Rodeo Algorithm and Adiabatic Quantum Computing for State Preparation of XX Heisenberg Model Hamiltonian

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Introduction

- We search for efficient ways to prepare specific states of a many-body quantum system on a quantum computer.
- Many methods exist, notably the Rodeo Algorithm [1], but have severe limitations.
- The Rodeo Algorithm has poor performance when the initial state overlap is low, hindering its applicability to the preparation of larger systems.

Model

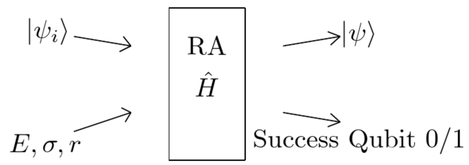
- Our model is the Heisenberg XX chain:

$$\hat{H} = -J\sum_{j=1}^{N}(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y).$$

- Conserves the sum of Pauli Z operators $\sum_{j=1}^{N}\sigma_j^z$
- Models a system of $n \leq N$ hard-core bosons occupying an N-site chain.
- We thus aim to minimize the energy over a subspace of the N-site wavefunctions:

Considering a system governed by the Hamiltonian above with N *sites, each possibly occupied by a spin-½ particle, prepare the minimal energy eigenstate with* n *particles.*

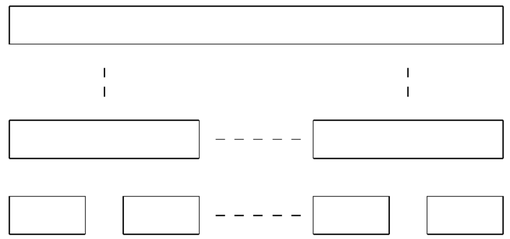
Rodeo Algorithm and Adiabatic Evolution



- Rodeo Algorithm (RA) suppresses eigenstates far from target energy.
- Works relatively fast given an initial state with a large overlap with the eigenstate corresponding to the target energy.
- Adiabatic Evolution (AE) slowly changes Hamiltonian to keep the state of the system in a given instantaneous eigenstate.
- Can be run for finite times, but only works perfectly as the runtime tends to infinity.
- Simulations of AE are done using TenPy [2], a Python implementation of tensor networks.
- RA implemented using Qiskit (See Poster # 46 by Onat Ayyildiz et. Al. for more details).

Fusion Technique

- Build many-body system from smaller blocks, that are easier to prepare.



- Fusion can be done with either RA or AE.
- Increases overlap between input state and target state.

Cost of Rodeo and Adiabatic Techniques

- Compare costs of using classic RA, adiabatic RA (ARA), and a fusional RA (FRA).
- The probability of success of a RA application with r cycles is given by:

$$P_r(E) = \sum_{\cdot} \frac{\left[1 + e^{-(E-E_i)^2\sigma^2/2}\right]^r |\langle E_i|\psi_I\rangle|^2}{2^r}.$$

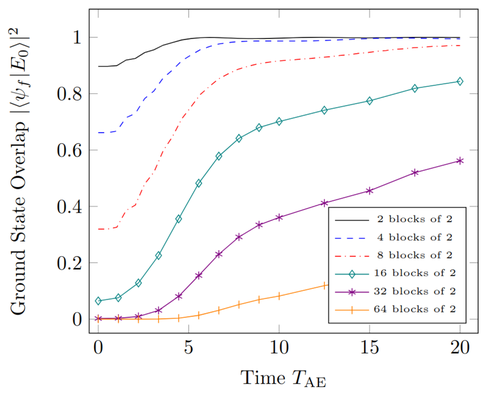
- Cost of RA is given by $c_{\mathrm{RA}} = \frac{r}{P_r(E)}$. and $T_{\mathrm{RA}} = c_{\mathrm{RA}}\sigma$.
- Assuming a perfect energy target guess and that the initial state's overlap with the target state is very close to 1, the number of rodeo cycles needed to reach within ε of the exact state will be given by:

$$r = \left\lceil \log_2\left(\frac{1-|\alpha|^2}{\epsilon}\right)\right\rceil$$

- Thus the total cost will simplify to: $c_{\mathrm{RA}} \approx \frac{\left\lceil \log_2\left(\frac{1-|\alpha|^2}{\epsilon}\right)\right\rceil}{|\alpha|^2}$.
- For the ARA, we have: $T_{\mathrm{ARA}} = c_{\mathrm{RA}}(\sigma + T_{\mathrm{AE}})$
- For the FRA,

$$T_{\mathrm{FRA}} = \sum_i \left[\sigma_i \prod_{j\geq i} c_j\right]$$

Pure Adiabatic Evolution with Fusional Method

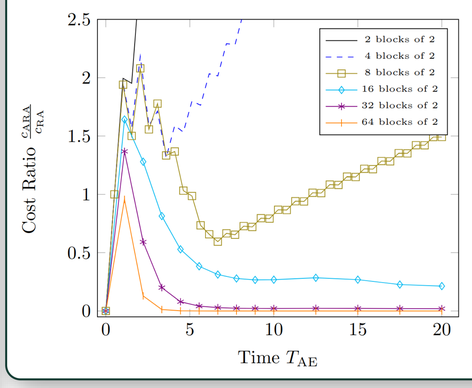


- One AE step, fusing n chains of 2 to a large chain of $2n$.
- Start with half-filled chains:

$$|\psi_i\rangle = \bigotimes_{i=1}^{N/2} |\psi_{N=2,n=1}\rangle$$

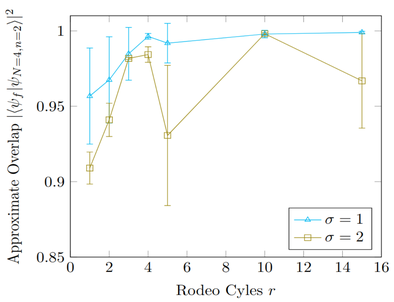
- Ground states found with DMRG.
- TEBD with $\delta t = 1$

Cost of Rodeo and Adiabatic Techniques combined



- Costs computed with approximation outlined earlier.
- Cost ratio below 1 indicates the ARA surpassing the RA performance.
- ARA quickly outperforms RA as more "bonds" are missing.

Going Past ARA: FRA and better?



Using verification RA to compute approximate overlap to fuse [2,2] into a chain of 4.

Verify ground state purity with a second RA application: probability of success of second RA ~ overlap with target state.

- Classical RA is very inefficient for larger systems once overlap gets small.
- For example, for a 128-qubit system, overlap is on the order 10^{-6}.
- Yet the FRA overcomes this limitation by ensuring the overlap is always quite high.
- Still needs to be tested, as numerous approximations are made, and it could still be possible to enhance FRA with AE.

Discussion and Outlook

- ARA clearly outperforms RA as more interaction terms are missing.
- FRA outperforms RA and possibly ARA in smaller systems.
- *Can AE be smartly applied in combination with RA to outperform FRA?*
- Compare RA, ARA, FRA methods with TenPy implementation of RA and scale for large systems.

Acknowledgments

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References

[1] K. Choi, D. Lee, J. Bonitati, Z. Qian, J. Watkins, Phys. Rev. Lett. **127**, 040505 (2021).
[2] J. Hauschild and F. Pollmann, SciPost Phys. Lect. Notes **5**, (2018).

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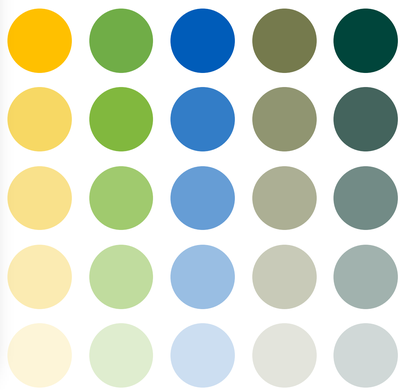

Primary color palette

Secondary color palette

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