Investigating the motion of a gravity car

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1 Experimental Design

1.1 Focus Question

How does the weight connected to a gravity car affect the distance it travels

1.2 Hypothesis

The weight will linearly affect the distance travelled by the gravity car

1.3 Theory

Let A the point at which the car starts, B the point at which at the weight touches the base of the car, and C the point at which the car stops. Also, let M the mass of the whole car, m the mass of the weight attached to the car, h the height from which the height is dropped, r the radius of the wheels, r_a the radius of the axle attached to the wheels, and m_w the mass of one of the wheels.

The distance AB depends on h, r, and r_a. When the weight falls by h, the axle will rotate by the same angular displacement θ as the wheels,

$$AB = \frac{r}{r_a}h \tag{1}$$

Next, the distance BC depends on the velocity v_B with which the car reaches B as well as the friction f which is applied to the car.

Ignoring the energy lost to heat, by conservation of mechanical energy,

$$\underbrace{\frac{1}{2}(M+m)v_B^2}_{\text{Translational Kinetic Energy of Car}} + \underbrace{4\cdot\frac{1}{2}I\omega^2}_{\text{Rotational Kinetic Energy of Wheels}} = \underbrace{mgh}_{\text{Gravitational Potential Energy of Weight}}$$

The moment of inertia of each wheel is $\frac{1}{2}m_w r^2$, and the angular velocity of each wheel is $\frac{u_b}{r}$. Substituting these values gives:

$$\frac{1}{2}(M+m)u_B^2 + 2I\omega^2 = mgh \Rightarrow$$
$$\frac{1}{2}(M+m)u_B^2 + 2\frac{1}{2}m_w r^2(\frac{u_b}{r})^2 = mgh \Rightarrow$$
$$\frac{1}{2}(M+m)u_B^2 + \frac{1}{r^2}u_b^2 = mgh$$

Solving for u_B, this becomes:

$$u_B = \sqrt{\frac{2mgh}{M+m_w+m}}$$

Then, BC will equal $\frac{u_B^2}{2a_f}$ where a_f is the acceleration of the car due to friction. a_f equals $\frac{\mu Mg}{M} = \mu g$, So,

$$\begin{aligned} BC &= \frac{u_B^2}{2a_f} = \frac{\frac{2mgh}{M+m_w+m}}{2\mu g} = \frac{mh}{\mu(M+m_w+m)} \\ &= \frac{h\cdot\mathbf{m}}{\mu(M+m_w)+\mu\cdot\mathbf{m}} \end{aligned}$$

The complete distance covered S covered by the car will, therefore be:

$$\boxed{\begin{aligned} S &= AB + BC \\ &= \frac{r}{r_a}h + \frac{h\cdot\mathbf{m}}{\mu(M+m_w)+\mu\cdot\mathbf{m}} \end{aligned}}$$

This means that the distance covered by the car will continuously increase with m, asymptotically approaching $\frac{r}{r_a}h + \frac{h}{\mu}$

1.4 Variables

Variables identified	Type of variable	Treatment
Mass on string	Independent	Increased in 20g increments
Distance covered	Dependent	5 measurements taken for each weight increment
Car	Controlled	The car remains the same in all aspects for each measurement
Tire width	Controlled	The same $21.97mm$ tires used for all measurements
Height	Controlled	The weight is always dropped from the same height from the platform of the car

1.5 Apparatus & Materials

- Lego pieces (for making the gravity car)
- Measured weights and weight holder
- String
- Data Logger
- Ultrasonic distance sensor
- Laptop

1.6 Procedure

1.6.1 Construction of the gravity car

1.6.2 Conducting the Experiment

1. Hang weight from string
2. Roll string on the axis until weight is at the height of the mark

3. Place car around 40cm from the sensor with an object to prevent it from moving. The exact distance does not matter since we are only measuring the distance travelled. It must be more than 40 cm, however, since that is the minimum distance the sensor can measure.

4. Start the logger

5. Remove the object so the car starts moving

6. When the car has completely stopped, stop the logger and record the data

2 Experimental Report

2.1 Modifications to the original design

- Because of the weight continuously falling off the car, a small amount of plasticine was added to the base of the car which would catch the weight.

- When the weight reached the base of the car, the string would start rolling up on the axis, stopping the car when it got fully stretched. The solution was to simply use a much longer piece of string. This had the side-effect of sometimes getting tangled up on the axle, however, all measurements in which that happened were discarded.

2.2 Data Collection

2.3 Data Processing

2.3.1 Overview

For processing the data, I will graph the relationship of the weight on the gravity car to the mean distance travelled in the five measurements with that weight. The error for each measurement is calculated as half the difference between the greatest and the least measurement, or: $\Delta\bar{s} = \frac{1}{2}(s_{max} - s_{min})$. The data will then be added to the program graph, which will allow us to derive a best-fit function for the data.

Table 1: Data

mass (kg) $\pm$ 0.0001	Distance (Measurement)				
	1st	2nd	3rd	4th	5th
0.04	1.263	1.330	1.315	1.254	1.281
0.06	1.651	1.700	1.643	1.596	1.643
0.08	1.858	1.922	1.920	1.935	1.905
0.10	1.978	1.944	1.944	1.883	1.859
0.12	1.949	1.961	1.984	1.944	1.959
0.14	1.972	1.969	1.971	2.023	2.021
0.16	2.031	2.097	2.041	2.046	2.055

2.3.2 Presentation

Table 2: Processed Data

weight (N)	Average	Error
0.392	1.289	0.0380
0.589	1.647	0.0534
0.785	1.908	0.0500
0.981	1.922	0.0626
1.18	1.959	0.0246
1.37	1.991	0.0318
1.57	2.054	0.0430

Figure 1: Weight-Distance graph with *cubic* trendline

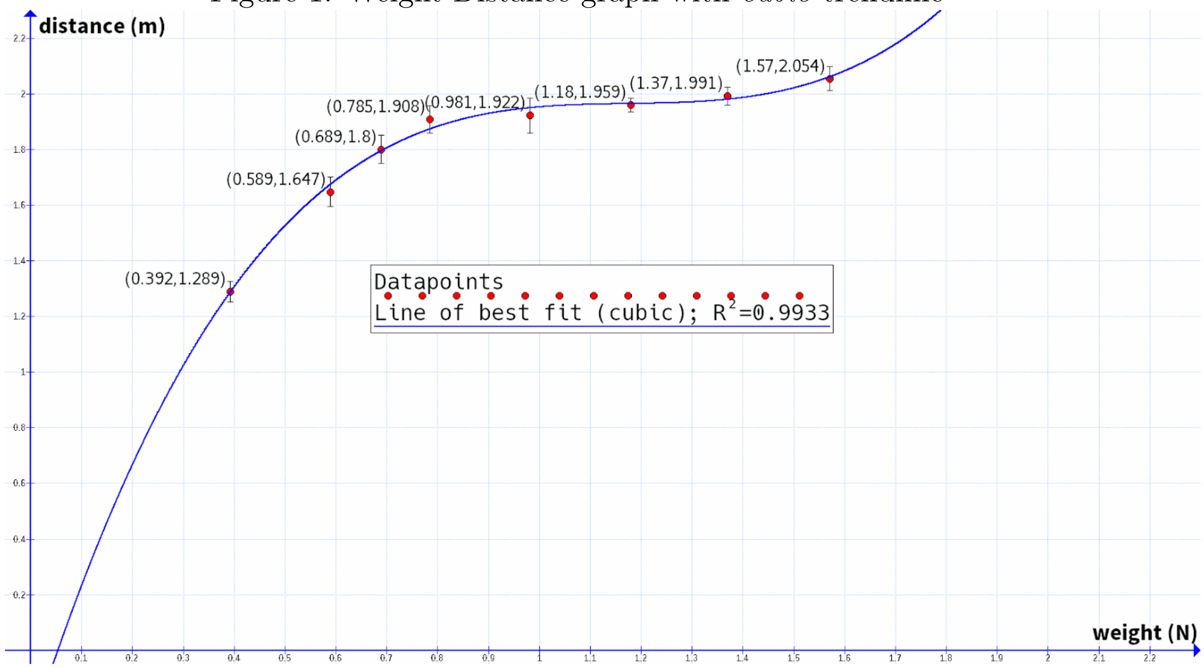

Figure 2: Weight-Distance graph with *linear* trendline

2.4 Conclusion & Evaluation

2.4.1 Conclusion & Justification

As it can be seen be seen in Figure 2, the relationship between the weight on the gravity car and the distance that it travels appears to be best modelled by the function (2) with a coefficient of determination of 0.993 (The values of the coefficients have been shortened to 3 S.F.).

$$1.37x^3 - 4.84x^2 + 5.71x - 0.289 \tag{2}$$

Figure 1 also shows that the data seems to refute the initial hypothesis, since the linear line of best fit does not go through the error bars of all data points.

2.4.2 Evaluating Procedures & Suggestions for Improvements

While the data gathered were fairly precise, there was a variety of possible improvements that could have further increased that precision:

- The way that the height from which the weight fell was measured was by having a small mark on the tower of the gravity car, and lining that mark up with the base of the weight each time. This method was, of course, very error-prone since lining up the weight with the mark was done completely manually. In order to improve the consistency of the fall height, a ruler or some other straight object could have been used to ensure that the weight fell exactly from the mark each time

- The plasticine, while effective in preventing the weight from falling over and stopping the car, did introduce another uncertainty into the experiment. On each iteration, the fall of the weight would slightly deform the plasticine, meaning that on the next iteration, the weight would have a slightly different distance to travel, by a few millimeters.

- It was observed after the completion of the experiment that the floor on which the experiment was conducted was tilted against the direction on which the car was travelling. While this incline did certainly affect the distance that the car travelled on each individual measurement, the fact that the incline was constant and that the car was, for all measurements, travelling in the same direction, meant that it most likely did not affect the overall behaviour of the distance travelled by the car relative to the weight used.