Optimal Virtual Machine Placement across Multiple Cloud Providers

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***Abstract*—Cloud computing provides users an efficient way to dynamically allocate computing resources to meet demands. Cloud providers can offer users two payment plans, i.e., reservation and on-demand plans for resource provisioning. Price of resources in reservation plan is generally cheaper than that in on-demand plan. However, since the reservation plan has to be acquired in advance, it may not fully meet future demands in which the on-demand plan can be used to guarantee the availability to the user. In this paper, we propose an optimal virtual machine placement (OVMP) algorithm. This algorithm can minimize the cost spending in each plan for hosting virtual machines in a multiple cloud provider environment under future demand and price uncertainty. OVMP algorithm makes a decision based on the optimal solution of stochastic integer programming (SIP) to rent resources from cloud providers. The performance of OVMP algorithm is evaluated by numerical studies and simulation. The results clearly show that the proposed OVMP algorithm can minimize users' budgets. This algorithm can be applied to provision resources in emerging cloud computing environments.**

I. Introduction

Cloud computing is a large scale distributed computing paradigm in which a pool of computing resources are available to the users via the Internet [2]. Computing resources, e.g., storage, computing power, platform, and software, are represented to users as accessible services. Infrastructure-as-a-Service (IaaS) is a computational service model applied in the cloud computing paradigm [3]. Virtualization technologies can be used to support computing resource access by the users in this model. Users can specify required software stack such as operating systems, software libraries, and applications; then package them all together into virtual machines (VMs). Finally, VMs will be hosted in a computing environment operated by third-party sites that we call *cloud providers*.

Cloud providers can offer customers two payment plans, i.e., reservation plan (e.g., pre-paid) and on-demand plan (e.g., pay-per-use). Amazon EC2 [5] and GoGrid [6] are, for instances, the cloud providers which provide IaaS services and offer reservation and on-demand plans to the customers. Generally, price of resources in reservation plan is cheaper than that in on-demand plan. However, customers need to subscribe a certain amount of resources in reservation plan in advance for future usage. As a result, an underprovisioning problem can occur when the amount of reserved resources is unable to fully meet the demands. Fortunately, this problem can be solved by subscribing resources in on-demand plan to fit the extra demands. However, such on-demand resources are more costly, , and the corresponding cost is called *on-demand cost*. An overprovisioning problem cannot be overlooked as well since the amount of reserved resources will be underutilized. The cost of idle reserved resource is generally referred to as *overprovisioning* or *oversubscribed cost*. Both on-demand and oversubscribed costs need to be minimized.

Both underprovisioning and overprovisioning problems of resource management in cloud computing environment are our motivation to explore an optimal strategy for the users. In this paper, we propose an optimal virtual machine placement (OVMP) algorithm to minimize the total cost due to buying reservation and on-demand plans of resource provisioning. With IaaS model, OVMP algorithm makes a decision to host a certain number of VMs on appropriate cloud providers. Uncertainty of future demands and prices of resources is taken into account to optimally adjust the tradeoff between on-demand and oversubscribed costs. The decision made by OVMP algorithm is obtained as the optimal solution from stochastic integer programming (SIP) formulation with two-stage recourse [23]. Extensive numerical studies and simulation in cloud computing environment are performed to evaluate the effectiveness of OVMP algorithm. The results show that OVMP algorithm can minimize the total cost, while requirements of both providers and customers are met.

II. Related Work

In cloud computing environments, the resources can be leased and rented as public utility such as gas, water, and electricity [1]. Three major computational service models were mentioned in [2], [3] i.e., Software-as-a-Service (SaaS), Platform-as-a-Service (PaaS), and Infrastructure-as-a-Service (IaaS). IaaS model has been identified as the most potential model for cloud computing. IaaS uses virtualization technologies (e.g., Xen hypervisor [4]) to efficiently manage computing workload. Amazon EC2 [5], GoGrid [6], Flexiscale [7], and Redplaid [8] are cloud providers which offer the services under IaaS model. Some free or open-source software for implementing IaaS are also available [9]–[12].

As virtualization is a core of cloud computing, the problem of VM placement becomes crucial [13]–[16]. This VM placement is usually relevant to server consolidation [17]. In [13], the broker-based architecture and algorithm for assigning VMs to physical servers were developed. In [14], techniques of VM placement and consolidation which leverage *min-max* and *shares* features provided by hypervisors were explored. In [15], a dynamic consolidation mechanism based

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on constraint programming was developed. Their consolidation was originally designed for homogeneous clusters. However, heterogeneity is common in a multiple cloud provider environment. Moreover, [13]–[15] did not consider uncertainty of future demands and prices. In [16], a dynamic VM placement was proposed. However, the placement in [16] is a heuristic which is not guaranteed to be optimal to minimize the cost.

Resource provisioning options were introduced in [18]. Resource provisioning strategies in distributed systems were addressed in [19]–[22]. In [19], a probabilistic advance reservation was proposed. This reservation relies on existing best effort batch schedulers which again cannot be guaranteed to be optimal. In [20], a concept of resource slot was proposed. The objective is to tackle uncertainty of resources availability. In [21], a binary integer program to maximize resource providers' revenues and utilization was formulated. Heuristic methods to solve this binary integer program were proposed. However, [19]–[21] did not consider uncertainty of future users' demands. In [22], an optimization framework for resource provisioning was developed. This framework considered multiple client QoS classes under uncertainty of workloads (e.g., demands of computing resources). The arrival pattern of workloads is estimated by using online forecasting techniques. In contrast, our work specifies that demands given probability distributions. Finally, these related works ignored uncertainty of future demands and prices. In addition, the price difference between reservation and on-demand plans was not taken into account.

Stochastic programming [23] has been developed to solve resource planning under uncertainty. Stochastic programming was applied to solve many problems in different fields, e.g., production planning, financial management, and capacity planning. However, to the best of our knowledge, the application of stochastic programming to computing resource provisioning has never been studied.

III. System Model and Assumption

As depicted in Fig. 1, the system model of cloud computing environment consists of four components, i.e., user, virtual machine (VM) repository, cloud providers, and cloud broker. The user has demands to host VMs[1]. Let $\mathcal{V} = \{V_1, V_2, \dots, V_{\text{last}}\}$ denote the set of VM classes. Let V_{last} denote the last VM class. A single VM class represents a distinct type of applications. For example, V_1 is a mail server and V_2 is a web server. For ease of management, the VM from the same owner can be identified into the same class. The number of VMs in each class can be different, depending on the demand from the user.

Let $\mathcal{P} = \{P_1, P_2, \dots, P_{\text{last}}\}$ denote the set of cloud providers. Let P_{last} denote the last cloud provider. Each cloud provider supplies a pool of resources to the user. In this work, the cloud provider supplies computing power (in a unit of CPU-hours), storage (in a unit of gigabytes or GBs), network bandwidth with regard to Internet data transfer (in a unit of GBs/day), and electric power (in a unit of kilowatt hour or kWh). For the rest of the paper, the variables with superscript of the letter inside parentheses, i.e., (h), (s), (n), and (e) correspond to the computing power, storage, network bandwidth, and electric power, respectively. Let $t_j^{(\mathrm{h})}$, $t_j^{(\mathrm{s})}$, and $t_j^{(\mathrm{n})}$ denote the maximum capacity of corresponding resource which cloud provider P_j can supply to the user. Let $r_i^{(\mathrm{h})}$, $r_i^{(\mathrm{s})}$, and $r_i^{(\mathrm{n})}$ denote the amount of corresponding resource required by a single VM under class V_i. Electric power consumed by a single VM of class V_i hosted by cloud provider P_j is defined as a function $\mathscr{W}_j(r_i^{(\mathrm{h})})$. Every cloud provider prepares facilities, i.e., virtual network switch, batch scheduler, and load balancer, to support hosted VMs of the user.

[1]For the rest of this paper, the terms *number of required virtual machines* and *demand* are interchangeable.

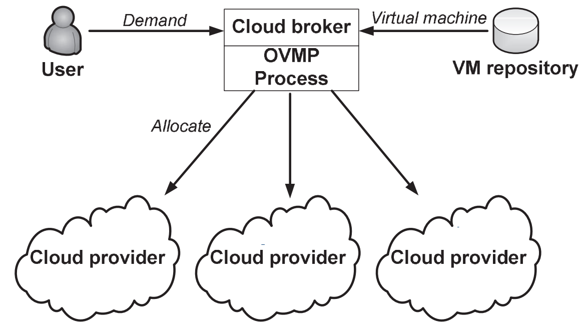

Fig. 1. Cloud computing environment.

The cloud broker (Fig. 1) is a centralized entity (e.g., server) located in the user's site. The broker is responsible for delivering VMs, stored in repository, to cloud providers. Furthermore, the OVMP algorithm (Fig. 1) is implemented in the broker. This algorithm is used to make optimal decision for the broker to reserve resources from and host VMs to any cloud providers.

A. Payment Plans and Provisioning Phases

We assume that every cloud provider offers the user two payment plans, i.e., reservation and on-demand plans. Cloud providers offer the price of resources which will be charged to the user when the resources are reserved or utilized. Price to provision resources in reservation plan is assumed to be cheaper than that in on-demand plan.

There are three phases of provisioning resources: reservation, utilization, and on-demand. First in the reservation phase, without knowing user's demand, the cloud broker provisions resources in the reservation plan. Then, the utilization phase starts when the reserved resources are used. However, if the demand exceeds the amount of reserved resources, the user can pay for additional resources in the on-demand plan, and then the on-demand phase starts. Based on the three phases, there are three costs associated with provisioning resources: reservation, utilization, and on-demand. Note that for the same resource, a sum of reservation and utilization costs is generally less than an on-demand cost. The objective of the cloud broker is to minimize all above costs while the demand of users is met.

As aforementioned, the cloud broker uses OVMP to obtain an optimal solution. In fact, the optimal solution is to reserve the optimal number of resources in the reservation phase. An optimal solution is obtained by solving and formulating a stochastic integer programming with two-stage recourse (discussed in Section IV). There are two stages of decision making: first stage and second stage. The *first stage* defines the number of VMs provisioned in reservation phase, while the *second stage* or *recourse* defines the number of VMs allocated in both utilization and on-demand phases. In other words, the second stage represents the actual number of VMs required by the user and actual prices defined by providers.

For cloud provider, price is defined in dollars ($) per unit of resource (e.g., $ per CPU-hour for computing power). For cloud provider P_j, let $c_j^{(\mathrm{h})}$, $c_j^{(\mathrm{s})}$, $c_j^{(\mathrm{n})}$, and $c_j^{(\mathrm{e})}$ denote the prices (i.e., costs to the user) of corresponding resources in reservation phase. Then, let $\overline{c}_j^{(\mathrm{h})}$, $\overline{c}_j^{(\mathrm{s})}$, $\overline{c}_j^{(\mathrm{n})}$, and $\overline{c}_j^{(\mathrm{e})}$ denote the costs of corresponding resource in utilization phase. With price uncertainty, costs of resources in on-demand phase can be random. This issue is discussed in Subsection III-B.

B. Uncertainty of Demands and Prices

Under uncertainty of demands, the number of required VMs by the user is not exactly known when the reservation of resource is made. Let $\mathcal{D}_i = \{d_{i1}, d_{i2}, \ldots, d_{i|\mathcal{V}|}\}$ denote the set of possible required numbers of VMs in class V_i. The set of all possible required number of VMs $\mathcal{D}$ in all classes can be obtained from the Cartesian product as follows:

$$\mathcal{D} = \prod_{V_i \in \mathcal{V}} \mathcal{D}_i = \mathcal{D}_1 \times \mathcal{D}_2 \times \cdots \times \mathcal{D}_{|\mathcal{V}|}. \tag{1}$$

Similarly, the price of resources in on-demand plan could be random. Let $\tilde{\mathcal{C}}_j^{(\mathrm{h})}$, $\tilde{\mathcal{C}}_j^{(\mathrm{s})}$, $\tilde{\mathcal{C}}_j^{(\mathrm{n})}$, and $\tilde{\mathcal{C}}_j^{(\mathrm{e})}$ denote the sets of possible prices of corresponding resource offered by cloud provider P_j in on-demand phase. Again, the set of all possible prices from provider P_j can be obtained from

$$\tilde{\mathcal{C}}_j = \tilde{\mathcal{C}}_j^{(\mathrm{h})} \times \tilde{\mathcal{C}}_j^{(\mathrm{s})} \times \tilde{\mathcal{C}}_j^{(\mathrm{n})} \times \tilde{\mathcal{C}}_j^{(\mathrm{e})}. \tag{2}$$

We assume that probability distributions for both demands in $\mathcal{D}$ and prices in $\tilde{\mathcal{C}}_j$ of all providers are known. These distributions can be obtained by using a statistical process to analyze historical data.

IV. Problem Formulation

A. Deterministic Integer Programming

If the number of required VMs in all classes is exactly known, all VMs can be subscribed for the certain amount of resources in reservation plan. Therefore, the on-demand resource is not needed, and on-demand cost is zero. A deterministic integer program can be formulated and solved for this VM provisioning as follows:

$$\text{Minimize:} \quad \sum_{V_i \in \mathcal{V}} \sum_{P_j \in \mathcal{P}} c_{ij}\ X_{ij}^{(\mathrm{r})} \tag{3}$$

$$\text{Subject to:} \quad \sum_{P_j \in \mathcal{P}} X_{ij}^{(\mathrm{r})} = v_i, \quad V_i \in \mathcal{V} \tag{4}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{h})}\ X_{ij}^{(\mathrm{r})} \leq t_j^{(\mathrm{h})}, \quad P_j \in \mathcal{P} \tag{5}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{s})}\ X_{ij}^{(\mathrm{r})} \leq t_j^{(\mathrm{s})}, \quad P_j \in \mathcal{P} \tag{6}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{n})}\ X_{ij}^{(\mathrm{r})} \leq t_j^{(\mathrm{n})}, \quad P_j \in \mathcal{P} \tag{7}$$

$$X_{ij}^{(\mathrm{r})} \in \{0, 1, \ldots\},\ V_i \in \mathcal{V},\ P_j \in \mathcal{P}. \tag{8}$$

Objective function in (3) is to minimize the total cost due to resource reservation. Decision variable $X_{ij}^{(\mathrm{r})}$ denotes the number of VMs in class V_i, allocated to provider P_j. c_{ij} denotes the cost, in reservation phase, charged by provider P_j for hosting one VM from class V_i. This cost c_{ij} can be defined as follows:

$$c_{ij} = c_j^{(\mathrm{h})} r_i^{(\mathrm{h})} + c_j^{(\mathrm{s})} r_i^{(\mathrm{s})} + c_j^{(\mathrm{n})} r_i^{(\mathrm{n})} + c_j^{(\mathrm{e})} \mathscr{W}_j(r_i^{(\mathrm{h})}). \tag{9}$$

The constraint in (4) ensures that the demand is met where v_i is the number of required VMs in class V_i. In (5) $-$ (7), the allocation of VMs for each class V_i must not exceed the resource capacity offered by provider P_j. The last constraint in (8) indicates that decision variable $X_{ij}^{(\mathrm{r})}$ takes a value from a set of non-negative integer number.

B. Stochastic Integer Programming

If demands and prices cannot be known precisely, the deterministic optimization formulation defined in (3) $-$ (8) is no longer applicable. Therefore, stochastic integer programming (SIP) with two-stage recourse is developed. The first stage defines the number of VMs provisioned in reservation phase, while the second stage defines the number of VMs allocated in both utilization and on-demand phases. In other words, the second stage represents the actual number of VMs required by the user and actual prices defined by providers. A SIP formulation can be expressed as follows:

$$\text{Minimize:} \quad \sum_{V_i \in \mathcal{V}} \sum_{P_j \in \mathcal{P}} c_{ij}\ X_{ij}^{(\mathrm{r})} + \mathscr{E}_\Omega[\mathscr{Q}(X_{ij}^{(\mathrm{r})}, \omega)] \tag{10}$$

$$\text{Subject to:} \quad (8).$$

The objective function (10) is to minimize the user's budget to allocate VMs in both stages or equivalently in all provisioning phases. Unlike (3), the objective function in (10) concerns both reservation and on-demand plans. The variable $X_{ij}^{(\mathrm{r})}$ denotes the number of VMs provisioned in the first stage. The expected cost in the second stage is defined as function $\mathscr{E}_\Omega[\mathscr{Q}(X_{ij}^{(\mathrm{r})}, \omega)]$, where $\omega \in \Omega = \mathcal{D} \times \prod_{P_j \in \mathcal{P}} \tilde{\mathcal{C}}_j$ denotes the set of possible demands and prices (called *realizations*, in general) observed in the second stage. For a given realization ω, the cost function $\mathscr{Q}(X_{ij}, \omega)$ can be expressed as follows:

$$\mathscr{Q}(X_{ij}^{(\mathrm{r})}, \omega) = \min_{Y=(X_{ij}^{(\mathrm{u})}(\omega), X_{ij}^{(\mathrm{o})}(\omega))} \mathscr{C}(Y) \tag{11}$$

where $Y \in \Upsilon(X_{ij}^{(\mathrm{r})}, \omega)$. In (11), composite variable Y consists of two variables, i.e., $X_{ij}^{(\mathrm{u})}(\omega)$ and $X_{ij}^{(\mathrm{o})}(\omega)$, which denote the number of VMs in class V_i allocated to provider P_j in utilization phase and on-demand phase, respectively. $\mathscr{C}(\cdot)$ denotes the cost function in the second stage. Set $\Upsilon(X_{ij}^{(\mathrm{r})}, \omega)$ controls the relationship between the numbers of VMs allocated in the first and second stages under the constraints as expressed in the following extended formulation:

Minimize: (10)

Subject to: (8)

$$X_{ij}^{(\mathrm{u})}(\omega) \le X_{ij}^{(\mathrm{r})}, \ V_i \in \mathcal{V}, \ P_j \in \mathcal{P} \tag{12}$$

$$\sum_{P_j \in \mathcal{P}} (X_{ij}^{(\mathrm{u})}(\omega) + X_{ij}^{(\mathrm{o})}(\omega)) \ge v_i(\omega), \ V_i \in \mathcal{V} \tag{13}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{h})}(X_{ij}^{(\mathrm{u})}(\omega) + X_{ij}^{(\mathrm{o})}(\omega)) \le t_j^{(\mathrm{h})}, P_j \in \mathcal{P} \tag{14}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{s})}(X_{ij}^{(\mathrm{u})}(\omega) + X_{ij}^{(\mathrm{o})}(\omega)) \le t_j^{(\mathrm{s})}, P_j \in \mathcal{P} \tag{15}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{n})}(X_{ij}^{(\mathrm{u})}(\omega) + X_{ij}^{(\mathrm{o})}(\omega)) \le t_j^{(\mathrm{n})}, P_j \in \mathcal{P}. \tag{16}$$

The constraint in (12) ensures that the number of VMs in utilization phase does not exceed that in reservation phase. The demand under a realization ω is governed by the constraint in (13), where $v_i(\omega)$ denotes the number of required VMs in the second stage. The constraints in (14) $-$ (16) ensure that the allocation of VMs in class V_i must be bounded by the resource capacity offered by provider P_j.

The SIP formulation of OVMP algorithm defined in (10), (8), (12) $-$ (16) can be transformed into a deterministic integer program called *deterministic equivalent* formulation by introducing two variables, i.e., $X_{ij}^{(\mathrm{u})}(m, d)$ and $X_{ij}^{(\mathrm{o})}(m, d)$. The deterministic equivalent SIP of OVMP algorithm is expressed in (17) $-$ (23). When demand $d \in \mathcal{D}$ (i.e., $\mathcal{D}$ is defined in (1)) and prices $m \in \tilde{\mathcal{C}}_j$ (i.e., $\tilde{\mathcal{C}}_j$ is defined in (2)) are realized, $X_{ij}^{(\mathrm{u})}(m, d)$ and $X_{ij}^{(\mathrm{o})}(m, d)$ denote the number of VMs allocated in utilization phase and on-demand phase, respectively. Both of them define recourse cost after d and m are observed.

The objective function in (17) indicates the user's budget to be minimized. c_{ij} in (9) is the cost of resources in the first stage. In the second stage, $\bar{c}_{ij}$ denotes the cost of resources charged by provider P_j to host VM in class V_i in utilization phase. In on-demand phase, $\tilde{c}_{ij}(m)$ denotes the cost of resources charged by provider P_j if prices $m = (\tilde{c}_j^{(\mathrm{h})}, \tilde{c}_j^{(\mathrm{s})}, \tilde{c}_j^{(\mathrm{n})}, \tilde{c}_j^{(\mathrm{e})}) \in \tilde{\mathcal{C}}_j$ are realized. Both costs are defined as follows:

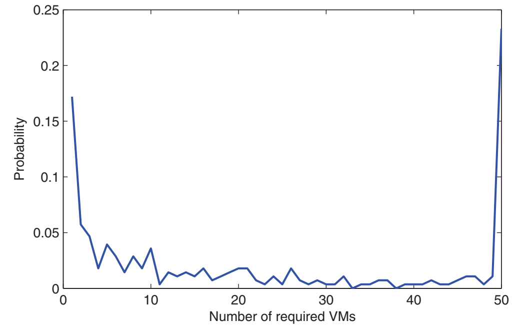

Fig. 2. Probability distribution of the test data.

$$\begin{aligned} \bar{c}_{ij} &= \bar{c}_j^{(\mathrm{h})} r_i^{(\mathrm{h})} + \bar{c}_j^{(\mathrm{s})} r_i^{(\mathrm{s})} \\ &\quad + \bar{c}_j^{(\mathrm{n})} r_i^{(\mathrm{n})} + \bar{c}_j^{(\mathrm{e})} \mathscr{W}_j(r_i^{(\mathrm{h})}) \end{aligned} \tag{24}$$

$$\begin{aligned} \tilde{c}_{ij}(m) &= \tilde{c}_j^{(\mathrm{h})} r_i^{(\mathrm{h})} + \tilde{c}_j^{(\mathrm{s})} r_i^{(\mathrm{s})} \\ &\quad + \tilde{c}_j^{(\mathrm{n})} r_i^{(\mathrm{n})} + \tilde{c}_j^{(\mathrm{e})} \mathscr{W}_j(r_i^{(\mathrm{h})}). \end{aligned} \tag{25}$$

In (17), there are two probabilities, i.e., $p(d)$ and $p_j(m)$. $p(d)$ denotes the probability if demand $d \in \mathcal{D}$ is realized. $p_j(m)$ denotes the probability if prices $m \in \tilde{\mathcal{C}}_j$ offered by provider P_j are realized. All constraints in (18) $-$ (23) consider both realizations of d and m. The constraints in (18) and (19) are respectively the same as those in (12) and (13). In (19), the VM allocation of the second stage is governed by $v_i(d)$, where $v_i(d)$ denotes the number of required VMs in class V_i if demand d is realized. The constraints in (20), (21), and (22) are equivalent to those in (14), (15), and (16), respectively. The constraint in (23) indicates that all decision variables takes the values from a set of non-negative integer number.

V. Performance Evaluation

A. Parameter Setting

We consider the system model of cloud computing as shown in Fig. 1 which consists of four cloud providers and three VM classes. The required number of VMs is assumed to be the same for all VM classes. The number of VMs required by each class is varied from 1 to 50 i.e., $\mathcal{D}_i = \{1, 2, \ldots, 50\}$. To simplify the evaluation, demand $\mathcal{D}$ is redefined by $\acute{\mathcal{D}}$ as follows:

$$\begin{aligned} \acute{\mathcal{D}} = \{ \ (d_1, \ d_2, \ d_3) \ \mid \ & d_1 \in \mathcal{D}_1, \ d_2 \in \mathcal{D}_2, \ d_3 \in \mathcal{D}_3 \\ & , \ d_1 = d_2 = d_3 \ \}. \end{aligned} \tag{26}$$

For demand $\mathcal{D}$, three probability distributions, i.e., normal distribution, uniform distribution, and distribution from test data are used in the evaluation to study an effect of uncertainty to the decision of OVMP algorithm and to the cost of user. Means of both normal and uniform distributions are set to 25.50. The variance of normal distribution is set to 12. The test data is obtained from Institute of High Performance Computing (IHPC) in Singapore [26]. The data is collected

$$\text{Minimize:} \quad \sum_{V_i \in \mathcal{V}} \sum_{P_j \in \mathcal{P}} c_{ij}\ X_{ij}^{(\mathrm{r})} + \sum_{V_i \in \mathcal{V}} \sum_{P_j \in \mathcal{P}} \sum_{m \in \mathcal{C}_j} \sum_{d \in \mathcal{D}} p_j(m)\ p(d) \left(\bar{c}_{ij}\ X_{ij}^{(\mathrm{u})}(m,d) + \tilde{c}_{ij}(m)\ X_{ij}^{(\mathrm{o})}(m,d) \right) \tag{17}$$

$$\text{Subject to:} \quad X_{ij}^{(\mathrm{u})}(m,d) \le X_{ij}^{(\mathrm{r})}, \quad V_i \in \mathcal{V},\ P_j \in \mathcal{P},\ m \in \mathcal{C}_j,\ d \in \mathcal{D} \tag{18}$$

$$\sum_{P_j \in \mathcal{P}} (X_{ij}^{(\mathrm{u})}(m,d) + X_{ij}^{(\mathrm{o})}(m,d)) \ge v_i(d), \quad V_i \in \mathcal{V},\ m \in \mathcal{C}_j,\ d \in \mathcal{D} \tag{19}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{h})}(X_{ij}^{(\mathrm{u})}(m,d) + X_{ij}^{(\mathrm{o})}(m,d)) \le t_j^{(\mathrm{h})}, \quad P_j \in \mathcal{P},\ m \in \mathcal{C}_j,\ d \in \mathcal{D} \tag{20}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{s})}(X_{ij}^{(\mathrm{u})}(m,d) + X_{ij}^{(\mathrm{o})}(m,d)) \le t_j^{(\mathrm{s})}, \quad P_j \in \mathcal{P},\ m \in \mathcal{C}_j,\ d \in \mathcal{D} \tag{21}$$

$$\sum_{V_i \in \mathcal{V}} r_i^{(\mathrm{n})}(X_{ij}^{(\mathrm{u})}(m,d) + X_{ij}^{(\mathrm{o})}(m,d)) \le t_j^{(\mathrm{n})}, \quad P_j \in \mathcal{P},\ m \in \mathcal{C}_j,\ d \in \mathcal{D} \tag{22}$$

$$X_{ij}^{(\mathrm{r})}, X_{ij}^{(\mathrm{u})}(m,d), X_{ij}^{(\mathrm{o})}(m,d) \in \{0,1,\dots\}, \quad V_i \in \mathcal{V},\ P_j \in \mathcal{P},\ m \in \mathcal{C}_j,\ d \in \mathcal{D} \tag{23}$$

TABLE I
PRICES OF RESOURCES IN EACH PROVISIONING PHASE DEFINED BY EACH CLOUD PROVIDER.

Provider	Computing Power			Storage			Network Bandwidth			Electric Power		
	R	U	O	R	U	O	R	U	O	R	U	O
P_1	0.20	0.10	0.40	0.30	0.20	0.60	0.15	0.05	0.25	0.10	0.10	0.20
P_2	0.20	0.10	0.50	0.30	0.20	0.70	0.15	0.05	0.30	0.10	0.10	0.20
P_3	0.10	0.10	0.50	0.20	0.10	0.70	0.10	0.05	0.30	0.10	0.10	0.20
P_4	0.10	0.10	0.60	0.20	0.10	0.80	0.10	0.05	0.35	0.10	0.10	0.20

from the actual usage of computing resources located in three clusters. We synthesize the data to fit our evaluation. After the synthesis, the mean of test data is 21.64. The probability distribution of test data is shown in Fig. 2.

Price of resources in on-demand plan is varied by the coefficients. Two coefficients are used to define two possible values of prices (i.e., $|\tilde{\mathcal{C}}_j| = 2$). The first coefficient indicates the normal price in on-demand plan (i.e., $\tilde{c}_{ij}(m)$). The other coefficient indicates the price of electric power resource with respect to the normal price. In this case, the prices of other resources remain constant. The probability to realize the first and the second coefficients are 0.7 and 0.3, respectively.

Computing power required by each VM in classes V_1, V_2, and V_3 is 12, 18, 24 CPU-hours, respectively. The storage required by VM in classes V_1, V_2, and V_3 is 20, 5, 10 GBs, respectively. The network bandwidth required by VM in classes V_1, V_2, and V_3 is 33.33, 66.67, 266.67 MBs/day, respectively. Cloud providers P_1 to P_4 offer the same maximum capacities of storage and network bandwidth which are 1,000 GBs and 6.67 GBs/day, respectively. The maximum capacity of computing power offered by providers P_1 and P_2 is 480 CPU-hours, while providers P_3 and P_4 offer 1,200 CPU-hours. The cost of power consumption is assumed to be $\mathscr{W}_j(r_i^{(\mathrm{h})}) = r_i^{(\mathrm{h})}$. The prices of resources in each payment plan are varied as defined in Table I. The three letters, i.e., R, U, and O, in the column titles indicate the prices in reservation phase, utilization phase, and on-demand phase, respectively.

B. Numerical Studies

1) Cost Structure: First, the cost structure is studied. To ease the illustration, a simple cloud computing environment is considered which consists of only one class V_1 and one provider P_4. The numbers of required VMs (i.e., demands) are random according to normal distribution. In Fig. 3, costs in the first and second stages, and total cost under different number of reserved VMs are presented. As expected, cost in the first stage increases, as the number of reserved VMs increases. However, cost in the second stage after knowing the actual demand decreases, as the number of reserved VMs increases since the user needs smaller number of on-demand VMs. In this case, the optimal solution can be determined (e.g., 31 reserved VMs as shown in Fig. 3). Clearly, even in this simple cloud computing environment, the optimal solution is not trivial to obtain due to uncertainty of demands. For example, the optimal solution is not the point where the cost in the second stage passes cost in the first stage. Therefore, the

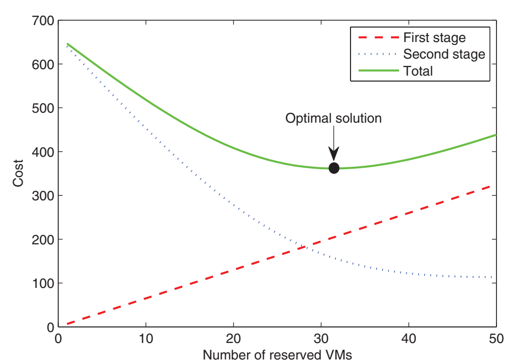

Fig. 3. The optimal solution in a simple cloud computing environment.

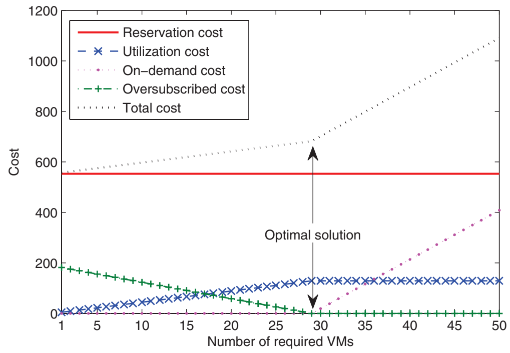

Fig. 4. Cost in different phases and with different demands.

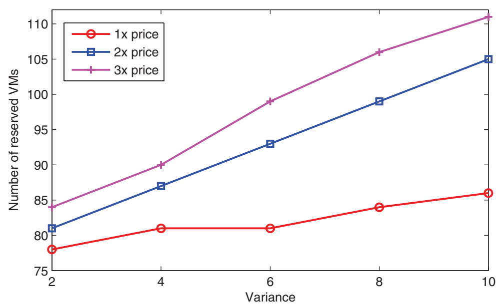

Fig. 5. Total cost under different variances and prices.

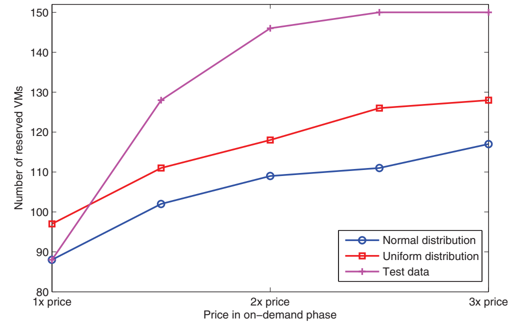

Fig. 6. Number of reserved VMs under different distributions and prices.

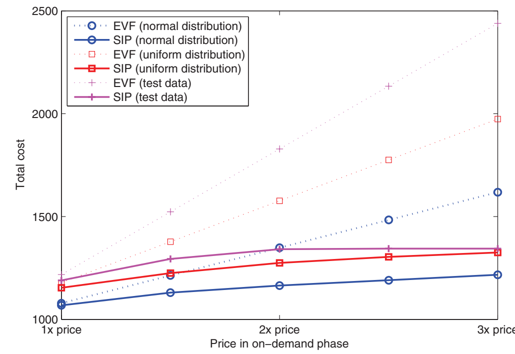

Fig. 7. Total cost from SIP and EVF.

SIP formulation for the OVMP algorithm would be required to guarantee the minimum cost to the users.

2) VM Placement in Different Phases: For the next experiments, SIP formulated in (17)−(23) is developed. The program is coded in GAMS [24] scripts. Each script inputs parameters defined in Subsection V-A. Some parameters are varied for different experiment scenarios. Then, all scripts are submitted to be processed on NEOS Server [25].

Fig. 4 shows the cost in the different phases under normal distribution. The oversubscribed cost is also shown. Given optimal number of reserved VMs to be 29, if the actual demand is one VM, 28 VMs will not be utilized. As a result, oversubscribed cost is highest. Although this is the worst case, it happens with small probability. As compared to cases close to mean = 25.50 (26 VMs), the probability is much higher. Although an on-demand cost increases since 30 VMs are required, the solution ensures that the cost is not too high. Another worst case is when 50 VMs are required. Again, this case happens with small probability. The probability to allocate resources in on-demand phase decreases as number of required VMs increases after it exceeds the optimal solution, due to the nature of normal distribution.

3) Impact of Variance in Random Demand: Then, the effect of randomness in demand is investigated. In Fig. 5, the variance of normal distribution for the demand is varied from 2 to 10. In this case, mean of the distribution is fixed to 25.50. Since three VM classes are considered, the maximum number of reserved VMs is 150. We observe that with the larger variance, the number of reserved VMs increases. Due to the nature of normal distribution, the larger variance increases the chance that the demand will be smaller or larger than the mean is high. Consequently, the optimal solution provisions more VMs in the first stage to ensure that the on-demand cost is minimized. In addition, an effect of different prices of resources in on-demand phase is shown in Fig. 5. Three prices of resources in on-demand plan are considered, i.e., normal price (1x), double price (2x), and triple price (3x). The last two prices are calculated by multiplying the normal price by coefficients of 2 and 3, respectively. We observe that with higher price of on-demand plan, the number of reserved VMs is larger. This result is due to the fact that cost in reservation plan becomes cheaper and total cost can be minimized if more number of VMs is reserved in advance.

4) Impact of Probability Distributions: The number of VMs provisioned in the first stage under three different probability distributions is shown in Fig 6. The different prices of resources in on-demand plan are also considered. In this case, the variance in normal distribution, uniform distribution, and distribution from test data are 12, 208.25, and 389.93, respectively. Similar to the observation in Fig. 5, the higher variance results in more number of VMs to be reserved. Therefore, the number of reserved VMs in the test data is highest, while that in the normal distribution is lowest.

TABLE II

NUMBER OF VMS AND PROVISIONING COST GIVEN BY DIFFERENT VM PLACEMENT.

Placement	Number of VMs			Cost ($)				
	Resv	Util	OnD	Resv	Util	OverS	OnD	Total
NoResv	0	0	64	0.00	0.00	0.00	2,964.21	2,964.21
MaxResv	150	64	0	1,017.20	393.38	594.96	0.00	1,410.58
EVF	66	39	26	415.80	191.59	171.95	1,174.41	1,781.80
SIP	146	63	1	971.60	314.57	560.05	47.65	1,333.82

5) Comparison between SIP and EVF: Next, we consider a simple approach to address uncertainty of stochastic parameters (e.g., demands and prices). This approach applies average values of such parameters and solves a deterministic integer program. We call this approach *expected-value formulation (EVF)*. The prices of resources in on-demand plan are varied and different probability distributions are considered. The comparison between EVF and SIP in terms of total cost is shown in Fig. 7. We observe that when the prices become higher, the total cost of EVF is much higher than that of SIP. For EVF, the number of VMs in the first stage is fixed by the average value of demand which is an approximation scheme. This average value cannot guarantee to achieve the minimum cost. Moreover, EVF cannot adapt to the change in price. On the other hand, SIP can always achieve the optimal solution for the number of reserved VMs given the varied and random prices to reduce the on-demand cost.

C. Simulation

A simulation program is developed for a cloud computing environment as shown in Fig. 1. The parameters in Subsection V-A are applied. The probability distribution of test data is used in the simulation. The optimal solution of SIP is obtained and used by OVMP algorithm. The simulation contains 1,000 iterations in which the demand and price are random in each iteration.

Table II shows the number of reserved VMs and total cost obtained from optimal solution of stochastic integer programming (SIP), expected-value formulation (EVF), maximum reservation placement (MaxResv), and non-reservation placement (NoResv). EVF is discussed in Subsection V-B. MaxResv uses a deterministic integer program to reserve the maximum number of VMs required in every VM class. NoResv does not provision any VMs in the first stage, and deterministic integer program is applied to obtain the solution for the second stage. In this Table II, the column titles Resv, Util, OnD, and OverS refer to the costs in reservation phase, utilization phase, on-demand phase, and oversubscribed cost, respectively.

Clearly, SIP achieves the lowest total cost, while NoRerv yields the highest total cost. Given the test data (Fig. 2), SIP reserves 146 VMs (from 150), while it allocates only one VM in the second stage. Although MaxResv reserves 150 VMs, it incurs higher cost than that of SIP. The oversubscribed cost of SIP is higher than that in EVF. However, the on-demand cost of SIP is much lower. This result shows the balance between the number of VMs to be acquired in the first and second stages in which SIP can provide the most optimal tradeoff.

VI. CONCLUSION

In this paper, we have proposed an optimal virtual machine placement (OVMP) algorithm to provision the resources offered by multiple cloud providers. The algorithm is based on an IaaS model which leverages virtualization technologies. This OVMP algorithm minimizes the total cost of resource provision in a cloud computing environment. The tradeoff between the advance reservation of resources and the allocation of on-demand resources is adjusted to be optimal. The optimal solution used by OVMP algorithm is obtained by formulating and solving stochastic integer programming with two-stage recourse. The performance evaluation of OVMP algorithm has been performed by numerical studies and simulation. Comparing to other selected placements, OVMP algorithm basd on stochastic integer programming can achieve the lowest total cost. Therefore, the proposed OVMP algorithm can be applied for efficient resource provisioning in emerging cloud computing environments. For the future work, stochastic integer programming with multiple stages will be formulated to solve a problem with many decision stages of resource provision.

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