A History of Mortality Modelling from Gompertz to Lee-Carter Everything in a single R package: MortalityLaws

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XXVIII International Population Conference
Cape Town, South Africa

October 31, 2017

What is Modelling?

Modellus

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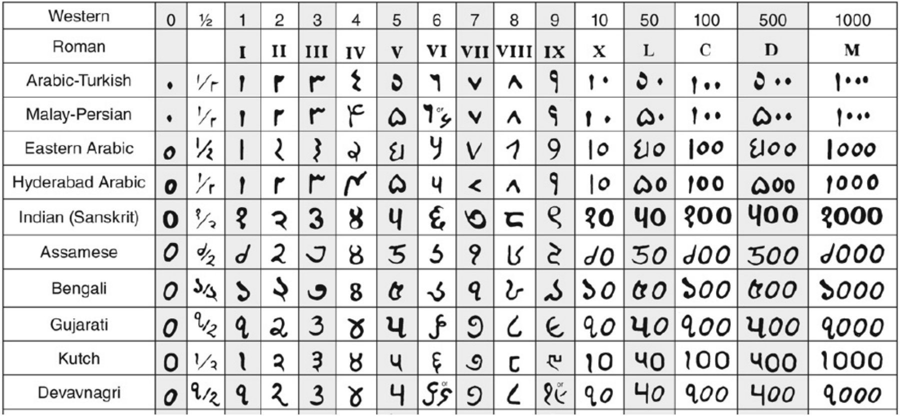
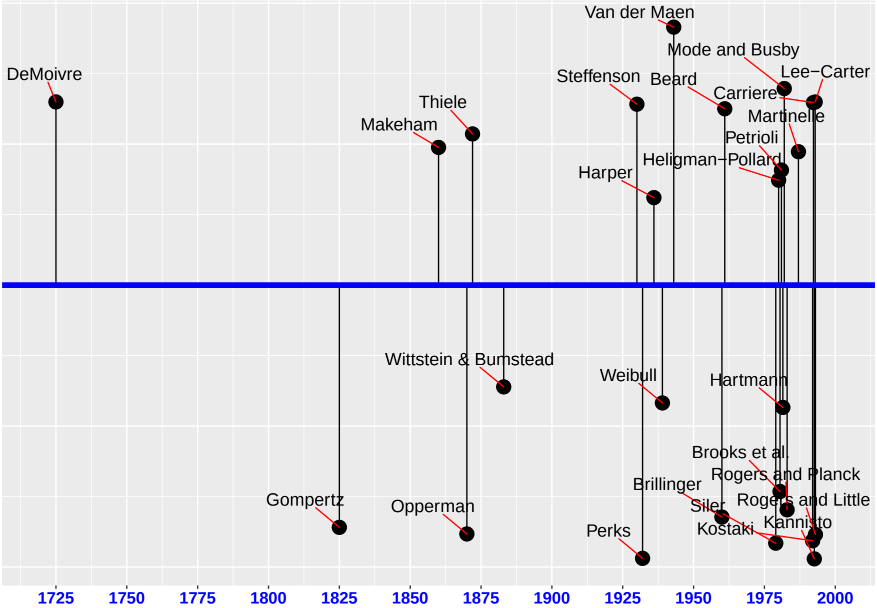


What is Modelling?

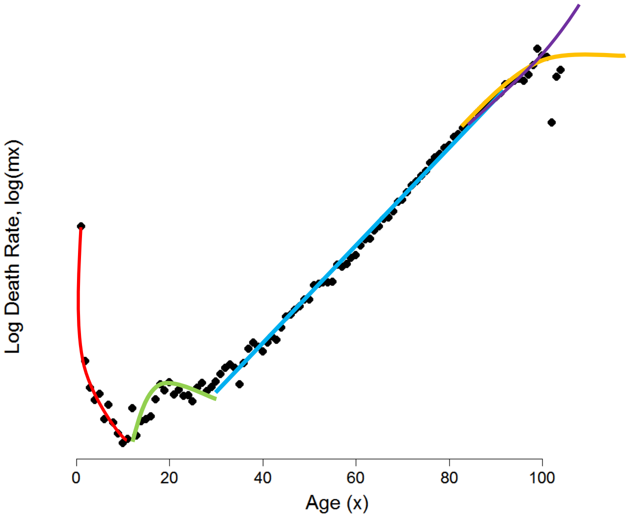
Mortality Modelling Timeline



Mortality models already implemented in the package

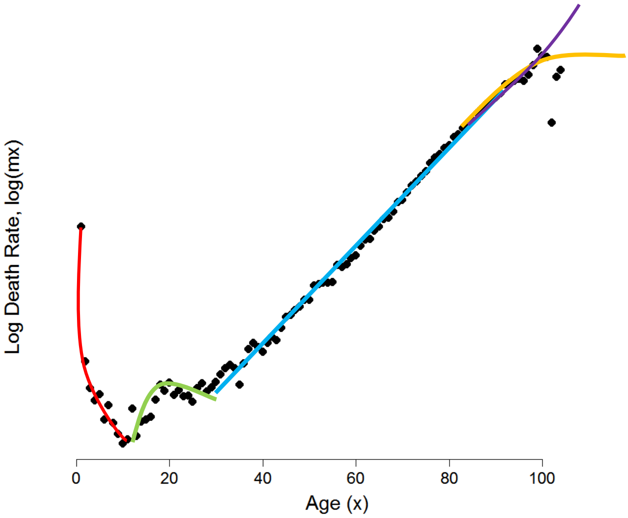
Mortality laws	Year	Predictor
DeMoivre	1725	$\frac{1}{(\omega - x)}$
Gompertz	1825	ae^{bx} or $\frac{1}{\sigma} exp \left\{ \frac{x-m}{\sigma} \right\}$
Makeham	1860	$ae^{bx} + c$
Opperman	1870	$\frac{a}{\sqrt{x}} + b + c\sqrt{x}$
Thiele	1872	$a_1 e^{-b_1 x} + a_2 e^{-\frac{1}{2} b_2 (x-c)^2} + a_3 e^{b_3 x}$
Wittstein & Bumstead	1883	$\frac{1}{m} a^{-(mx)^n} + a^{-(M-x)^n}$
Weibull	1939	$\frac{1}{\sigma} \left(\frac{x}{m} \right)^{\frac{m}{\sigma} - 1}$
...	...	...
Siler	1979	$a_1 e^{-b_1 t} + a_2 + a_3 e^{b_3 t}$
Heligman - Pollard	1980	$A^{(x+B)^C} + De^{-E(\ln x - \ln F)^2} + GH^x$
Kannisto	1998	$\frac{ae^{bx}}{1+ae^{bx}} + c$

Age Pattern of Human Mortality



Infant mortality
Accident hump
Adult mortality
Old-age mortality

Age Pattern of Human Mortality



Weibull
Opperman
Inverse-Gompertz
Gompertz
Makeham
Kannisto
Quadratic

Thiele
Wittstein
Heligman-Pollard
Carriere
...

Mortality Modelling & Forecasting in R

- **Demography** (Hyndman 2014)
  Lee-Carter model and several of its variants
- **ilc** (Butt, Haberman, and Shang 2014)
  Lee-Carter with cohorts and Lee-Carter under a Poisson framework
- **Lifemetrics** - open source R code (Cairns 2007)
  CBD and extensions
- **StMoMo** (Villegas et al. 2016)
  Stochastic Mortality Modelling (GLMs)
- **MortalityLaw** ...

- INSTALLATION from CRAN

```
install.packages(''MortalityLaws'')
```

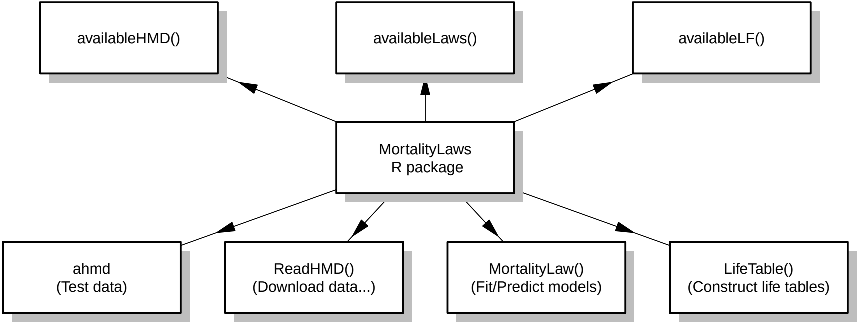
Make sure you have the most recent version of R and

- Repository & Development version on GitHub

```
https://github.com/mpascariu/MortalityLaws
```

- HELP
  All functions are documented in the standard way, which means that once you load the package using `library(MortalityLaws)` you can just type `?MortalityLaw` to see the help file.

Structure


availableHMD()
availableLaws()
availableLF()
MortalityLaws
R package
ahmd
(Test data)
ReadHMD()
(Download data...)
MortalityLaw()
(Fit/Predict models)
LifeTable()
(Construct life tables)

Model fitting: Data

Download data from HMD using `ReadHMD(...)` function

```
# Download life tables for female population
HMD_LT_f <- ReadHMD(what = "LT_f",
                    countries = "USA",
                    interval = "1x1",
                    username = username,
                    password = password,
                    save = FALSE)
HMD_LT_f
```

Model fitting: Data

```
  |++++++++++++++++++++++++++++++++++++++++++++++++++| 100% elapsed = 09s      :Downloading USA
HMD download completed!

> HMD_LT_f
Human Mortality Database (www.mortality.org)
Downloaded by: pascariu.marius@outlook.com
Download Date: Mon Oct 30 15:50:25 2017
Type of data: LT_f
Countries included: USA

Data:
     country Year Age     mx     qx   ax    lx   dx    Lx      Tx    ex
1        USA 1933   0 0.0545 0.0522 0.21 1e+05 5224 95850 6278342 62.78
2        USA 1933   1 0.0089 0.0088  0.5 94776  837 94357 6182492 65.23
3        USA 1933   2  0.004  0.004  0.5 93939  377 93750 6088135 64.81
4        USA 1933   3 0.0029 0.0029  0.5 93562  268 93428 5994385 64.07
5        USA 1933   4 0.0022 0.0022  0.5 93294  208 93190 5900957 63.25
...      <NA>  ...  ...    ...    ...  ...   ...  ...   ...     ...   ...
9209     USA 2015 106 0.5393 0.4248  0.5   255  108   201     446  1.75
9210     USA 2015 107 0.5708 0.4441  0.5   147   65   114     245  1.67
9211     USA 2015 108 0.6019 0.4626  0.5    82   38    63     131   1.6
9212     USA 2015 109 0.6321 0.4803  0.5    44   21    33      68  1.55
9213     USA 2015 110 0.6613      1 1.51    23   23    34      34  1.51
> |
```

Example: Heligman-Pollard Model - USA, 1990

$$q_x/p_x = A^{(x+B)^C} + De^{-E(\ln x - \ln F)^2} + GH^x$$

Fit the model using: `MortalityLaw(...)`

```r
fit.HP <- MortalityLaw(x    = 0:100,
                       Dx   = Dx1990,
                       Ex   = Ex1990,
                       law  = "HP",
                       opt.method = "LF2")
```

Check output object: `ls(...)`

```
> ls(fit.HP)
[1] "coefficients"    "fitted.values"    "goodness.of.fit"
[4] "info"            "input"            "opt.diagnosis"
[7] "residuals"
>
```

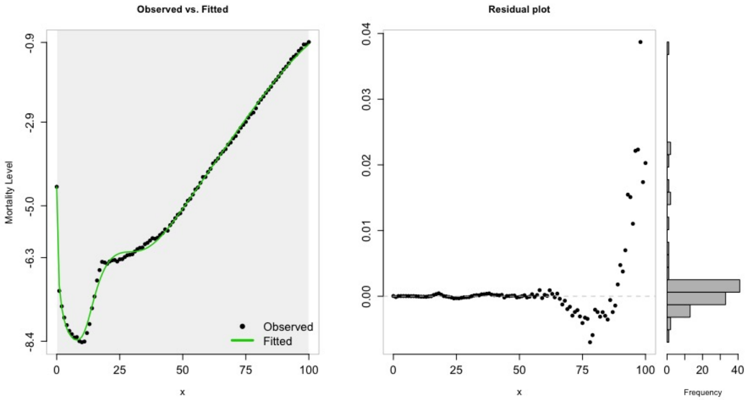
Example: Heligman-Pollard Model - USA, 1990

Summary : `summary(...)`

```
> summary(fit.HP)
Heligman-Pollard:
q[x]/p[x] = A^[(x + B)^C] + D exp[-E log(x/F)^2] + G H^x

Call: MortalityLaw(x = 0:100, Dx = Dx1990, Ex = Ex1990, law = "HP",
    opt.method = "LF2")

Deviance Residuals:
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-0.00700 -0.00029  0.00001  0.00122  0.00021  0.03871

Fitted values: mx
Coefficients:
       A        B        C        D        E       F_        G        H
 0.00087  0.03059  0.12718  0.00141  5.30002 24.50710  0.00006  1.09706
> |
```

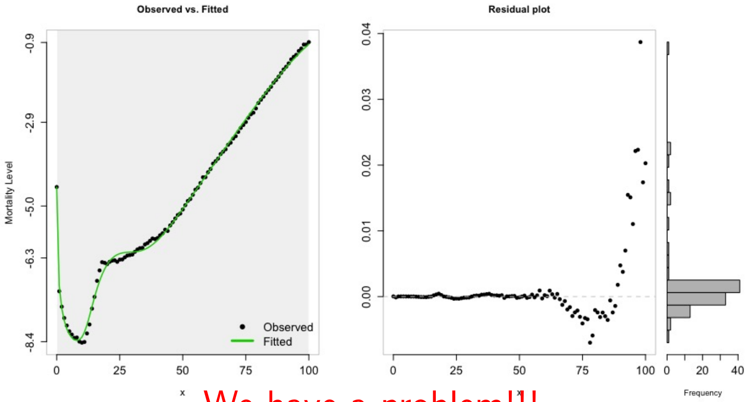
Example: Heligman-Pollard Model - USA, 1990

Generic plot function : `plot(...)`



Example: Heligman-Pollard Model - USA, 1990

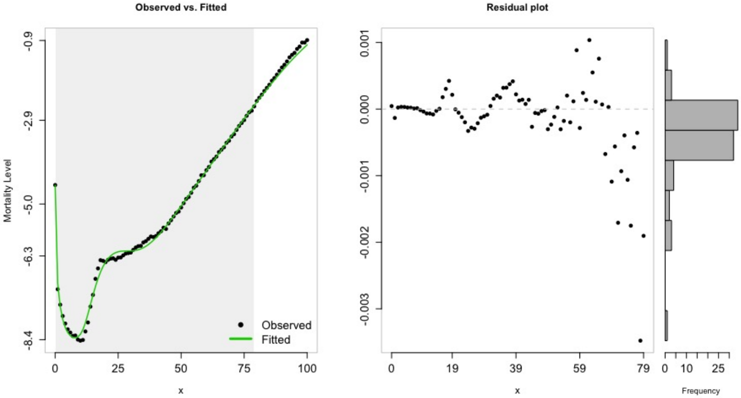
Generic plot function : `plot(...)`



We have a problem!!!

Model fitted on the 0-75 age-range

```
fit.HP2 <- MortalityLaw(x = 0:100, Dx = Dx1990, Ex = Ex1990, law = "HP", opt.method = "LF2",
                        fit.this.x = 0:75)
```



Objective or loss functions

Find parameter estimates by minimizing any of the functions below:

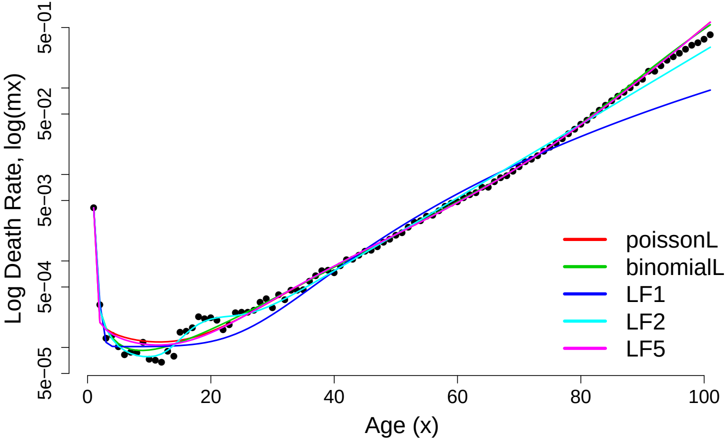
Name	Function
Poisson Log-Likelihood	$-\sum_x \{D_x \log \widehat{\mu}_x - E_x^c \widehat{\mu}_x\} + c$
Binomial Log-Likelihood	$-\sum_x \left\{D_x \log\left[1 - e^{-\widehat{\mu}_x}\right] - [E_x^c - D_x]\,\widehat{\mu}_x\right\} + c$
Loss Function 1 (LF1)	$\left(1 - \frac{\widehat{\mu}_x}{\mu_x}\right)^2$
LF2	$\log\left(\frac{\widehat{\mu}_x}{\mu_x}\right)^2$
LF3	$\frac{(\mu_x - \widehat{\mu}_x)^2}{\mu_x}$
LF4	$(\mu_x - \widehat{\mu}_x)^2$
LF5	$(\mu_x - \widehat{\mu}_x)\log\left(\frac{\mu_x}{\widehat{\mu}_x}\right)$
LF6	$\lvert\mu_x - \widehat{\mu}_x\rvert$

Optimization procedures

1. **Nelder-Mead** method - approximates a local optimum of a problem with n variables when the objective function varies smoothly and is unimodal. Implemented in **stats** R package, called in **optim** function. Nelder & Mead (1965)
2. **PORT routines** - provides unconstrained optimization and optimization subject to box constraints for complicated functions. See **nlminb** function, **stats** package.
3. **Levenberg-Marquardt** algorithm - **damped least-squares** method. Check **nls.lm** function in **minpack.lm**. Levenberg(1944); Marquardt(1963).

Heligman-Pollard applied to E&W 2010

$$q_x/p_x = A^{(x+B)^C} + De^{-E(\ln x - \ln F)^2} + GH^x$$

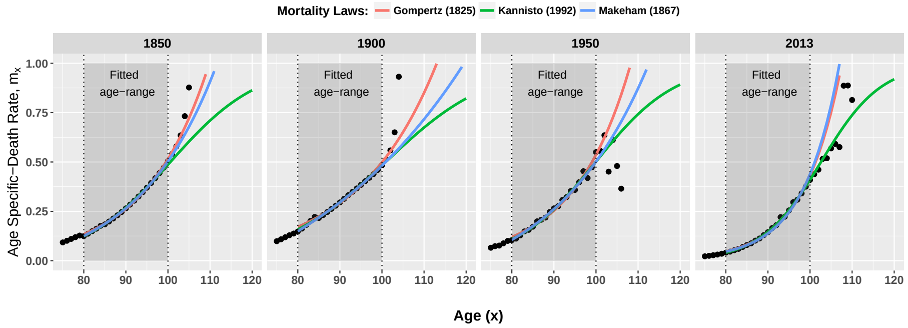


More mortality laws

Observed and fitted death rates between age 0 and 100 for female population in England & Wales

Old-age mortality

Observed and fitted old-age mortality for female population in England & Wales



1. Fitting parametric model
2. Smoothing data
3. Eliminating or/and reducing errors
4. Construct full/abridge life tables
5. Facilitate comparisons of mortality improvement
6. Forecasting

Reproducible research:

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