**Force of Mortality**

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# Synonyms

hazard rate mortality intensity instantaneous risk of mortality instantaneous death rate failure rate

# Definitions

The force of mortality is a continuous function of age and can be defined as the instantaneous effect of mortality at a certain age. If we denote by the number of survivors at age in a population, the force of mortality can be specifically described as the ratio of the rate of change of to the value of . Mathematically, then,

(1)

where represents differentiation with respect to age and indicates the rate of decrease of the survivorship over an infinitesimally small increment of age. In formal demography and actuarial science, the force of mortality is denoted by the Greek letter *µ* and is a positive measure over the entire age range. Alternatively, the force of mortality can be regarded as the behavior of a death rate , over a small duration *n* converging to zero,

. (2)

It is natural to think about the force of mortality in terms of the expected number of deaths in a population (Preston et al., 2001). The difference, within a tiny time frame of length *n*, in the number of survivors in a cohort aged exactly years old, is given by the expected number of deaths, which can be written algebraically as follows:

(3)

Following equation (1) it can be shown that the ratio of survivors at any age to the survivors at age is determined solely by the aggregate mortality between those two ages: a relationship of great importance in the theory of stable populations and the life table framework. This is done by writing the survival probability at age *x* over the period *n*, , as a function of *µ*:

(4)

Age-specific mortality data are usually available in discrete intervals (e.g. for 1, 5 or 10-year age classes). As a consequence, in practice, the force of mortality for non-integer ages can be derived from age-specific death rates by assuming either 1) a constant force of mortality within the age interval, or 2) a uniform distribution of deaths (i.e. increasing hazard rate), or 3) a decreasing failure rate also known as the Balducci assumption (Pitacco et al., 2009).

# Overview

The age-pattern of human mortality has a characteristic shape, as the force of mortality decreases quickly after birth, reaches its lowest point during childhood around ages 8–14 and then increases with age, with a *hump* around young adult ages marking accidental and maternal mortality (see examples in Figure 1). At the oldest ages, e.g. after age 100, the force of mortality starts to level off as the risk of dying remains approximately constant, forming the so-called *mortality plateau*. Nevertheless, such behavior of the mortality curve at advanced ages is based on scarce data, as the number of people reaching very old ages was relatively small until recently. Population ageing is providing new opportunities for the improvement of knowledge about the force of human mortality beyond age 100.

Given the variation of mortality by age, one of the main roles of the force of mortality is to provide a tool for a fundamental statement of assumptions about the behavior of individual mortality as a function of the attained age. This was of such importance in the past, that researchers and actuaries tried to represent mortality by means of mathematical functions (Smith, 1948) in their efforts to discover the universal law governing human mortality (analogous to, for instance the law of gravitation in physics). For this reason, it is common to find the term *mortality laws* in the scientific literature, which refers to parametric models of

FEMALES

MALES

0

30

60

90

110

0

30

60

90

110

0.0001

0.0010

0.0100

0.1000

1.0000

**Age−specific death rate**

Time interval

1900 −

09

1950 −

59

2010 −

16

**Age (Years)**

Note: Data presented on a logarithmic scale

**Figure 1:** *Mortality decline and postponement over age in France, based on age-specific death rates for selected historical periods, by sex. Source: Life tables from the Human Mortality Database (2019)*

mortality. Although a perfect fit to the entire mortality experience over age has never been achieved, some of these attempts are of particular interest.

The Gompertz model provides an excellent example. In the early nineteenth century, Gompertz (1825) made the first noteworthy proposal to describe mortality, by speculating that “it is possible that death may be the consequence of two generally co-existing causes; the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or an increased inability to withstand destruction.” He showed that, for various human populations, arithmetic increases in age were consistently accompanied by geometric increases in mortality between ages 20 and 60 (Olshansky and Carnes, 1997). He concluded that as the ageing process takes place, the inability to slow down physiological deterioration increases nearly exponentially.

An important modification of the model proposed by Gompertz was made by Makeham (1860). He assumed that, in addition to the part of the human age-range where the force of mortality increases geometrically as the individuals grow old, a baseline mortality level operating with equal intensity at all ages can be identified. Both Benjamin Gompertz and Matthew Makeham were British actuaries trying to provide solutions to the practical work involving mortality statistics, such as smoothing data, eliminating or reducing errors, aiding inferences from incomplete data, facilitating comparisons of mortality between various groups and populations, forecasting and life annuity pricing.

In addition to the two models mentioned above, numerous other mortality laws have been proposed during the past two centuries, attempting to cover either the entire experience from infancy to old age or distinct parts of it, such as life spent in retirement. Tabeau (2001) provides an overview of the most notable contributions.

The concept of the force of mortality plays an important role in the theory of life contingencies and actuarial science. For instance, information about changes in mortality intensities is essential for determining feasible ages of retirement and public policies needed to maintain a viable social security system in the context of transition to an ageing society. Insurance companies and private pension funds rely on mortality data for accurately determining the present value of their future liabilities and the adequate level of capital required to ensure their solvency under stressed market conditions, characterized among other factors by significant short and long-term demographic changes.

Moreover, information on mortality levels by age is fundamental for the identification of needs in intervention, for instance in the public health domain. Since the start of the *statistical revolution* in the 18th and the 19th centuries–when increased efforts began to be made for improving the collection and quality of population data and statistics–, the availability of accurate estimates of the force of mortality in a given population is essential for the implementation of efficient health measures and policies. For example, given the consideration of ageing as something problematic for both individuals and societies (Victor 2004), an understanding of the dynamics in the force of mortality at the oldest ages is essential for informed social policies.

# Key Research Findings

Most populations around the world have experienced impressive reductions in death rates at most ages during the last centuries and decades (see examples in Figure 1). Those reductions have triggered unprecedented improvements in human survival: the rise in life expectancy, referred to as the “crowning achievement of the modern era” by Riley (2001), is a major demographic phenomenon with deep individual and social implications.

In the developed countries, where the demographic transition started earlier and proceeded slowly, various historical phases of change in the force of mortality can usually be distinguished, based on the ages and causes of death contributing the most to reductions in overall mortality (i.e. to gains in life expectancy). During the early stages in the 19th and early 20th century, important reductions in infectious mortality at the youngest ages (0 to 14) were achieved, generating rapid gains in life expectancy. This decline was accompanied by reductions in young adult mortality in the first half of the 20th century, especially from tuberculosis. From about the middle of the 20th century onwards, important reductions in old age mortality (65+) have contributed the most to further (albeit slower) gains in life expectancy. The *cardiovascular revolution* has been key in the later development (Vallin and Meslé, 2001). The Epidemiologic Transition (Omran, 1971) and the Health Transition (e.g. Vallin and Meslé 2009) formalize and illustrate such trajectories, although the former theory does not include the current phase of improvements in old age mortality. In the developing countries, where the demographic transition started later and has proceeded at a fast pace, very rapid gains in life expectancy have been achieved, as large reductions in mortality at young and old ages are taking place simultaneously.

The sustained reductions in mortality referred to above, especially the ongoing improvements at old ages, have sparked academic discussion about the *biological limits to human lifespan*. For instance, various studies that make hypotheses about future mortality improvements conclude that such a limit exists, and it will soon be reached (e.g. Olshansky et al. 1990). On the other hand, Oeppen and Vaupel (2002) showed that the idea of being close to such a ceiling is not supported by empirical evidence, as asserted limits to life expectancy have repeatedly been broken since the 1920s and maximum recorded life expectancy levels have continued to increase at a constant pace of “2.5 years per decade for a century and a half”.

In addition to variation by age and over time, research has shown that levels of death rates may differ considerably between population subgroups according to certain attributes such as sex (women experience lower mortality and live longer than men, e.g. Zarulli et al. 2018), marital status (married men tend to have lower mortality than their non-married counterparts, e.g. Rendall et al. 2011), education and socio-economic status (mortality is generally lower among highly educated and wealthy individuals, e.g. Lleras-Muney 2005; Valkonen 2006), urban/rural place of residence (e.g. higher urban than rural mortality in the past vs. lower urban mortality in the present, e.g. Woods 2003; Van De Poel et al. 2009; Vierboom and Preston 2020), lifestyle (e.g. impact of smoking and obesity on health and survival, e.g. Mehta and Chang 2009; Lindahl-Jacobsen et al. 2016), and migration status (e.g. the *healthy migrant* effect and the *salmon bias* hypothesis, see Abraido-Lanza et al. 1999) among other characteristics.

# Future Directions of Research

Many aspects of health and human mortality at advanced ages remain unclear due to the limitations imposed by the data available (e.g. Grundy 1997). However, with the rapid increase in the share of the population reaching older and older ages, new opportunities are emerging for improving understanding in those areas. For example, new models to fit, explain and forecast the force of mortality after ages 90 or 100 are of great interest, motivated by the desire to accurately reflect past experience while also providing plausible future mortality trajectories (Heuveline and Clark, 2011). Additionally, thanks to advances in computational capacity for the storage, linkage and analysis of large databases, future research will shed light on the characteristics (genetic and environmental) associated with longer lifespans.

Furthermore, age-specific mortality improvements are not constant over time, as they depend on a multitude of factors (see Cutler et al. 2006). Considering that current gains in life expectancy are increasingly driven by reductions in old-age mortality, postponement of senescence will have to take place in order for life expectancy to continue rising at the same rate (Vaupel, 2010).

Finally, developments in age-specific mortality, especially at old ages, and their impact on life expectancy will most likely continue, fueling the debate on the biological limits to human lifespan.

# Summary

The force of mortality is a continuous function representing the instantaneous death rate. Given the characteristic age-pattern of human mortality, numerous mathematical models have been proposed for the estimation of mortality levels as a function of attained age. Impressive reductions in the force of mortality at most ages have been achieved during the last centuries and decades. As more and more people reach advanced ages, current gains in life expectancy are increasingly driven by reductions in old-age mortality. Understanding past and present trends in the force of mortality is essential for taking informed decisions on social and health policies. Sustained reductions in old-age mortality and population ageing are bringing new opportunities for improving understanding on health and mortality at advanced ages.

# Cross–references

Mortality modelling

Probability of dying

Demographic transition theories

Maximum lifespan Mortality leveling

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