Rodeo Algorithm for Calculating Ground State Energies of 2D Systems

Sirac O. Ayyildiz, Dean Lee, Matthew Patkowski, Jacob Watkins
Facility for Rare Isotope Beams (FRIB), Michigan State University, East Lansing, MI 48824 USA

Introduction

- Quantum computing holds promise in tackling many unsolved problems in quantum many-body physics.
- It allows for arbitrary linear combinations of tensor products of qubits.
- This capability enables the storage of exponentially more information compared to classical bits.
- Calculating the ground states and corresponding energies is crucial for understanding the properties of the system.
- This calculation for large systems is computationally costly.
- The Rodeo Algorithm [1] can be used to calculate the ground state energy by performing an energy sweep.
- We aim to maintain accuracy of the state by calculating the ground states of smaller subsystems.
- These smaller subsystems are then used to calculate the ground states of larger systems composed of these subsystems (block partitioning)

Background

- 2D Heisenberg Model:

$$H = c_{XX} \sum_{\langle \mathbf{n},\mathbf{n}' \rangle} X_{\mathbf{n}} X_{\mathbf{n}'} + c_{YY} \sum_{\langle \mathbf{n},\mathbf{n}' \rangle} Y_{\mathbf{n}} Y_{\mathbf{n}'} + c_{ZZ} \sum_{\langle \mathbf{n},\mathbf{n}' \rangle} Z_{\mathbf{n}} Z_{\mathbf{n}'} + h_Z \sum_{\mathbf{n}} Z_{\mathbf{n}},$$

Coupling coefficients; Magnetic field coefficient; Sum Pauli matrices over nearest neighbors on all sites; Sum of Pauli Z for all sites

- This model represents a 2D lattice of localized spins, which can be mapped onto the hard-core boson model using creation and annihilation operators.
- The Rodeo Algorithm [1] is a more efficient variation of the phase estimation algorithm [2].
- The system undergoes time evolution, controlled by the ancilla qubit.
- Controlled reversal gates are used to increase efficiency
  - Operators that anti-commute with time evolution operators allow for forward and backward time evolution.
  - Increases efficiency of numerical time evolution by at least a factor of 2.

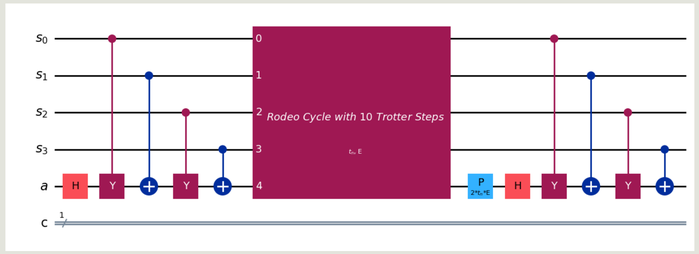
Methods

- Used second-order Trotter-Suzuki Decomposition to approximate the time evolution of the Hamiltonian.
- Implement Rodeo Algorithm repeatedly within a target energy range.
- Measure ancilla qubit after each cycle.
- Evaluate the probability that the ancilla qubit is in the $|0\rangle$ state.

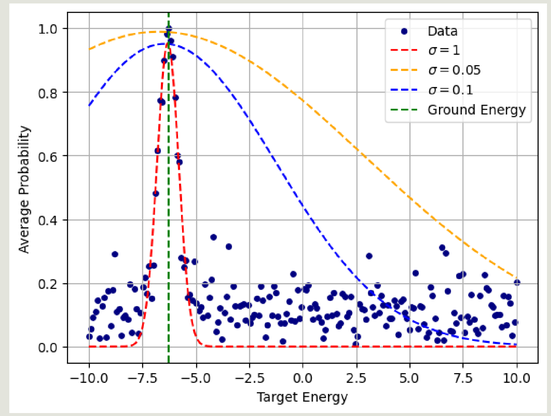
$$P_{0^N}(E|\{t_k\}) = \prod_{k=1}^{N} \cos^2\left[\frac{t_k}{2}(E_{\rm obj} - E)\right]$$

Probability of 0 over all cycles; Time sample; Target Energy; Eigenvalue of Hamiltonian

- Centered around the peaks from the first scan, perform a second scan with larger σ for the time sampling, to increase the resolution.
- Compare results with classically computed eigenvalues using tensor networks for larger more complex systems.
  - Decompose the global Hilbert space into local tensors.
  - Perform operations on smaller subsystems.
  - Density Matrix Renormalization Group (DMRG) optimizes the MPS representation to find ground states efficiently.



Circuit Diagram of Rodeo Algorithm One cycle of the Rodeo Algorithm applied to a system initialized to be $|\Psi\rangle$. The ancilla qubit is initialized into the $|0\rangle$ state. The time evolution and phase get depend t_n, which are sampled from a Gaussian distribution with a standard deviation σ.



Energy Sweep for Rodeo Algorithm Results for the ground state of a 2x2 lattice, using 3 Rodeo Cycles. When the target energy is more than 1/ σ away from the eigenvalues, the possibility of measuring the system in the $|0\rangle$ state is 1/2, thus the background value for the probability of success is 0.125.

Future Goals

- Expand the use of the Rodeo Algorithm to evaluate ground states of larger 2D systems by:
  - Splitting systems into subsystems and simultaneously evaluating the ground state for each subsystem
  - Building up the larger system using the smaller subsystem
- Explore whether quantum adiabatic evolution will help in achieving ground states of larger systems/building up larger systems from smaller ones
- Simplify and make the model more efficient, observing how it will affect the accuracy of results.
- Explore the advantages of quantum computers in this computation over classical methods such as tensor networks.

Acknowledgements

We acknowledge funding from the Department of Energy grant DE-SC0023658 and National Science Foundation grant PHY-2310620.

References

[1] Choi, Kenneth, et al. “Rodeo Algorithm for Quantum Computing.” Physical Review Letters, 23 July 2021, https://doi.org/10.1103/physrevlett.127.040505.
[2] Kitaev, A, and L Landau. Quantum Measurements and the Abelian Stabilizer Problem. 1995.
[3] Qian, Zhengrong, et al. “Demonstration of the Rodeo Algorithm on a Quantum Computer.” The European Physical Journal A, 20 July 2024, https://doi.org/10.1140/epja/s10050-024-01373-9.
[4] Ostmeyer, Johann. “Optimised Trotter Decompositions for Classical and Quantum Computing.” Journal of Physics A: Mathematical and Theoretical, 26 June 2023, https://doi.org/10.1088/1751-8121/acde7a.

FRIB FACILITY FOR RARE ISOTOPE BEAMS

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics and used resources of the Facility for Rare Isotope Beams (FRIB) Operations, which is a DOE Office of Science User Facility under Award Number DE-SC0023633.

U.S. DEPARTMENT OF ENERGY | Office of Science

MICHIGAN STATE UNIVERSITY