 1.25\pi\text{s}^{-1}$ and the wave number $k = \frac{2\pi}{\lambda} = 2.5 \times 10^{-4}\pi$. So we have the equation

$$\mathbf{u}(x,t) = 0.04\sin(1.25\pi t - 2.5 \times 10^{-4}\pi x)$$

Solution (b)

$\sin(x) = cos(x - \frac{\pi}{2})$. So we can write

$$\mathbf{u}(x,t) = 0.04\cos(1.25\pi t - 2.5 \times 10^{-4}\pi x - \frac{\pi}{2})$$

Why we would do this is beyond me

Solution (c)

We plug in

$$\begin{aligned}
\mathbf{u}(6000, 30) &= 0.04\sin(1.25\pi 30 - 2.5 \times 10^{-4}\pi 6000) \\
&= 0.04\sin(\frac{150}{4}\pi - \frac{6}{4}\pi) \\
&= 0.04\sin(\frac{144}{4}\pi) = 0.04\sin(36\pi) = 0
\end{aligned}$$

Solution (d)

Due to $\mathbf{u}(x,t)$ having only one spatial dimension we only need to calculate

$$\begin{aligned}
\mathbf{e}_{xx} &= \frac{\partial \mathbf{u}(x,t)}{\partial x} \\
&= 0.04 * (-2.5 \times 10^{-4}\pi)\cos(1.25\pi t - 2.5 \times 10^{-4}\pi x) \\
&= -\pi \times 10^{-5}\cos(1.25\pi t - 2.5 \times 10^{-4}\pi x)
\end{aligned}$$

Given that cosine takes values on $[-1, 1]$. This funciton achieves the maximum of $kA = \pi \times 10^{-5}$

Problem 3

Consider two types of monochromatic plane waves propagating in the x direction in a uniform medium:

(a) P wave in which $u_x = A\sin(\omega t - kx)$,

(b) S wave with displacements in the y direction, i.e., $u_y = A\sin(\omega t - kx)$

For each case, derive expressions for the non-zero components of the stress tensor.

Solution (a)

Of all the partial derivatives of $\mathbf{u}$, only the x derivative of the x component $\partial_x u_x = -kA\cos(\omega t - kx)$ is non-zero. We will avoid writing out the full expression to save space, writing the strain and stress in terms of $\partial_x u_x$. We get the strain tensor:

$$\mathbf{e} = \begin{bmatrix} \partial_x u_x & \frac{1}{2}\partial_x u_x & \frac{1}{2}\partial_x u_x \\ \frac{1}{2}\partial_x u_x & 0 & 0 \\ \frac{1}{2}\partial_x u_x & 0 & 0 \end{bmatrix}$$

We see that $\mathrm{tr}(\mathbf{e}) = \partial_x u_x$ so eqn. 2.30 from Shearer gives us the stress tensor:

$$\boldsymbol{\tau} = \begin{bmatrix} (\lambda + 2\mu)\partial_x u_x & \mu\partial_x u_x & \mu\partial_x u_x \\ \mu\partial_x u_x & \lambda\partial_x u_x & 0 \\ \mu\partial_x u_x & 0 & \lambda\partial_x u_x \end{bmatrix} = \partial_x u_x \begin{bmatrix} (\lambda + 2\mu) & \mu & \mu \\ \mu & \lambda & 0 \\ \mu & 0 & \lambda \end{bmatrix}$$

Solution (b)

For the S wave, only the y component of $\mathbf{u}$ is non-zero and it only depends on x. Therefore, of the partial derivatives of the components of $\mathbf{u}$, only $\partial_x u_y = -kA\cos(\omega t - kx)$ is non-zero. This gives us the strain tensor:

$$\mathbf{e} = \begin{bmatrix} 0 & \frac{1}{2}\partial_x u_y & 0 \\ \frac{1}{2}\partial_x u_y & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We see that $\mathrm{tr}(\mathbf{e}) = 0$ so eqn. 2.30 from Shearer gives us the stress tensor:

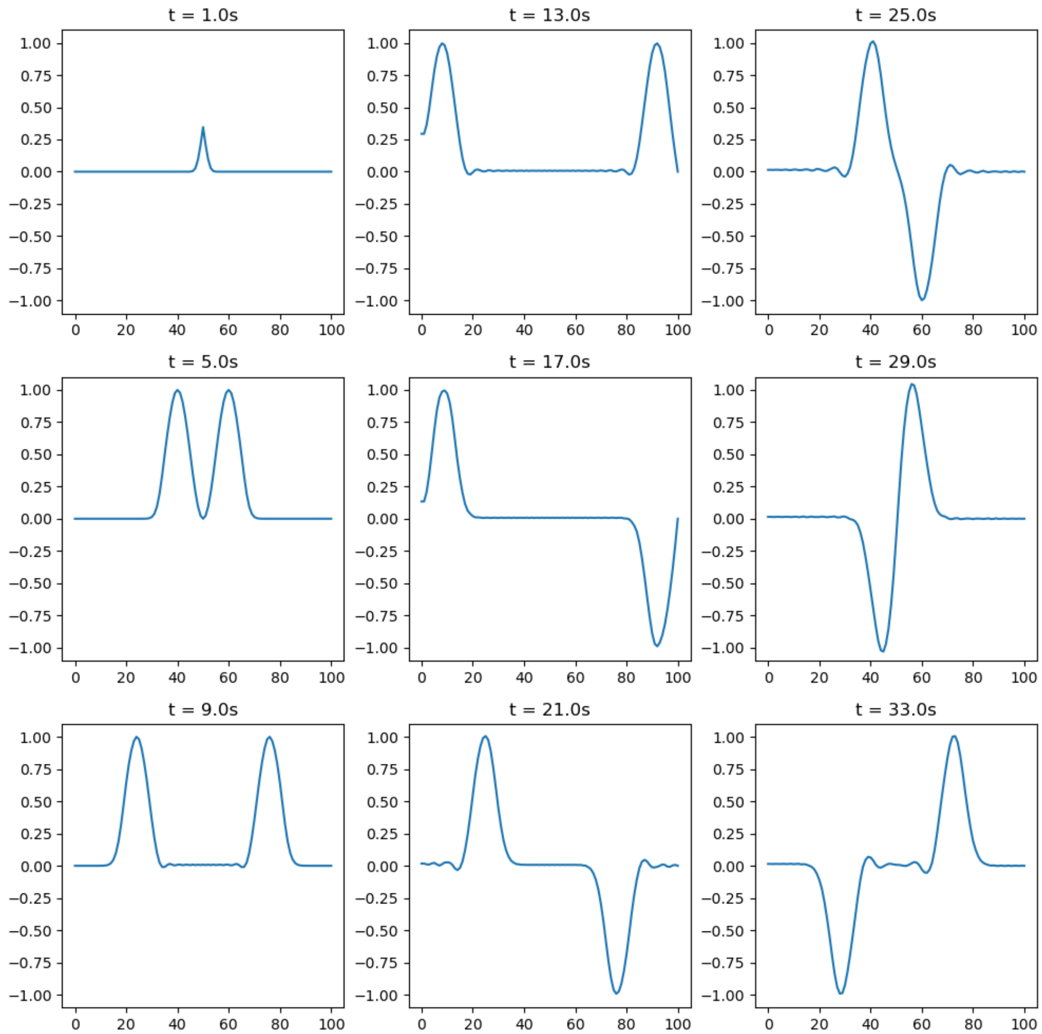
$$\boldsymbol{\tau} = \begin{bmatrix} 0 & \mu\frac{1}{2}\partial_x u_y & 0 \\ \mu\frac{1}{2}\partial_x u_y & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Problem 7

We use the code from the book as a guide and flesh out the following script

```
import numpy as np
import matplotlib.pyplot as plt

t = 0
dx = 1
dt = 0.1
tlen = 5
beta = 4
u1 = np.zeros(101)
u2 = np.zeros(101)
u3 = np.zeros(101)
fig, axs = plt.subplots(3,3, figsize=(10,10))
pltno = 0

while t <= 33:
    t += dt

    for i in range(1,100):
        rhs = (beta**2)*(u2[i+1] - 2*u2[i] + u2[i-1])/dx**2
        u3[i] = (dt**2)*rhs + 2*u2[i] - u1[i]

    u3[0] = u3[1]
    u3[100] = 0
    if t <= tlen:
        u3[50] = np.sin(np.pi * t / tlen)**2

    u1 = np.copy(u2)
    u2 = np.copy(u3)

    if np.abs((t % 4) - 1.0) < 1e-4:
        py = pltno // 3
        px = pltno % 3
        axs[px, py].plot(np.arange(101), u2)
        axs[px, py].set_title("t = " + str(np.round(t,decimals=0)) + "s")
        axs[px, py].set_ylim(-1.1, 1.1)
        pltno += 1

fig.tight_layout()
plt.savefig('problemset2/problems/figures/P7output.png')
```

The output images are as follows



We see that The pulse hitting the stress free boundary is reflected as is but the pulse hitting the fixed boundary has its displacement mirrored as it is reflected back. When the pulses meet they pass through each other.