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1. $xy' + 3y = 2x^5 \quad y(2) = 1$

$y' + \frac{3}{x}y = 2x^4$

$I(x) = e^{\int 3/x \, dx} = e^{3\ln x} = e^{\ln x^3} = x^3$

$x^3y' + 3x^2y = 2x^7$

$(x^3y)' = 2x^7$

$x^3y = \frac{1}{4}x^8 + C$

$y = \frac{1}{4}x^5 + \frac{C}{x^3}$

$1 = \frac{32}{4} + \frac{C}{8}$

$1 = 8 + \frac{C}{8}$

$8 = 64 + C$

$-56 = C$

$y = \frac{1}{4}x^5 - \frac{56}{x^3}$

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2. $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 & 0 \\ 3 & 2 & 0 \\ -7 & -5 & 3 \end{bmatrix}$

$\det(A) = \begin{vmatrix} -2 & 2 \\ 0 & 2 \end{vmatrix} = -2(2) - 2(0) = -4$

$C = B^T$ $\det(C) = \det(B)$ $\det(C) = -1 \begin{vmatrix} 2 & 0 \\ -5 & 3 \end{vmatrix}$

$= -(2(3) - 0(-5)) = -6$

$\det(A^T B^{-1} A) = \det(A^T)\det(B^{-1})\det(A)$

$\det(A^T) = \det(A) = -4$

$\det(B^{-1}) = 1/\det(B) = -1/6$

$\det(A^T B^{-1} A) = (-4)(-1/6)(-4) = -8/3$

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3. $x_1 + 2x_2 + 7x_3 - 9x_4 + 31x_5 = 0$
$2x_1 + 4x_2 + 7x_3 - 11x_4 + 13x_5 = 0$
$3x_1 + 6x_2 + 5x_3 - 11x_4 - 19x_5 = 0$

$$\begin{bmatrix} 1 & 2 & 7 & -9 & 31 \\ 2 & 4 & 7 & -11 & 13 \\ 3 & 6 & 5 & -11 & -19 \end{bmatrix} \xrightarrow[R_3 - 3R_1 \to R_3]{R_2 - 2R_1 \to R_2} \begin{bmatrix} 1 & 2 & 7 & -9 & 31 \\ 0 & 0 & -7 & 7 & -49 \\ 0 & 0 & -16 & 16 & -112 \end{bmatrix}$$

$$\xrightarrow[-\frac{1}{7}R_2 \to R_2]{} \begin{bmatrix} 1 & 2 & 7 & -9 & 31 \\ 0 & 0 & 1 & -1 & 7 \\ 0 & 0 & -16 & 16 & -112 \end{bmatrix} \xrightarrow[R_3 + 16R_2 \to R_3]{R_1 - 7R_2 \to R_1} \begin{bmatrix} 1 & 2 & 0 & -2 & -18 \\ 0 & 0 & 1 & -1 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 + 2x_2 - 2x_4 - 18x_5 = 0$
$x_1 = -2x_2 + 2x_4 + 18x_5$
$x_1 = -2r + 2s + 18t$

$x_3 - x_4 + 7x_5 = 0$
$x_3 = x_4 - 7x_5$
$x_3 = s - 7t$

$(-2r + 2s + 18t, r, s - 7t, s, t)$
$= r(-2, 1, 0, 0, 0) + s(2, 0, 1, 1, 0) + t(18, 0, -7, 0, 1)$

A basis for the solution space is
$\{(-2,1,0,0,0), (2,0,1,1,0), (18,0,-7,0,1)\}$.

The dimension of the solution space is 3

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4. $y^{(4)} - 3y''' + 3y'' - 3y' + 2y = 0$ $\quad y(0)=4 \quad y'(0)=-1 \quad y''(0)=-3 \quad y'''(0)=-3$

$r^4 - 3r^3 + 3r^2 - 3r + 2 = 0$

$(r^2+1)(r-1)(r-2)=0$

$r^2+1=0 \qquad r=1 \qquad r=2$

$r^2=-1$

$r=\pm 1$

$r_1=1,\ r_2=2,\ r_3=-i,\ r_4=i$

$y = C_1e^x + C_2e^{2x} + \cancel{e^x}(C_3\cos x + C_4\sin x)$

$-C_3\sin x + C_4\cos x$

$y' = C_1e^x + 2C_2e^{2x} \cancel{+e^x(C_3\cos x + C_4\sin x - C_3\sin x + C_4\cos x)}$

$-C_3\cos x - C_4\sin x$

$y'' = C_1e^x + 4C_2e^{2x} \cancel{+e^x(2C_4\cos x - 2C_3\sin x)}$

$+C_3\sin x - C_4\cos x$

$y''' = C_1e^x + 8C_2e^{2x} \cancel{+e^x(2C_4\cos x - 2C_4\sin x - 2C_3\cos x - 2C_3\sin x)}$

$4 = C_1 + C_2 + C_3 \qquad -1 = C_1 + 2C_2 \cancel{+C_3} + C_4$

$-3 = C_1 + 4C_2 \cancel{+2C_4} - C_3 \qquad -3 = C_1 + 8C_2 \cancel{-2C_3 + 2C_4} - C_4$

$$\cancel{\begin{bmatrix} 1 & 1 & 1 & 0 & 4 \\ 1 & 2 & 1 & 1 & -1 \\ 1 & 4 & 0 & 2 & -3 \\ 1 & 8 & -2 & 2 & -3 \end{bmatrix}} \xrightarrow{\text{RREF}} \cancel{\begin{bmatrix} 1 & 0 & 0 & 0 & 13 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}}$$

$\cancel{C_1 = 13 \quad C_2 = -3 \quad C_3 = -6 \quad C_4 = -2}$

$\cancel{y = 13e^x - 3e^{2x} + e^x(-6\cos x - 2\sin x)}$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 4 \\ 1 & 2 & 0 & 1 & -1 \\ 1 & 4 & -1 & 0 & -3 \\ 1 & 8 & 0 & -1 & -3 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \quad \begin{matrix} C_1=3 \\ C_2=-1 \\ C_3=2 \\ C_4=-2 \end{matrix}$$

$$y = 3e^x - e^{2x} + 2\cos x - 2\sin x$$

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5. $A = \begin{bmatrix} -3 & 1 & -2 \\ -10 & 4 & -6 \\ -2 & 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\det(A - I\lambda) = \begin{vmatrix} -3-\lambda & 1 & -2 \\ -10 & 4-\lambda & -6 \\ -2 & 1 & -1-\lambda \end{vmatrix} = -\lambda^3 + \lambda = 0$

$-\lambda(\lambda^2 - 1) = 0$

$-\lambda(\lambda+1)(\lambda-1) = 0$

$\lambda_1 = -1 \quad \lambda_2 = 1 \quad \lambda_3 = 0$

$\lambda = -1$
$A + I = \begin{bmatrix} -2 & 1 & -2 \\ -10 & 5 & -6 \\ -2 & 1 & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $x = t/2 \quad z = 0$

$(t/2, t, 0)$

$(t/2)(1, 2, 0)$

$\vec{v}_1 = (1, 2, 0)$

$\lambda = 1$
$A - I = \begin{bmatrix} -4 & 1 & -2 \\ -10 & 3 & -6 \\ -2 & 1 & -2 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ $x = 0, \quad y = 2t$

$(0, 2t, t) = t(0, 2, 1)$ $\vec{v}_2 = (0, 2, 1)$

$\lambda = 0$
$A = \begin{bmatrix} -3 & 1 & -2 \\ -10 & 4 & -6 \\ -2 & 1 & -1 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $x = -t \quad y = -t$

$(-t, -t, t) = t(-1, -1, 1)$

$\vec{v}_3 = (-1, -1, 1)$

$X^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 2R_1 \to R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$

$\xrightarrow{R2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & -2 & 1 & 0 \end{array}\right] \xrightarrow[R_3 - 2R_2 \to R_3]{} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 & 1 & -2 \end{array}\right]$

$\xrightarrow[-R_3 \to R_3]{} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 & 2 \end{array}\right] \xrightarrow{\substack{R_1 + R_3 \to R_1 \\ R_2 - R_3 \to R_2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 2 \\ 0 & 1 & 0 & -2 & 1 & -1 \\ 0 & 0 & 1 & 2 & -1 & 2 \end{array}\right]$

$$X = \begin{bmatrix} 3 & -1 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 2 \end{bmatrix}$$

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6. $\vec{x}' = \begin{bmatrix} -2 & -2 \\ 2 & -6 \end{bmatrix} \vec{x}$ $\quad \vec{x}(0) = (1,0)$

$A = \begin{bmatrix} -2 & -2 \\ 2 & -6 \end{bmatrix}$ $\quad \det(A - \lambda I) = \begin{vmatrix} -2-\lambda & -2 \\ 2 & -6-\lambda \end{vmatrix} = 0$

$\lambda^2 + 8\lambda + 16 = 0$

$(\lambda + 4)^2 = 0$

$\lambda_1 = \lambda_2 = -4$

$\lambda = -4$

$A + 4I = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ $\quad x = t$

$(t,t) = t(1,1)$

$\vec{v}_1 = (1,1)$

$\vec{v}_2 = (1,0)$

$\vec{v}_1 = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \cancel{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

$\vec{x}(t) = C_1 e^{-4t} \overset{(2,2)}{\cancel{(1,1)}} + C_2 e^{-4t} \left(\overset{(2,2)}{\cancel{(1,1)}} + t(1,0) \right)$

$(1,0) = C_1 \overset{(2,2)}{\cancel{(1,1)}} + C_2 (1,0)$

$(1,0) = C_2 (1,0) \quad C_1 = 0 \quad C_2 = 1$

$\vec{x}(t) = e^{-4t} \left(\overset{(2,2)}{\cancel{(1,1)}} + t(1,0) \right)$

$\vec{x}(t) = e^{-4t} \left((2,2) + t(1,0) \right)$

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7. $A=\begin{bmatrix}-3 & 7 & 2\\ -2 & 6 & 2\\ -3 & 3 & 2\end{bmatrix}$

$$\det(A-\lambda I)=\begin{vmatrix}-3-\lambda & 7 & 2\\ -2 & 6-\lambda & 2\\ -3 & 3 & 2-\lambda\end{vmatrix}=0$$

$$-\lambda^3+5\lambda^2-2\lambda-8=0$$

$$-(\lambda-4)(\lambda-2)(\lambda+1)=0$$

$$\lambda_1=-1,\quad \lambda_2=2,\quad \lambda_3=4$$

$\lambda=-1$

$$A+I=\begin{bmatrix}-2 & 7 & 2\\ -2 & 7 & 2\\ -3 & 3 & 3\end{bmatrix}\xrightarrow{RREF}\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}\quad x=t\quad y=0$$

$(t,0,t)=t(1,0,1)$

$\vec{v}_1=(1,0,1)$

$\lambda=2$

$$A-2I=\begin{bmatrix}-5 & 7 & 2\\ -2 & 4 & 2\\ -3 & 3 & 0\end{bmatrix}\xrightarrow{RREF}\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 0 & 0 & 0\end{bmatrix}\quad x=-t\quad y=-t$$

$(-t,-t,t)=-t(1,1,-1)$

$\vec{v}_2=(1,1,-1)$

$\lambda=4$

$$A-4I=\begin{bmatrix}-7 & 7 & 2\\ -2 & 2 & 2\\ -3 & 3 & -2\end{bmatrix}\xrightarrow{RREF}\begin{bmatrix}1 & -1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\quad x=t\quad z=0$$

$(t,t,0)=t(1,1,0)$

$\vec{v}_3=(1,1,0)$

a)

$$\cancel{\Phi(t)=\begin{bmatrix}e^{-t} & 0 & e^{-t}\\ e^{2t} & e^{2t} & -e^{2t}\\ e^{4t} & e^{4t} & 0\end{bmatrix}^T}\qquad \cancel{\Phi(0)^{-1}=\begin{bmatrix}1/2 & 1/2 & 0\\ -1/2 & -1/2 & 1\\ 1/2 & -1/2 & 0\end{bmatrix}}$$

$$\Phi(t)=\begin{bmatrix}e^{-t} & e^{2t} & e^{4t}\\ 0 & e^{2t} & e^{4t}\\ e^{-t} & -e^{2t} & 0\end{bmatrix}$$

$$\Phi(0)=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 1 & -1 & 0\end{bmatrix}\quad \Phi(0)^{-1}=\begin{bmatrix}1 & -1 & 0\\ 1 & -1 & -1\\ -1 & 2 & 1\end{bmatrix}$$

b)

$$\cancel{e^{At}=\Phi(t)\Phi(0)^{-1}=\begin{bmatrix}e^{-t} & 0 & e^{-t}\\ e^{2t} & e^{2t} & -e^{2t}\\ e^{4t} & e^{4t} & 0\end{bmatrix}\begin{bmatrix}-\frac{1}{2} & \frac{1}{2} & 0\\ -\frac{1}{2} & -\frac{1}{2} & 1\\ -\frac{1}{2} & -\frac{1}{2} & 0\end{bmatrix}}$$

$$\cancel{=\begin{bmatrix}0 & 0 & 0\\ \frac{1}{2}e^{2t} & \frac{1}{2}e^{2t} & e^{2t}\\ 0 & 0 & e^{4t}\end{bmatrix}}$$

$$\begin{bmatrix}e^{-t}+e^{2t}-e^{4t} & -e^{-t}-e^{2t}+2e^{4t} & -e^{2t}+e^{4t}\\ e^{2t}-e^{4t} & -e^{2t}+2e^{4t} & -e^{2t}+e^{4t}\\ e^{-t}-e^{2t} & -e^{-t}+2e^{2t} & e^{2t}\end{bmatrix}$$

$$\Phi(t)\Phi(0)^{-1}=\begin{bmatrix}e^{-t} & e^{2t} & e^{4t}\\ 0 & e^{2t} & e^{4t}\\ e^{-t} & e^{-2t} & 0\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 1 & -1 & -1\\ -1 & 2 & 1\end{bmatrix}=\begin{bmatrix}\cancel{e^{-t}+e^{2t}-e^{4t}} & \cancel{-e^{-t}-e^{2t}} & \\ \cancel{e^{2t}-e^{4t}} & & \\ \cancel{e^{-t}-e^{2t}} & & \end{bmatrix}$$

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8. $y'' + y = \cos t \quad y(0) = -1 \quad y'(0) = 0$

$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{\cos t\}$

$s^2 Y - sy(0) - y'(0) + Y = \frac{s}{s^2+1}$

$s^2 Y + s + Y = \frac{s}{s^2+1}$

$Y(s^2+1) + s = \frac{s}{s^2+1}$

$Y(s^2+1) = \frac{s}{s^2+1} - s$

$Y = \left(\frac{s}{s^2+1}\right)\left(\frac{1}{s^2+1}\right) - \frac{s}{s^2+1}$

$Y = \left(\frac{s}{s^2+1}\right)\left(\frac{1}{s^2+1}\right) - \frac{s}{s^2+1}$

$\frac{s}{(s^2+1)^2} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+1}$

$s = (s^2+1)(As+B+Cs+D)$

$A = A+C \quad B = B+D$

$s = (s^2+1)(As+B)$

$s = As^3 + Bs^2 + As + B$

no

$\mathcal{L}^{-1}(Y) = \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) * \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$

$y = \cos t * \sin t - \cos t$

$y = \int_0^t \cos\tau \sin(t-\tau)\, d\tau - \cos t$

$y = \frac{t}{2}\sin t - \cos t$

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9. $(2x+1)y' + 3y = 0$

$y = \sum_{n=0}^{\infty} c_n x^n \qquad y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$

$(2x+1)\sum_{n=1}^{\infty} n c_n x^{n-1} + 3\sum_{n=0}^{\infty} c_n x^n = 0$

$\sum_{n=1}^{\infty} 2n c_n x^n + \sum_{n=1}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} 3 c_n x^n = 0$

$\sum_{n=0}^{\infty} 2n c_n x^n + \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n + \sum_{n=0}^{\infty} 3 c_n x^n = 0$

For $n \geq 0$, $2n c_n + (n+1)c_{n+1} + 3c_n = 0$

$c_{n+1} = \frac{-2nc_n - 3c_n}{n+1}$

$c_{n+1} = -\frac{(2n+3)c_n}{n+1}$

$c_1 = -\frac{3c_0}{1} = -3c_0$

$c_2 = -\frac{5c_1}{2} = \frac{15}{2}c_0$

$c_3 = -\frac{7c_2}{3} = -\frac{35}{2}c_0$

$c_4 = -\frac{9c_3}{4} = \frac{315}{8}c_0$ ← not needed, first 4 nonzero

$y = c_0\left(1 + \frac{15}{2}x - \frac{35}{2}x^3 + \frac{315}{8}x^4 + \cdots\right)$

c_1?

$y = c_0\left(1 - 3x + \frac{15}{2}x^2 - \frac{35}{2}x^3 + \cdots\right)$

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10. $y' = x + 3y \qquad y(1) = 3$

$y_1 = 3 + \int_1^x (t + 9)\,dt = 3 + \left[\frac{t^2}{2} + 9t\right]_1^x$

$= 3 + \frac{x^2}{2} + 9x - \frac{1}{2} - 9 = \frac{1}{2}x^2 + 9x - \frac{13}{2}$

$y_2 = 3 + \int_1^x \left(t + 3\left(\frac{1}{2}t^2 + 9t - \frac{13}{2}\right)\right)dt$

$= 3 + \int_1^x \left(\frac{3}{2}t^2 + 28t - \frac{39}{2}\right)dt$

$= 3 + \left[\frac{1}{2}t^3 + 14t^2 - \frac{39}{2}t\right]_1^x$

$= 3 + \frac{1}{2}x^3 + 14x^2 - \frac{39}{2}x - \frac{1}{2} - 14 + \frac{39}{2}$

$= \frac{1}{2}x^3 + 14x^2 - \frac{39}{2}x + 8$

$y_3 = 3 + \int_1^x \left(t + 3\left(\frac{1}{2}t^3 + 14t^2 - \frac{39}{2}t + 8\right)\right)dt$

$= 3 + \int_1^x \left(\frac{3}{2}t^3 + 42t^2 - \frac{115}{2}t + 24\right)dt$

$= 3 + \left[\frac{3}{8}t^4 + 14t^3 - \frac{115}{4}t^2 + 24t\right]_1^x$

$= 3 + \frac{3}{8}x^4 + 14x^3 - \frac{115}{4}x^2 + 24x - \frac{3}{8} - 14 + \frac{115}{4} - 24$

$= \frac{3}{8}x^4 + 14x^3 - \frac{115}{4}x^2 + 24x - \cancel{\frac{1}{2}} - \frac{53}{8}$

$y_4 = 3 + \int_1^x \left(t + 3\left(\frac{3}{8}t^4 + 14t^3 - \frac{115}{4}t^2 \overset{+24t}{\cancel{9t}} - \cancel{\frac{13}{2}}\frac{53}{8}\right)\right)dt$

$= 3 + \int_1^x \left(\frac{9}{8}t^4 + 42t^3 - \frac{345}{4}t^2 \overset{+73t}{\cancel{-26t}} - \frac{159}{8}\cancel{\frac{39}{2}}\right)dt$

$= 3 + \left[\frac{9}{40}t^5 + \frac{21}{2}t^4 - \frac{115}{4}t^3 \overset{+\frac{73}{2}t^2 - \frac{159}{8}t}{\cancel{-13t^2 - \frac{39}{2}t}}\right]_1^t$

$= 3 + \frac{9}{40}t^5 + \frac{21}{2}t^4 - \frac{115}{4}t^3 \overset{+\frac{73}{2}t^2 - \frac{159}{8}t}{\cancel{-13t^2 - \frac{39}{2}t}} - \frac{9}{40} - \frac{21}{2} + \frac{115}{4} \overset{-\frac{73}{2} + \frac{159}{8}}{\cancel{+13 + \frac{39}{2}}}$

$= \frac{9}{40}t^5 + \frac{21}{2}t^4 - \frac{115}{4}t^3 \cancel{-13t^2 + \frac{201}{8}t + \frac{2140}{40}}$

$x \quad x \quad x \quad + \frac{73}{2}x^2 - \frac{159}{8}x + \frac{22}{5}$

$Y_4 = \frac{9}{40}x^5 + \frac{21}{2}x^4 - \frac{115}{4}x^3 + \frac{73}{2}x^2 - \frac{159}{8}x + \frac{22}{5}$

Estimated Grade

1	10/10	
2	10/10	
3	10/10	
4	6/10	Treated $r = \pm bi$ as $r = a \pm bi$
5	8/10	Used calculator for det and RREF
6	8/10	Used defective eigenvector (1,1) instead of (2,2)
7	6/10	Used $\Phi(t)^T$ as $\Phi(t)$
8	10/10	
9	9/10	Skipped c_1
10	~~8/10~~ 7/10	Two arithmetical errors compounded, used + instead of × at end

Total 84/100

Final Grade: ≈ 79% (B)