Seismology

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Homework 1

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Problem 1

Assume that the horizontal components of the 2-D stress tensor are

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} \\ \tau_{yx} & \tau_{yy} \end{bmatrix} = \begin{bmatrix} -30 & -20 \\ -20 & -40 \end{bmatrix} \text{MPa}$$

(a) Compute the normal and shear stresses on a fault that strikes 10° east of north.

(b) Compute the principal stresses, and give the azimuths (in degrees east of north) of the maximum and minimum compressional stress axes.

(a) Solution

We have

$$\hat{\mathbf{n}} = \begin{bmatrix} \cos(10^\circ) \\ -\sin(10^\circ) \end{bmatrix} \approx \begin{bmatrix} 0.9848 \\ -0.1736 \end{bmatrix}$$

And

$$\mathbf{t}(\hat{\mathbf{n}}) = \boldsymbol{\tau}\hat{\mathbf{n}} \approx \begin{bmatrix} -30 & -20 \\ -20 & -40 \end{bmatrix} \begin{bmatrix} 0.9848 \\ -0.1736 \end{bmatrix} \approx \begin{bmatrix} -26.0713 \\ -12.7502 \end{bmatrix}$$

This gives us

$$t_N = \mathbf{t}(\hat{\mathbf{n}}) \cdot \hat{\mathbf{n}} \approx \begin{bmatrix} -26.0713 \\ -12.7502 \end{bmatrix} \cdot \begin{bmatrix} 0.9848 \\ -0.1736 \end{bmatrix} \approx -23.4611$$

And

$$t_S = \sqrt{\|\mathbf{t}(\hat{\mathbf{n}})\|^2 - t_N^2} \approx 17.0837$$

(b) Solution

To find the principal stresses we need the eigenvalues of $\boldsymbol{\tau}$. Using Python we find them to be $\lambda_1 = -55.6155$ and $\lambda_2 = -14.3845$. The associated eigenvectors are

$$\mathbf{u}^{(1)} \approx \begin{bmatrix} 0.6154 \\ 0.7882 \end{bmatrix}, \quad \text{and} \quad \mathbf{u}^{(2)} \approx \begin{bmatrix} 0.7882 \\ -0.6154 \end{bmatrix}$$

The azimuth in degrees east of north of the maximum compressional stress axis is

$$\sin^{-1}\left(\frac{\mathbf{u}_x^{(1)}}{\|\mathbf{u}^{(1)}\|}\right) \approx \sin^{-1}\left(\frac{0.6154}{1}\right) \approx \sin^{-1}(0.6154) = 37.9819^\circ$$

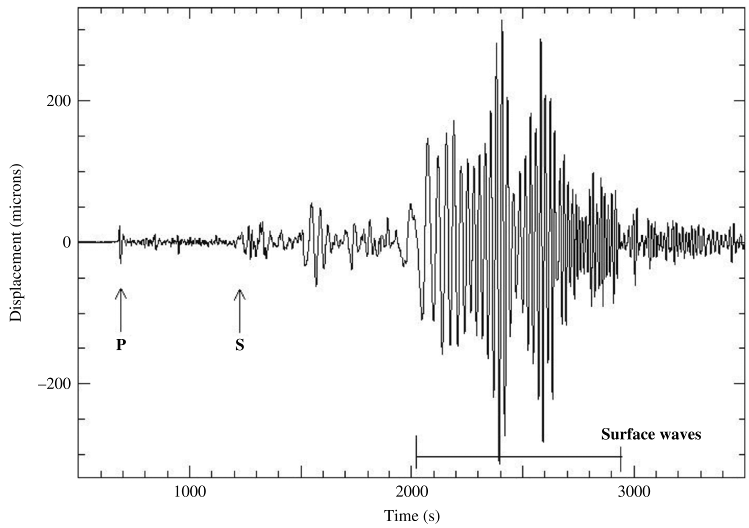
As for the minimum compressional stress axis it will be perpendicular to the maximum compressional stress axis and inspection reveals that the azimuth in degrees east of north is $37.9819^\circ + 90^\circ = 127.9819$

Problem 3

The figure shows a vertical-component seismogram of the 1989 Loma Rieata earthquate recorded in Finland.

(a) Estimate the dominant period T, of the surface wave from its first ten cycles. Then compute the frequency $f = 1/T$.

(b) Make an estimate of the *maximum* surface-wave strain recorded at this site. Hints: 1 micron = 10^{-6}m, assume the Rayleigh surface wave phase velocity at the dominant period is 3.9 km/s.



(a) Solution

Counting the first 10 wavetroughs we see that $10T \approx 350$s so $T \approx 35$s and $f = 1/T \approx 2.9 \times 10^{-2}$Hz

(b) Solution

We approximate the displacement with a wave

$$\mathbf{u}(\mathbf{x}, t) = A\sin[2\pi f(t - x/c)]$$

Giving us displacement

$$\frac{\partial u_z}{\partial x} = \frac{-2\pi f A}{c}\cos[2\pi f(t - x/c)]$$

Since the range of cosine is $[-1, 1]$ we have the maximum displacement

$$d_{max} = \frac{2\pi f A}{c}$$

From the figure we estimate $A \approx 300$ micron $= 3 \times 10^{-4}$m. With the previously calculated f and the given value for c this gives us a maximum displacement of 1.4×10^{-8}

Problem 4

Show that the principal stress axes always coincide with the principal strain axes for isotropic media. In other words, show that if $\mathbf{x}$ is an eigenvector of $\mathbf{e}$, then it is also an eigenvector of $\boldsymbol{\tau}$.

Solution

We use the relationship

$$\boldsymbol{\tau} = \lambda \mathrm{tr}(\mathbf{e})\mathbf{I} + 2\mu\mathbf{e}$$

and the definition of eigenvectors/eigenvalues of a matrix, $\mathbf{ex} = a\mathbf{x}$, using a rather than the more traditional λ to avoid confusion. Note that the trace of a matrix is a scalar.

$$\begin{aligned}
\boldsymbol{\tau}\mathbf{x} &= (\lambda \mathrm{tr}(\mathbf{e})\mathbf{I} + 2\mu\mathbf{e})\mathbf{x} \\
&= \lambda \mathrm{tr}(\mathbf{e})\mathbf{I}\mathbf{x} + 2\mu\mathbf{e}\mathbf{x} \\
&= \lambda \mathrm{tr}(\mathbf{e})\mathbf{x} + 2\mu a\mathbf{x} \\
&= (\lambda \mathrm{tr}(\mathbf{e}) + 2\mu a)\mathbf{x} \\
&= b\mathbf{x}
\end{aligned}$$

So $\mathbf{x}$ is an eigenvector of $\boldsymbol{\tau}$ with the eigenvalue of $b = (\lambda \mathrm{tr}(\mathbf{e}) + 2\mu a)$.

From eqn. 2.30 in the book we can calculate that $\mathrm{tr}(\boldsymbol{\tau}) = 3\lambda \mathrm{tr}(\mathbf{e}) + 2\mu \mathrm{tr}(\mathbf{e})$ meaning that

$$\mathrm{tr}(\mathbf{e}) = \frac{\mathrm{tr}(\boldsymbol{\tau})}{3\lambda + 2\mu}$$

giving us the eiginvalue of $\boldsymbol{\tau}$, b corresponding to the eigenvector $\mathbf{x}$

$$b = \frac{\lambda}{3\lambda + 2\mu}\mathrm{tr}(\boldsymbol{\tau}) + 2\mu a$$

Problem 7

What is the P/S velocity ratio for a rock with a Poisson's ratio of 0.30?

Solution

Take the wave speeds of P and S waves, α and β and find the ratio

$$\frac{\alpha}{\beta} = \frac{\sqrt{\frac{\lambda+2\mu}{\rho}}}{\sqrt{\frac{\mu}{\rho}}} = \sqrt{\frac{\lambda+2\mu}{\mu}} = \sqrt{\frac{\lambda}{\mu}+2}$$

Now we look at the ratio of Lamès parameters in terms of Poisson's ratio

$$\frac{\lambda}{\mu} = \frac{\frac{E\nu}{(1+\nu)(1-2\nu)}}{\frac{E}{2(1+\nu)}} = \frac{2\nu}{1-2\nu}$$

Plugging this back in we get

$$\frac{\alpha}{\beta} = \sqrt{\frac{2\nu}{1-2\nu} + \frac{2-4\nu}{1-2\nu}} = \sqrt{\frac{2-2\nu}{1-2\nu}}$$

Plugging in the value of Poisson's ratio

$$\frac{\alpha}{\beta} = \sqrt{\frac{2-0.6}{1-0.6}} = \sqrt{\frac{1.4}{0.4}} = \sqrt{\frac{7}{2}} \approx 1.9$$

Problem 9

Using values from the PREM model, compute values for the bulk modulus on both sides of

(a) the core-mantle boundary (CMB)
(b) the inner-core boundary

Express your answers in pascals.

Solution

We start by deriving an equation for the bulk modulus in terms of the speed of P and S waves. From the given formulas we have:

$$\beta^2 = \frac{\mu}{\rho}, \quad \mu = \rho\beta^2$$

$$\alpha^2 = \frac{\lambda + 2\mu}{\rho} = \frac{\lambda + 2\rho\beta}{\rho}, \quad \lambda = \rho\alpha^2 - 2\rho\beta^2$$

Now

$$\kappa = \lambda + \frac{2}{3}\mu = \rho\alpha^2 - 2\rho\beta^2 + \frac{2}{3}\rho\beta^2 = \rho(\alpha^2 - \frac{4}{3}\beta^2)$$

Now that we have the equation, we need to think about units. To get the answer in Pa we have to transform the units in the table to SI units, this happens to be a factor of a 1000 for each parameter, resulting in a total factor of 10^9. We can therefore calculate with the units in the table and get the answer in GPa.

We also note that S waves do not probagate in liquid so we can see the estimated boundaries in the table from that. Finally we can do the calculations

(a)

Above the boundary

$$5.57 \times (13.72^2 - \frac{4}{3} \times 7.26^2) = 657 \text{ GPa}$$

Below the boundary

$$9.90 \times (8.06^2 - \frac{4}{3} \times 0^2) = 643 \text{ GPa}$$

(b)

Above the boundary

$$12.17 \times (10.36^2 - \frac{4}{3} \times 0^2) = 1306 \text{ GPa}$$

Below the boundary

$$12.76 \times (11.03^2 - \frac{4}{3} \times 3.50^2) = 1344 \text{ GPa}$$