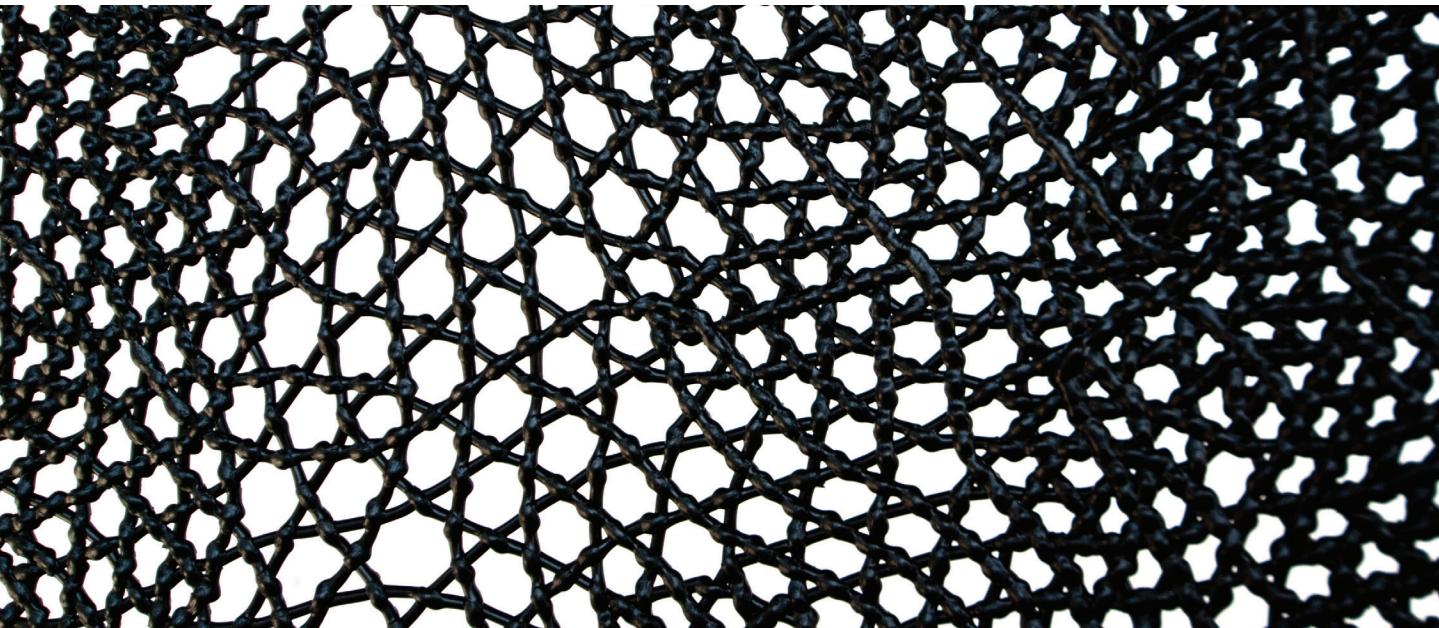


# Topologically Optimized and Functionally Graded Cable Nets

New Approaches through Robotic Additive Manufacturing

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## ABSTRACT

Recent advancements in the realm of additive manufacturing technologies have made it possible to directly manufacture the complex geometries that are resultant from topological optimization and functionally graded material processes. Topological optimization processes are well understood and widely used within the realm of structural engineering and have been increasingly adopted in architectural design and research. However, there has been little research devoted to the topological optimization of cable nets and their fabrication through robotic additive manufacturing.

This paper presents a design framework for the optimization of additively manufactured tensile cable nets that attempts to bridge between these two domains by reframing the scale of topological optimization processes. Instead of focusing solely on the topology optimization at the macro-scale of cable nets, this research develops a method to optimize the meso-scale topology and defines metamaterial units with different properties that are to be aggregated into a complex whole. This reorientation from the formal towards the material domain signals an engagement with morphogenetic modes of design that find formal expression through bottom-up material processes. In order to further investigate the emerging potentials of this reorientation, the presented method is validated through physical deformation tests and applied to the design of a furniture-scale case study project realized through the use of robotic additive manufacturing of elastomeric materials.

1 Detail of a robotic additively manufactured, functionally graded cable-net topology.

## INTRODUCTION

Building upon prior research by the authors that developed tools and techniques for the robotic extrusion of thermoplastic elastomers for programmable cable nets (McGee et al. 2017), this case study is part of a larger research project that investigates the architectural opportunities afforded by functionally graded materials. This paper reports on a developed method that enables the design and fabrication of complex cable net topologies that are functionally graded in response to anticipated stresses in the overall net. In functionally graded materials one or multiple material ingredients change gradually over the surface or volume to produce changes in the overall property of the material (Miyamoto 1999). It has been well observed that natural materials change their geometry in response to environmental conditions and stresses to make the most economical use of material. A number of architects have leveraged this concept in order to create cohesive material systems with gradients of material properties for additional functionality (Oxman, Tsai, and Firstenberg 2012).

### Traditional Approaches to Topological Optimization

Topological optimization processes work similarly, in that the algorithm attempts to place structural material only where it is needed within a larger boundary condition to satisfy the stress placed on the element. While many methods for this process exist, the two most common are the SIMP (solid isotropic material with penalization) and the BESO (bi-directional evolutionary topology optimization) methods (Deaton and Grandhi 2014). The SIMP method, developed by Bendsøe (1989) and Zhou and Rozvany (1991) independently, is a density-based method wherein each finite element in the system is assigned a density through interpolation that minimizes the overall weight of the system. These interpolated densities are then iteratively steered towards either a solid or void state through a penalization process. The BESO method, which is a further development of the ESO method established by Xie and Steven (1993), is a hard-kill algorithm from which underutilized elements are gradually removed and reinforcing elements are added to the system in order to arrive at an optimized form. The BESO method uses only binary or solid and void distinctions, so it is better suited to the design of objects for additive manufacturing than the SIMP method, which inevitably leaves some elements with densities just shy of the binary distinction (Brackett, Ashcroft, and Hague 2011).

### Extending Topological Optimization: From Macro to Meso

This research bridges these two domains by reframing the scale of topological optimization processes. Instead of focusing on the topology optimization at the macro-scale of the cable net, namely the design and specification of each

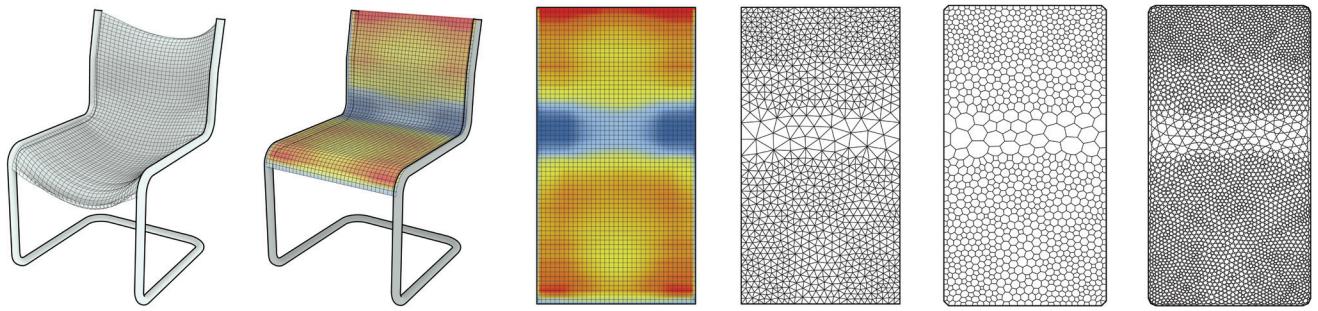
individual load-bearing element as would be typical in the previous approaches, this research develops a method to optimize the meso-scale topology and defines metamaterial units with different properties to be aggregated into a complex whole (Lee, Singer, and Thomas 2012). This reorientation from the formal towards the material domain signals an engagement with morphogenetic modes of design that find formal expression through bottom-up material processes. The interactions between the base material's properties and the anticipated load conditions drive the optimization process, which specifies differential densities in the cable net topology to deposit material where it is needed most. In this way, the optimization process is similar to the SIMP method, but without the need for a penalization routine to drive the system towards binary solid or void states due to the ability to print differentiated meso-scale topologies (Figure 1). Thus, rather than presenting a generic optimization approach, the method is enabled by and tailored to the capacities of robotic additive manufacturing, leveraging the ability to create complex single-bead topologies with high precision to produce metamaterial constructs not possible by other fabrication methods.

## METHODS

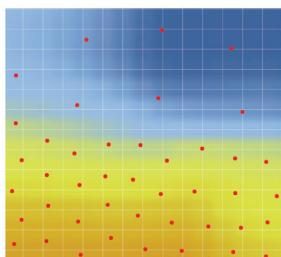
### Functionally Graded Topology Construction

The design process begins by constructing a regular square-celled mesh of the desired design surface at an appropriately high resolution. Through the use of a particle-spring physics system, the deformation of this base mesh under a typical design load is simulated, and a mapping of the strain of each edge member in the surface is produced. This simulated strain data is the basis for the differential nature of the functionally graded cable net, as the optimization sequence attempts to define the material densities for each area in relation to the strain data (Figure 2). This is accomplished by subdividing the design surface into a series of 40 x 40 mm subsurfaces and averaging the strain data of the simulated mesh edges that are contained within each subsurface. The strain data in each subsurface is averaged separately for the transverse and longitudinal directions, or U and V, in order to introduce anisotropy later in the process if necessary. The average strain data is translated into a new domain defined by a minimum and maximum material density that is determined by the optimization sequence.

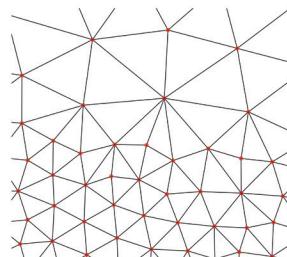
New topologies that correspond to the specified material density for each subsurface are generated by either a random uniform distribution method or a subdivision method. The random uniform distribution method distributes a particular number of points randomly in the subsurface and then utilizes Delaunay triangulation to



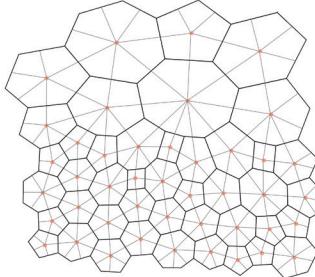
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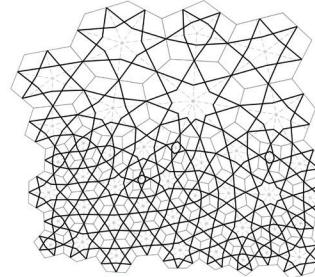
1. Determine point density by stress



2. Create Delaunay mesh



3. Produce mesh dual graph



4. Generate weaving patterns within each cell

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2 Diagram depicting topology construction process from base mesh particle-spring simulation to continuous toolpathing.

3 Detail diagram of translation from irregular-valence Delaunay topology to continuous toolpathing through algorithmic weaving process.

produce the topology, while the subdivision method uses the transverse and longitudinal strain data to separately specify grid subdivisions of each direction of the surface. At this point, a buffer area sized relative to the local density is introduced to blend between the subsurfaces and stitch the topologies back into a cohesive whole.

#### Continuous Toolpathing for Robotic Additive Manufacturing

While the outlined method thus far results in cable nets with differential densities, it also produces topologies that contain an uneven valence structure, making it impossible to fabricate them using a single-bead network with one continuous toolpath (Dreifus et al. 2017). Delaunay triangulation regularly produce vertices within the topology that are connected with an odd number of edges, and these conditions constitute dead ends in robotic additive manufacturing processes. While it is possible to start and stop the extrusion mid print, this is undesirable as it leads to slow prints and poor print quality. By eliminating odd vertices, the topology can be translated into a single continuous toolpath. This is achieved by an algorithmic weaving operation within each cell of the dual graph of the differential density mesh. The weaving operation replaces each edge of the dual graph with a vertex at the midpoint, connecting this vertex to its neighboring vertices within the cell with new edges (Figure 3). Since each edge, with the exception of the exterior edges, is shared between exactly two faces, the result is a topology where every vertex is

connected to four edges. For structural performance superior to joints in which two segments attempt to meet at a point, the new edges are not joined internally within their cells, but rather across cells to produce a continuous toolpath with full crossover joints at every intersection.

#### Meso-optimization of Functionally Graded Topology

The optimization process determines the minimum and maximum material densities that will minimize material usage of the continuous toolpath cable net topology while meeting a given performance criteria. This process is achieved through the use of a combination of multivariable solvers. The algorithmic solvers have control over the parameters that define the bounds of the material density range, in addition to two further parameters that can produce nonlinearity in the mapping from strain to density. To establish the fitness of each individual in the optimization process, the maximum deformation of each individual topology is simulated using finite element analysis, with the same typical design load that informed the initial strain mapping. The individual's maximum deformation is compared against an allowable deformation threshold, and those individuals that do not meet this criterion are penalized in relation to their difference from the threshold. This deformation score is combined with an estimation of the total mass of the printed topology to produce a single optimization objective to be minimized. Using the NLOpt library (Johnson 2008), a global evolutionary algorithm (da Silva Santos, Gonçalves, and Hernández-Figueroa 2010)

first searches for a coarsely optimal solution, which is then locally refined through the use of the Subplex algorithm (Rowan 1990). When the fluctuations in the final fitness criteria have fallen below 0.001 for the typically double-digit scores, the resultant topology is considered optimized.

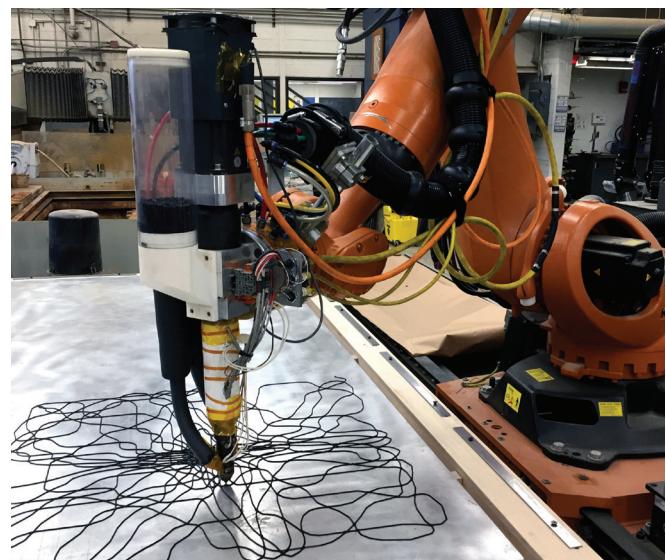
### Robotic Additive Manufacturing

This fully optimized, continuous toolpath cable net is then fabricated by extruding a thermoplastic elastomer material through a custom end effector for a KUKA KR120 7-axis robot (Figure 4). The extrusion takes place on a 1.2 x 2.4 m heated aluminum bed to help produce fully fused joints at all crossings of the cable net, a process that is further facilitated by the inclusion of explicit pressing motions at each intersection. The material used in this research is a thermoplastic polyurethane (TPU) with a durometer of 85A, which bonds well to itself at normal printing temperatures and exhibits impressive flexibility and strength but limited elasticity. The properties of this material are encoded into the particle-spring and FEA simulations through the physical testing of a standard calibration net to ensure that the digital simulations utilized in the optimization method perform as the physical topologies do.

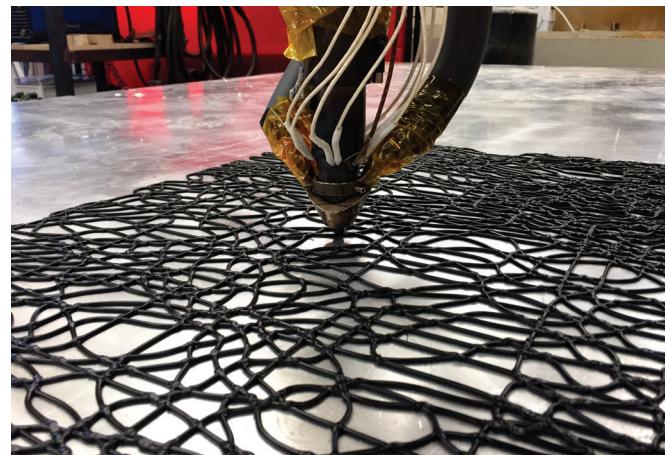
## RESULTS

The developed method is employed and tested in the design of a series of functionally graded cable nets. The first is for a test rig measuring 850 x 850 mm and which is tested with a number of standardized weights to accurately gauge performance across different printed configurations, while the second is a reimagining of the seat and back of Marcel Breuer's iconic 1928 Cesca chair.

The test rig cable nets are designed to resist the load of a 7.25 kg weight placed in the center of the surface, and the optimizations were set up to produce a surface that would deform less than 60 mm. An optimized cable net was designed using both the random uniform distribution and subdivision methods to compare the performance of each method. The resultant optimized random uniform distribution net weighed only nominally more than its subdivision counterpart. Additionally, a standard square-celled cable net with the same overall weight as the average of the two optimized nets was also fabricated as a base comparison with the same material usage. The subdivision method result exhibited the best performance by deforming 90 mm under load, as compared to 97 mm for the random uniform distribution method net, suggesting that the ability to control the anisotropy and direction of the cable net topology helps deliver greater structural efficiency. The square-celled comparison net deformed 112 mm under load, which results in an overall increase in performance of



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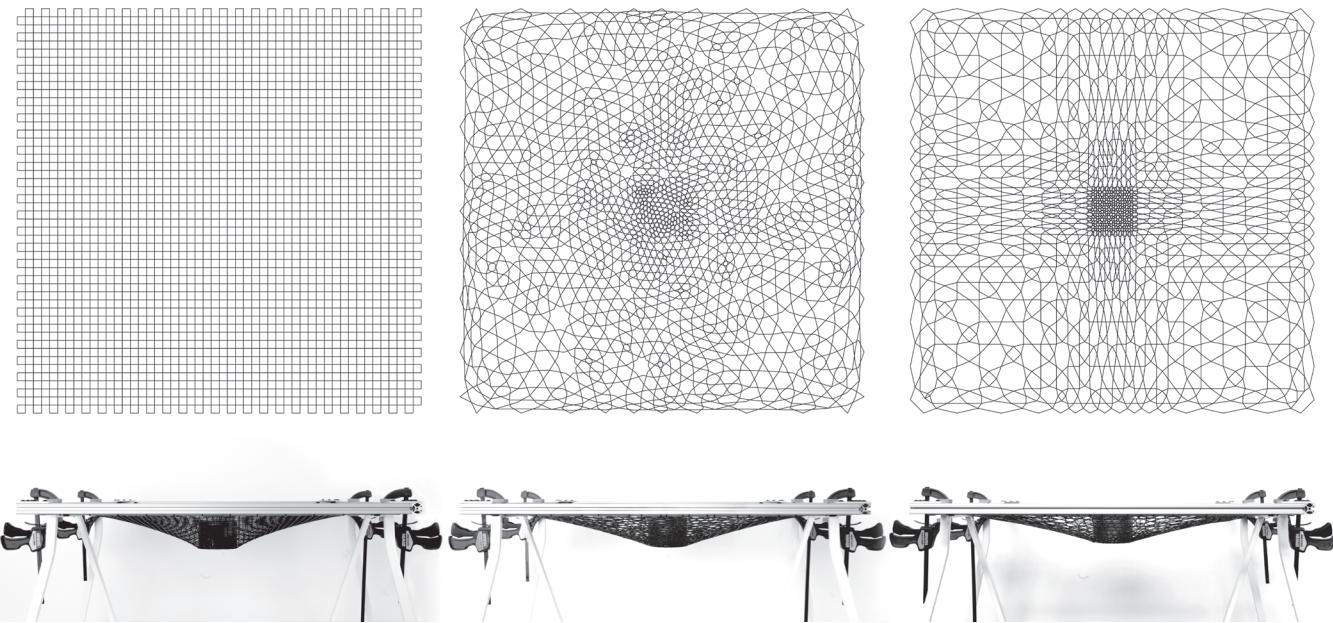
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4 Robotic additive manufacturing workcell towards the beginning of the additive manufacturing process.

5 Detail of robotic additive manufacturing process.

24.4% for the subdivision method and 15.5% for the random uniform distribution method. While neither of the cable nets matched the design criteria, this can be attributed to a need for further refinement of the physical material properties within the FEA simulation, as each physical piece exceeded its simulated deformations in nearly equal proportion.

The Cesca chair surface transforms the discrete seat and back of the original design into a continuous surface that follows the contours of the frame from the top of the chair to just below the knee. The chair surface was simulated and optimized for the stresses imparted by an average person's seated weight and form. The optimized topology places additional material adjacent to the frame of the chair; especially in areas near the top of the back and across the seat, while the structure is most sparse in the transition zone between seat and back (Figure 6). The densities of the subsurface



6 (Top) Plan figures and (bottom) physical deformation results of test rig cable nets: (L to R) base square-celled, optimized random uniform distribution method, and optimized subdivision method.

areas range from a minimum of  $0.815 \text{ kg/m}^2$  to a maximum of  $2.61 \text{ kg/m}^2$ , where a density of  $2.84 \text{ kg/m}^2$  would represent a completely solid cell at the same extrusion height. The algorithmic weaving pattern of the continuous toolpath topology harkens back to Breuer's woven cane seating surfaces in the original chair design, though conceived and fabricated through 21st-century technologies.

## CONCLUSION

This research offers a method for producing functionally graded cable nets through an alternative to the hard-kill topological optimization protocols typically employed for additive manufacturing. The method engages with the design of the materiality of the cable net by optimizing the meso-structure of the surface, remaining agnostic to the placement of individual edges and focusing instead on the specification of differential material properties through metamaterial constructs. The design of the cable nets is also achieved by a novel technique for algorithmic mesh weaving, which is capable of translating an irregular-valence topology into continuous-line, even-valence networks for robotic extrusion. This research is demonstrated through the fabrication of a series of functionally graded cable nets with varying local densities in anticipation of the stresses they are likely to see during their functional life, creating a materially efficient structure.

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