Effect-Driven Parsing

Formal studies on a categorical approach to semantic parsing

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General Introduction

This work, based on [BC25] aims to provide a categorical formalization of a type and effects system for semantic interpretation of the natural language.

We will develop a graphical formalism for semantic type-driven parsing that explains how to derive the meaning of a sentence from the meaning of its words.

Types in Semantics of Natural Languages

Expression	Type	λ-Term
planet	$\mathtt{e} \to \mathtt{t}$	$\lambda x.\mathbf{planet}\, x$
	Generalizes to **common nouns**	
carnivorous	$(\mathtt{e} \to \mathtt{t})$	$\lambda x.\mathbf{carnivorous}\, x$
	Generalizes to **predicative adjectives**	
skillful	$(\mathtt{e} \to \mathtt{t}) \to (\mathtt{e} \to \mathtt{t})$	$\lambda p.\lambda x.px \wedge \mathbf{skillful}\, x$
	Generalizes to **predicate modifier adjectives**	
Jupiter	$\mathtt{e}$	$\mathbf{j} \in \mathrm{Var}$
	Generalizes to **proper nouns**	
sleep	$\mathtt{e} \to \mathtt{t}$	$\lambda x.\mathbf{sleep}\, x$
	Generalizes to **intransitive verbs**	

Handling Non-Determinism

What should be the type of expressions such as **a cat** or **Jupiter, a planet**?

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Since we should be able to use **a cat** and **the cat** interexchangebly - from a syntax point of view - they should have the same type.

We use *effects* to do the difference between:

$$\mathbf{a\ cat} = \{c \mid \mathbf{cat}\, c\}$$

$$\mathbf{the\ cat} = x \text{ if } \mathbf{cat}^{-1}(\top) = \{x\} \text{ else } \#$$

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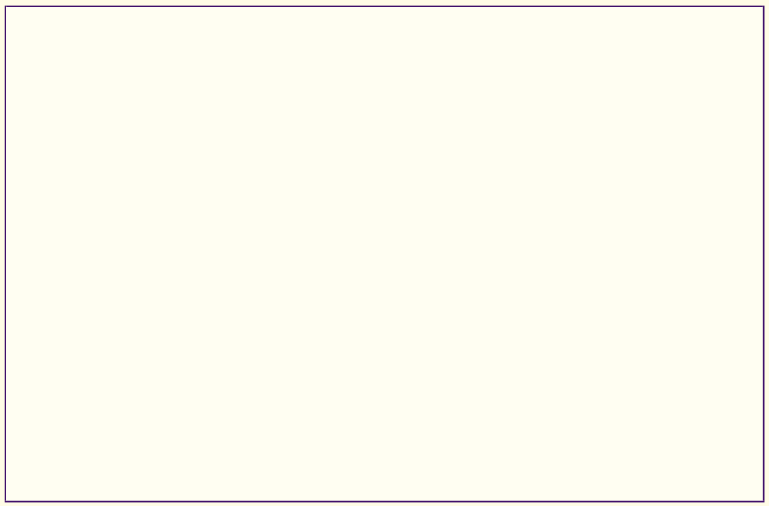
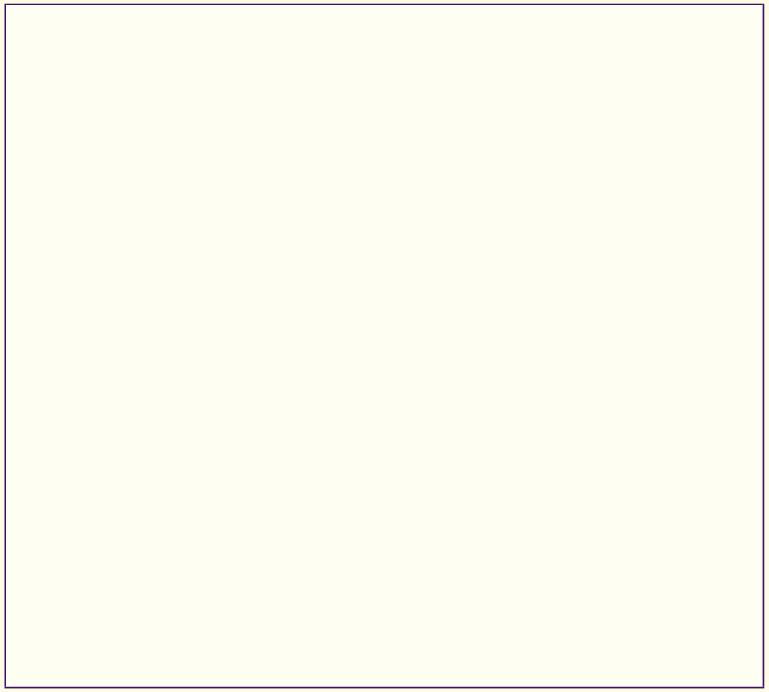
Side-Effects



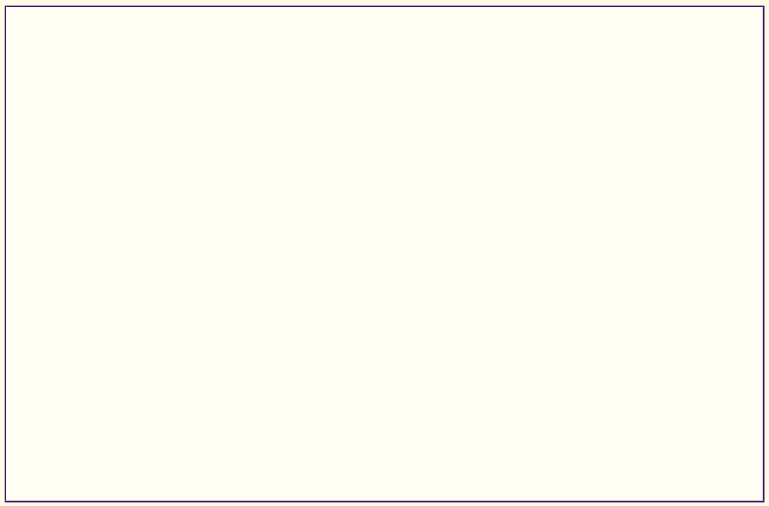
A pure program.



```
def
↪   add(
↪   y):
print("I
↪   LOVE
↪   CHOM
return
↪   x
↪   +
↪   y
```

An impure program

Side-Effects



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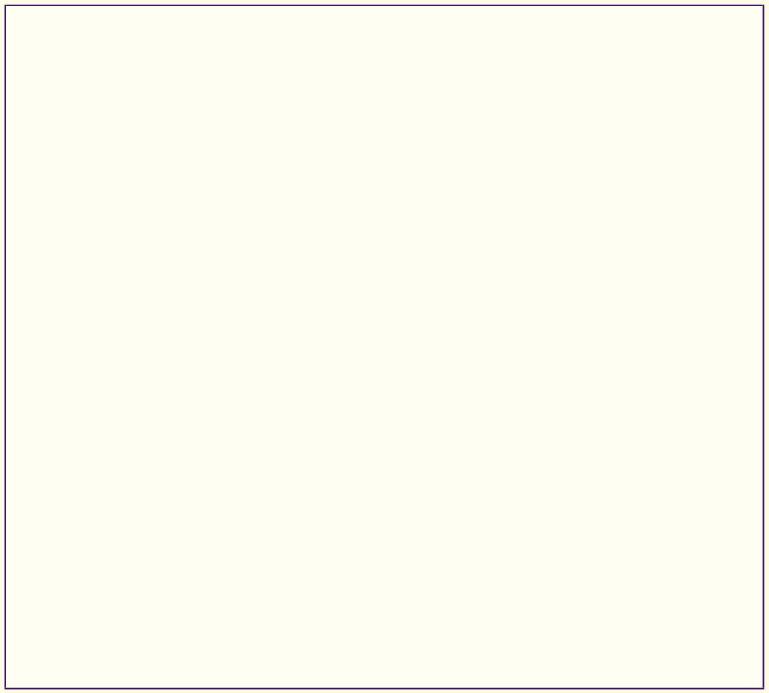

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An impure program

The addition of the `print` statement modifies the behaviour of the programs: we do not know what actually happens to the memory state of the computer.

Category Theory 101

- A category $\mathcal{C}$ is a structure with things called objects, and ways to go between things called morphisms or arrows.

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- Objects represent the set of objects of a certain type and arrows represent ways to go from one type to another language. The type of a function is then an object that represents the set of arrows between $A \to B$.

Category Theory 10201

- A functor from a category to another is a morphism between categories. It translates types as well as function between types.

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$$\begin{array}{ccc} A & \xrightarrow{\varphi} & B \\ {\scriptstyle F}\downarrow & & \downarrow{\scriptstyle F} \\ FA & \xrightarrow[F\varphi]{} & FB \end{array}$$

Functors represent modifications of a type: they represent effects.

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$$\begin{array}{ccc} FA & \xrightarrow{\theta_A} & GA \\ {\scriptstyle F\varphi}\downarrow & & \downarrow{\scriptstyle G\varphi} \\ FB & \xrightarrow[\theta_B]{} & GB \end{array}$$

Category of Endofunctors

- A monadic effect is a type of effect that can be created from an object, without losing information.
- When an object bears two of the same monadic effect, it can be transformed to only bear one instance of the effect.

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- When an object bears two of the same monadic effect, it can be transformed to only bear one instance of the effect.

Mathematically, we have two natural transformations $\eta : \mathrm{Id} \Rightarrow M$ and $\mu : MM \Rightarrow M$ called unit and multiplication or join.

I'm FREE! Forget it.

An adjunction between two functors $L \dashv R$ is a pair of natural transformations $\eta : \mathrm{Id} \Rightarrow L \circ R$ and $\varepsilon : R \circ L \Rightarrow \mathrm{Id}$.

It mimics the behaviour of a bijection for functor composition.

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A classical example is the Read - Write adjunction. It mimics the behaviour of the anaphora: once we have wrote data next to a denotation, reading said data makes us go back to the beginning, or almost.

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A Type and Effect System

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We consider $\bar{\mathcal{C}}$ the categorical closure of $\mathcal{C}$ under the action of $\mathcal{F}(\mathcal{L})^*$. We close it for the cartesian product and exponential of $\mathcal{C}$. $\bar{\mathcal{C}}$ represents all possible combinations of a sequence of effects and a base type, contains functions and products.

Typing Rules

We then have typing judgements for basic combinations:

$$\frac{\Gamma \vdash x : \tau \qquad \Gamma \vdash F \in \mathcal{F}(\mathcal{L})}{\Gamma \vdash Fx : F\tau}\,\textsf{Cons}$$

$$\frac{\Gamma \vdash x : F\tau_1 \qquad \Gamma \vdash \varphi : \tau_1 \to \tau_2}{\Gamma \vdash \varphi x : F\tau_2}\,\texttt{fmap}$$

$$\frac{\Gamma \vdash x : \tau_1 \qquad \Gamma \vdash \varphi : \tau_1 \to \tau_2}{\Gamma \vdash \varphi x : \tau_2}\,\textsf{App}$$

Typing Rules

We then have typing judgements for basic combinations:

$$\frac{\Gamma \vdash x : A\tau_1 \qquad \Gamma \vdash \varphi : A\,(\tau_1 \to \tau_2)}{\Gamma \vdash \varphi x : A\tau_2}\text{<*>}$$

Typing Rules

Typing judgements for natural transformations:

$$\frac{\Gamma \vdash x : \tau}{\Gamma \vdash x : A\tau}\texttt{pure/return}$$

$$\frac{\Gamma \vdash x : MM\tau}{\Gamma \vdash x : M\tau}\texttt{>>=}$$

More generally:

$$\forall F \overset{\theta}{\Longrightarrow} G, \qquad \frac{\Gamma \vdash x : F\tau \qquad \Gamma \vdash G : S' \subseteq \star \qquad \tau \in S'}{\Gamma \vdash x : G\tau}\texttt{nat}$$

To ensure termination and decidability, we prevent the use of the unit rule out of the blue, more on why that is fine later.

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Language

To present the language, we of course need the syntax of the language, as well as an increased model of our lexicon.

Lexicon

Expression	Type	λ-Term
it	$\mathtt{Ge}$	$\lambda g.g_0$
$\cdot$, **a** $\cdot$	$\mathtt{e} \to (\mathtt{e} \to \mathtt{t}) \to \mathtt{We}$	$\lambda x.\lambda p.\,\langle x, px\rangle$
which	$(\mathtt{e} \to \mathtt{t}) \to \mathtt{Se}$	$\lambda p.\,\{x \mid px\}$
the	$(\mathtt{e} \to \mathtt{t}) \to \mathtt{Me}$	$\lambda p.x$ if $p^{-1}(\top) = \{x\}$ else $\#$
a	$(\mathtt{e} \to \mathtt{t}) \to \mathtt{De}$	$\lambda p.\lambda s.\,\{\langle x, x +\!\!+ s\rangle \mid px\}$
every	$(\mathtt{e} \to \mathtt{t}) \to \mathtt{Ce}$	$\lambda p.\lambda c.\forall x, px \Rightarrow cx$

Higher-Order Constructs

Using the notion of functors, we can also implement higher-order semantic constructions in our lexicon, such as the future, without caring about morphological markers:

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Using the notion of functors, we can also implement higher-order semantic constructions in our lexicon, such as the future, without caring about morphological markers:

$$\mathbf{future}\left(\mathbf{be}\left(\mathbf{I}, \mathbf{a\ cat}\right)\right) \xrightarrow{\beta} \mathbf{be}\left(\mathbf{future}\left(\mathbf{I}\right), \mathbf{a\ cat}\right) \xrightarrow{\beta} \mathbf{be}\left(\mathbf{future}\left(\mathbf{I}\right), \mathbf{a\ cat}\right)$$

Those constructs are integrated by using natural transformations explaining their propagation through other effects, as those are purely semantic predicates.

Higher-Order Constructs

For the plural, this gives:

CN(P)	$\Gamma \vdash p : (\mathtt{e} \to \mathtt{t})$	$\Pi(p) = \lambda x.\,(px \wedge \lvert x\rvert \geq 2)$
ADJ(P)	$\Gamma \vdash p : (\mathtt{e} \to \mathtt{t})$	$\Pi(p) = \lambda x.\,(px \wedge \lvert x\rvert \geq 2)$
	$\Gamma \vdash p : (\mathtt{e} \to \mathtt{t}) \to (\mathtt{e} \to \mathtt{t})$	$\Pi(p) = \lambda \nu.\lambda x.\,(p\,(\nu)\,(x) \wedge \lvert x\rvert \geq 2)$
NP	$\Gamma \vdash p : \mathtt{e}$	$\Pi(p) = p$
	$\Gamma \vdash p : (\mathtt{e} \to \mathtt{t}) \to \mathtt{t}$	$\Pi(p) = \lambda \nu.p\,(\Pi\nu)$
IV(P)/VP	$\Gamma \vdash p : \mathtt{e} \to \mathtt{t}$	$\Pi(p) = \lambda o.\,(po \wedge \lvert x\rvert \geq 2)$
TV(P)	$\Gamma \vdash p : \mathtt{e} \to \mathtt{e} \to \mathtt{t}$	$\Pi(p) = \lambda s.\lambda o.\,(p\,(s)\,(o) \wedge \lvert s\rvert \geq 2)$

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Handlers

A handler for an effect F is a natural transformation $F \Rightarrow \mathrm{Id}$.

Handlers should also be exact inverses to monadic and applicative units: this justifies semantically why we can remove the usage of the unit rule out of certain situations.

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1. Language-Defined Handlers, which are defined with adjunctions and comonads, for example. Those arise from fundamental properties of the considered effects.
2. Speaker-dependant handlers, which are considered when retrieving the denotation from a sentence from under the effects that arose in the computation of its meaning. Those need to be considered dependent on the speaker because for example of the multiple ways to solve non-determinism.

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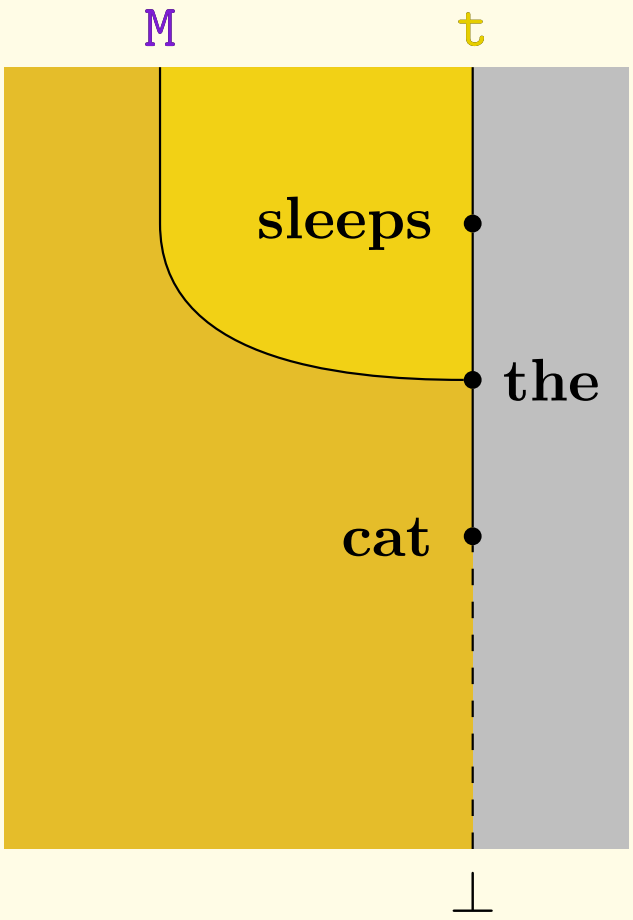
String Diagrams in Denotations I

A string diagram is a representation of the side-effects and types of a sentence across its computation.

The lines are functors (effects or base types), the nodes are natural transformations.

String Diagrams in Denotations II



This diagram for example represents the sentence *The cat sleeps*. The order

String Diagram Equivalence

String diagrams will be the formalism we use to compute equality between denotations, and especially handling the denotations.

Theorem 3.1 — **Theorem 3.1 [Sel10], Theorem 1.2 [JS91]** A well-formed equation between morphism terms in the language of monoidal categories follows from the axioms of monoidal categories if and only if it holds, up to planar isotopy, in the graphical language.

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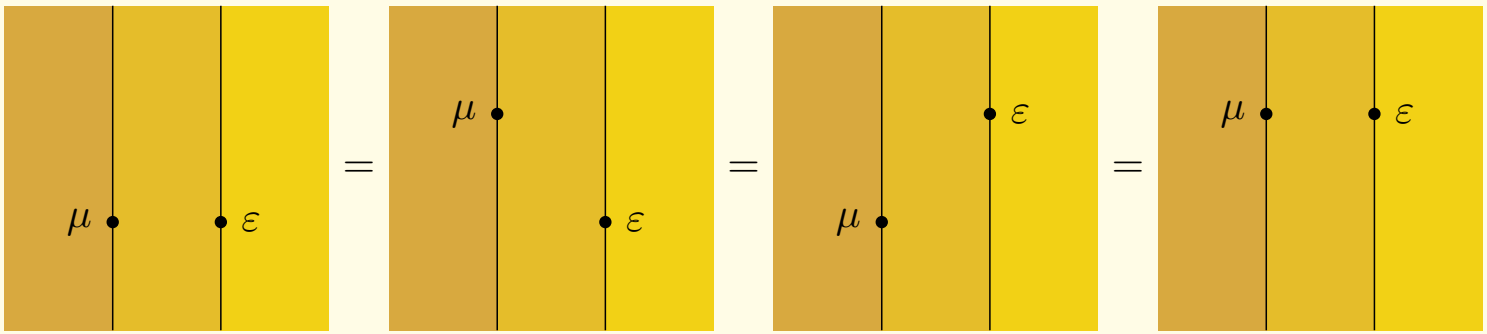
Conversion Software, version 7.0 I

Every property of the functors, monads, natural transformations, adjunctions and more can be explained in terms of commutative diagrams, but also as string diagrams.

First, the elevator equations are a consequence of 3.1:



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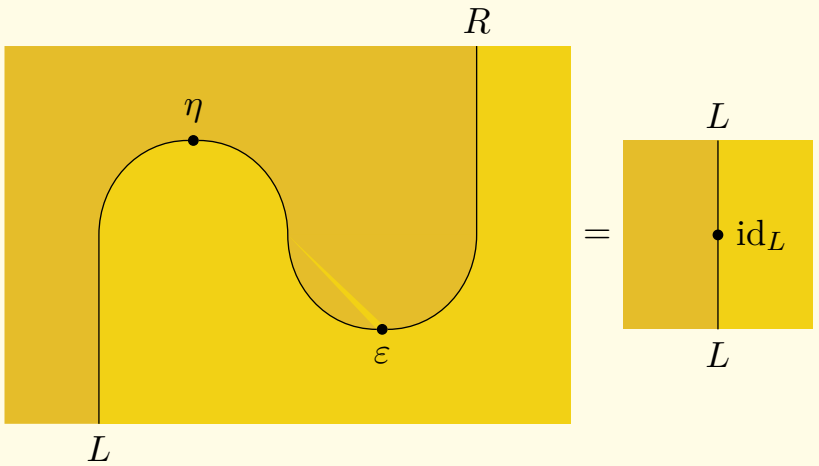
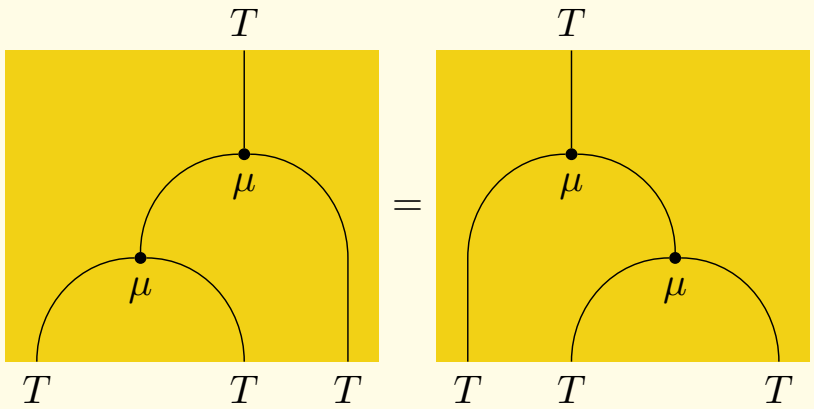
Conversion Software, version 7.0 II

The Snake equations are a rewriting of the properties of an adjunction:



Conversion Software, version 7.0 III

The Monadic equations are a rewriting of the properties of a monad:



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Reduction Scheme

[DV22] proposed a combinatorial description to check in linear time for equality under Theorem 3.1.

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Theorem 3.3 — **Confluence** Our reduction system is confluent and therefore defines normal forms:

1. Right reductions are confluent and therefore define *right* normal forms for diagrams under the equivalence relation induced by exchange.
2. Equational reductions are confluent and therefore define *equational* normal forms for diagrams under the equivalence relation induced by exchange.

Polynomial Time Reductions

Theorem 3.4 — **Normalization Complexity** Reducing a diagram to its normal form is done in quadratic time in the number of natural transformations in it.

This is accomplished using a formalism based on [DV22].

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String Diagram Combination

Given our grammar, we could build parsing trees, but that would blur the actual usefulness of our grammar and our string diagrammatic representation of sentences.

We thus consider diagrams whose 1-cells are objects in $\bar{\mathcal{C}}$, i.e. types and effects and whose natural transformations are the combinators of our grammar.

Bibliography I

[BC25] Dylan Bumford and Simon Charlow. *Effect-Driven Interpretation: Functors for Natural Language Composition*. Mar. 2025. (Visited on 24/04/2025).

[DV22] Antonin Delpeuch and Jamie Vicary. *Normalization for Planar String Diagrams and a Quadratic Equivalence Algorithm*. Jan. 2022. DOI: 10.48550/arXiv.1804.07832. arXiv: 1804.07832. (Visited on 31/03/2025).

Bibliography II

[JS91] André Joyal and Ross Street. "The Geometry of Tensor Calculus, I". In: *Advances in Mathematics* 88.1 (July 1991), pp. 55–112. ISSN: 00018708. DOI: 10.1016/0001-8708(91)90003-P. (Visited on 28/03/2025).

[Sel10] Peter Selinger. "A Survey of Graphical Languages for Monoidal Categories". In: vol. 813. 2010, pp. 289–355. DOI: 10.1007/978-3-642-12821-9_4. arXiv: 0908.3347 [math]. (Visited on 03/04/2025).