Authors' response to reviewer 1

Thank you for reviewing the manuscript "The Linear Link: Deriving Age-Specific Death Rates from Life Expectancy" submitted for publication in the MDPI journal Risks. We appreciate the time and effort that dedicated to providing feedback on our manuscript and are grateful for the insightful comments and valuable improvements to our paper. We have incorporated most of the proposed suggestions. Those changes are highlighted within the manuscript. Please see below, in blue, for a point-by-point response to your comments and concerns.

Open Review

(x) I would not like to sign my review report
( ) I would like to sign my review report

English language and style

( ) Extensive editing of English language and style required
( ) Moderate English changes required
(x) English language and style are fine/minor spell check required
( ) I don't feel qualified to judge about the English language and style

	Yes	Can be improved	Must be improved	Not applicable
Does the introduction provide sufficient background and include all relevant references?	(x)	( )	( )	( )
Is the research design appropriate?	( )	(x)	( )	( )
Are the methods adequately described?	( )	( )	(x)	( )
Are the results clearly presented?	( )	(x)	( )	( )
Are the conclusions supported by the results?	(x)	( )	( )	( )

Review of 'The Linear Link: Deriving Age-Specific Death Rates from Life Expectancy'

Review

This paper proposes a method for deriving age-specific death rates from life expectancy estimations. This method is akin to the commonly used "model life tables" of demography, which permit the derivation of a schedule of mortality rates replicating a given value of life expectancy. Although short, the paper flows well and is very clearly written. Furthermore, I praise the authors for making their code available on Github – this is a very good practice which should be taken up more frequently in the actuarial literature. The method proposed in the paper is very closely related to the method presented in Ševcíková et al. (2016) which uses a Lee-Carter (LC) based approach to reconstruct mortality schedules. Therefore, I think that the authors need to be more specific about the differences and advantages of the Linear Link (LL) method in comparison to the method described in Ševcíková et al. (2016 Algorithm 1). I think there differences and advantages to the

LL but this need to be articulated better. Also, some technical imprecision in the paper can mislead the reader to taking the two methods as being the same. I elaborate on this below in my main comments

Main comment 1

The only essential difference between the method in Ševcíková et al. (2016) [1]and the LL method is that while Ševcíková et al. (2016) assume a LC formulation so that

$$\log m_{xt} = a_x + \nu_x \kappa_t, \tag{1}$$

the LL method assumes

$$\log m_{xt} = \beta_x \log e_{\theta,t} + \nu_x \kappa_t. \tag{2}$$

Apart from that the two methods are essentially the same as both:

1. use a Kannisto approach to extend mortality to older ages,
2. estimate ax (in Ševcíková et al. (2016)), β_x (in the LL), v_x and k_x using historical data
3. use the estimated parameters of a_x (in Ševcíková et al. (2016)), β_x (in the LL) and v_x to solve
1. numerically for a k_t value that matches a given (future) estimated value of life expectancy $e_{\theta,\tau}$.
4. Reconstruct the mortality scheduled using Equation (1) or (2) and the values estimated in steps 2 and 3.

In summary, the only difference is assuming $\beta_x \log e_{\theta,\tau}$ rather than a_x. This similarities and difference should be made more explicitly and the advantages should be better justified.

Authors` response: The reviewer has very eloquently outlined the similarities between the two methods of estimating mortality. It is also worth mentioning that a key difference is given by the estimation procedures employed in the to models. If in Ševcíková et al. (2016) we have a Lee-Carter type model in which the future level of mortality is given by the extrapolation of a mortality index, k[t], using time series approaches, the Linear-Link methods makes use of a total different strategy. Namely, the life expectancy is imposed a priori and the mortality pattern is derive subsequently following an optimization procedure by adjusting the k parameter, which is no longer a time index but an adjustment factor. And indeed the anchoring of our mortality estimates to the life expectancy level provides a clear advantage.

Figures 1 and 2 and the discussion around them partially serve as motivation for choosing $\beta_x \log e_{\theta,\tau}$rather than a_x but the advantages of this choice should be highlighted in the result

[1] As defined by Equation 1-3 and Algorithm 3 in Ševcíková et al. (2016)

section. To me the main advantage of the LL method is that exploiting the linear relationship between mortality and life expectancy, it produces mortality profiles that are less distorted around the age dimensions when forecasted far into the future (as illustrated in Figure 7).

Authors` response: We are thankful for this suggestion. First we have computed the Pearson correlation coefficient for each age group in Figure 1 and Figure 2 to emphasize the linearity between life expectancy and death rates and also make a distinction between the linearity at adult ages and the one exhibited at young or very old ages. And second, we highlighted the choice of the Linear-link model in the results section as noted in the comments.

Main comment 2

There are some impressions in the paper that can mislead the reader not to appreciate the differences between the LL method and LC based method as in Ševcíková et al. (2016).

In Equation 2, I think the dependence of k on time should be made explicit, that is, replace k with k_t. This is be because in both your OLS and MLE algorithms you are indeed estimating a time specific k_t rather than a single k_t for all t as stated in equation 2 and in lines 110-111.

Authors` response: We choose to maintain the k notation in order to avoid confusing the reader that we are considering a time index subject to extrapolation with a time-series model like in the case of Lee-Carter model or as in Ševcíková et al. (2016).

There are also some problems with the MLE method described in Appendix B and implemented in (https://github.com/mpascariu/MortalityEstimate/blob/master/R/fun_LinearLink.R. It is imprecise to assume that $a_x = \beta_x \log e_{\theta,\tau}$ as done in line 307[2] as we cannot make a time independent quantity equal to a time dependent quantity. This also makes the transformation in line 311-312 from a_x to $\beta_x$incorrect and line 343 of the code of function PoissonMLE inappropriate.

The correct way of adapting the MLE approach in Brouhns, Denuit, and Vermunt (2002) is by directly taking the derivative of the log-likelihood with respect to β_x and thus derive an updating procedure specific to β_x. From equation (2) it can easily be derived that the updating relationship for β_x should read:

$$\hat{\beta}_x(\omega+1) = \hat{\beta}_x(\omega) - \frac{\sum_t (D_{x,t} - \hat{D}_{x,t}) \log e_{\theta,t}}{-\sum_t \hat{D}_{x,t} (\log e_{\theta,t})^2}$$

Authors` response: Initially we derived the MLE procedure from the one proposed for the Lee-Carter model. As the reviewer suggested it makes sense to be written in terms of β_x instead of a_x which is not a defined parameter in our model. We agree that the derivation shown above is correct and we are happy to adopt it our manuscript and modify our source code provided in Github in

[2] The same problem occurs in line 127 in the main text

short time. However, because the life expectancy term is a constant in the likelihood and is not a parameter that we are estimating the outcome of the procedure will be the same.

References

Brouhns, Natacha, Michel Denuit, and J. K. Vermunt. 2002. “A Poisson log-bilinear regression approach to the construction of projected lifetables.” Insurance: Mathematics and Economics 31 (3): 373–93.

Ševcíková, Hana, Vladimíra Kantorová, Patrick Gerland, and Adrian E Raftery. 2016. “Age-Specific Mortality and Fertility Rates for Probabilistic Population Projections.” In Dynamic Demogrpaphic Analysis, 285–310. https://doi.org/10.1007/978-3-319-26603-9.