Problem set

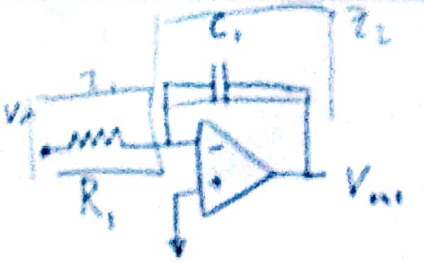
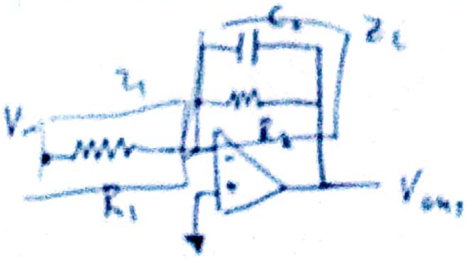
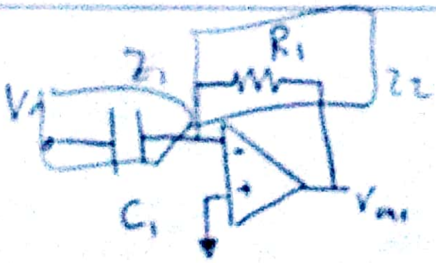
Analysis of filters with complex impedance

For each of the circuits in the table, figure out the complex number that represents the ratio of the output voltage divided by the input voltage. Write your final result in the tables. For each circuit in the table, create a plot of the magnitude of this complex number as a function of frequency. Put both the magnitude and frequency on a logscale. For each circuit use:

- $R1 = 1\ k\Omega$ $\quad C1 = 1\ \mu F$
- $R2 = 10\ k\Omega$ $\quad C2 = 0.1\ \mu F$

Remember, the impedance of a resistor and capacitor are

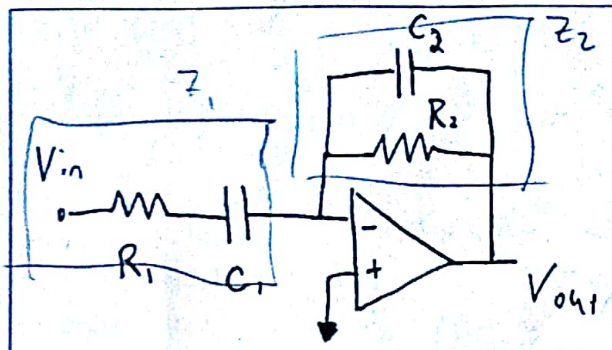
- $Z_R = R$
- $Z_C = \frac{1}{j\omega C}$

	$V_{out} = \frac{-Z_2}{Z_1}$ $\quad V_{out} = \frac{-1}{j\omega RC}$ $Z_2 = \frac{1}{j\omega C}$ $Z_1 = R_1$
	$V_{out} = \frac{-Z_2}{Z_1} = \left(-\frac{R_2}{R_1}\right)\left(\frac{j\omega C}{1+j\omega C}\right)$ $Z_1 = R_1$ $Z_2 = \frac{1}{1/R_2 + j\omega C} = \frac{R_2 j\omega C}{\frac{j\omega C + R_2}{R_2}}$
	$V_{out} = \frac{-Z_2}{Z_1}$ $\quad = -R_1 j\omega C$ $Z_1 = \frac{1}{j\omega C}$ $\quad V_{out} = -j\omega RC$ $Z_2 = R_1$

$\int dt \quad \int e^{j\omega t} dt$

$\frac{1}{j\omega} e^{j\omega t}$

$$\frac{R_2}{1+j\omega R_2C_2}\cdot\frac{j\omega C_1}{1+j\omega R_1C_1} \qquad \frac{1}{(1/R_2+j\omega C_2)(R_1+1/j\omega C_1)} \qquad \frac{R_2\, j\omega C_1}{(1+j\omega R_2C_2)(1+j\omega R_1C_1)}$$

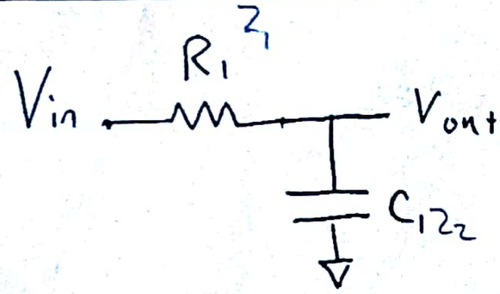


$$V_{out} = \frac{-Z_2}{Z_1} = -\frac{\frac{1}{1/R_2 + j\omega C_2}}{R_1 + 1/j\omega C_1}$$

$$Z_1 = R_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = \frac{1}{1/R_2 + j\omega C_2} = -\frac{R_2 j\omega C_2}{R_2 + j\omega C_2 R_2} = \left(\frac{-j\omega R_2C_2}{1+j\omega R_2C_2}\right)$$

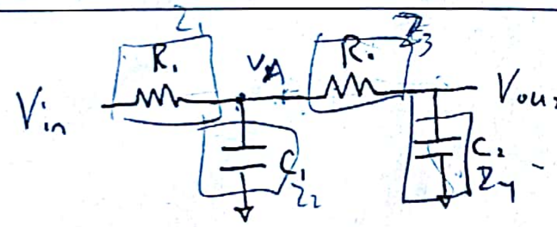
$$\left(\frac{-j\omega R_2C_2}{1+j\omega R_2C_2}\right)\left(\frac{j\omega R_1C_1}{R_1+1/j\omega R_1C_1}\right)$$



$$V_{out} = \frac{Z_2}{Z_1+Z_2} = \frac{1/j\omega C_1}{R_1 + 1/j\omega C_2}$$

$$Z_1 = R_1$$

$$Z_2 = \frac{1}{j\omega C_1} = \frac{1}{1+j\omega R_1C_1}$$



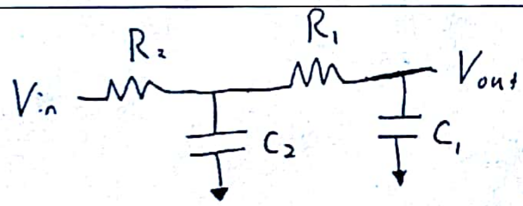
$$\frac{Z_2}{Z_1+Z_2} + Z_3$$

$$\frac{Z_3}{\frac{Z_2}{Z_1+Z_2} + Z_4}$$

$$V_A - V_{in} = Z_T I$$

$$\frac{V_A - V_{in}}{Z_1} = \frac{-V_A}{Z_2} + \frac{V_{out} - V_A}{Z_3}$$

$$\frac{-V_{out}}{Z_4} = \frac{V_{out} - V_A}{Z_3}$$



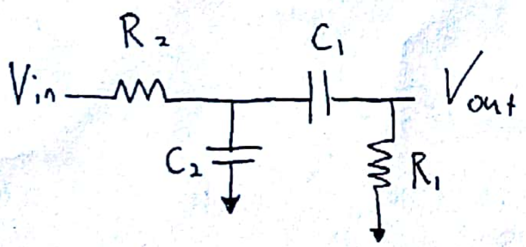
$$\frac{V_1}{V_{in}} = \frac{Z_2}{Z_1+Z_2}$$

$$\frac{V_{out}}{V_1} = \frac{Z_4}{Z_3+Z_4}$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_2 \cdot Z_4}{(Z_1+Z_2)(Z_3+Z_4)}$$

$$-V_{out}\frac{Z_3}{Z_4} = V_{out} - V_A$$

$$V_A = V_{out}(1 + Z_3/Z_4)$$



$$\frac{V_{out}(1+Z_3/Z_4) - V_{in}}{Z_1} = \frac{-V_{out}(1+Z_3/Z_4)}{Z_2} + \frac{V_{out} - V_{out}(1+Z_3/Z_4)}{Z_3}$$

$$V_{out}(1 - (1+Z_3/Z_4))$$

$$V_{out}(Z_3/Z_4)$$

$$\frac{V_{out}(1+Z_3/Z_4) - V_{in}}{Z_1} = \frac{-V_{out}(1+Z_3/Z_4)}{Z_2} - \frac{V_{out}}{Z_4}$$