CS 101: Computer Programming and Utilization

11-Matrices
Multidimensional Arrays

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Activity – Load balancing

There are two trucks, A and B, each having packages with different weights. We wish to balance the load in both the trucks by swapping exactly one pair of packages between them.

Write a program to determine if such load balancing is possible for a given set of input weights, and if yes, identify which packages to swap. Assume that all weights are integers (e.g. 85 Kg, 23 Kg, 7 Kg, etc.).

Discuss: pseudo-code; Run: demo11-balance.cpp

Load balancing trucks – basic idea

If x is element of A, and y element of B, we need to find x and y such that, if x and y are exchanged, the resulting sums match.

sumA +y -x = sumB -y + x (after exchanging x and y)

or y = (sumB-sumA)/2 + x

For some i, if we are looking at A[i]; then for some j, we must have B[j] = (sumB -sumA)/2 + A[i]

Multidimensional Arrays

We can use arrays with more than one dimension

int A[50][40];

- Declares a 2D array with 50 rows and 40 columns
- Each element is accessed by a reference requiring two index expressions, e.g., A[i][j] = 37;
  - Row index ‘i‘, can have a value from 0 to 49,
  - Column index ‘j’ can have a value from 0 to 39
  - Violations may lead to silent garbage
- All rules for index expression apply to index for each dimension

Storage: row major

- Internally, no distinction between [3][5] and [15]
- cellNumber(rx, cx) = rx * cols + cx
- Base address of imat[rx][cx] is base address of imat plus 4 * cellNumber(rx, cx)

	0	1	2	3	4
0	Cell 0, Byte 0	1, 4	2, 8	3, 12	4, 16
1	5, 20	6, 24	7, 28	8, 32	9, 36
2	10, 40	11, 44	12, 48	13, 52	14, 56… 59

Matrix manipulations

2D arrays are commonly used to represent Matrices and are found in programs such as:

- Matrix multiplication
  - Given: m x n matrix A; n x p matrix B, the matrix product C = AB will be m × p matrix.
  - $C[i][j] = \sum_{k=0}^{n-1} A[i][k] * B[k][j]$

- Solving linear equations by Gaussian elimination

Basic operations

- Initialize square matrix to identity

```
for (int rx = 0; rx < rows; ++rx) {
  for (int cx = 0; cx < cols; ++cx) {
    dmat[rx][cx] = (rx == cx)? 1 : 0;   }   }
```

- Better code uses two loops (why?)
  - In first rx,cx loop set all elements to zero

```
for (int rx=0; rx<rows; ++rx) {
  for (int cx=0; cx<cols; ++cx) { dmat[rx][cx]=0; } }
```

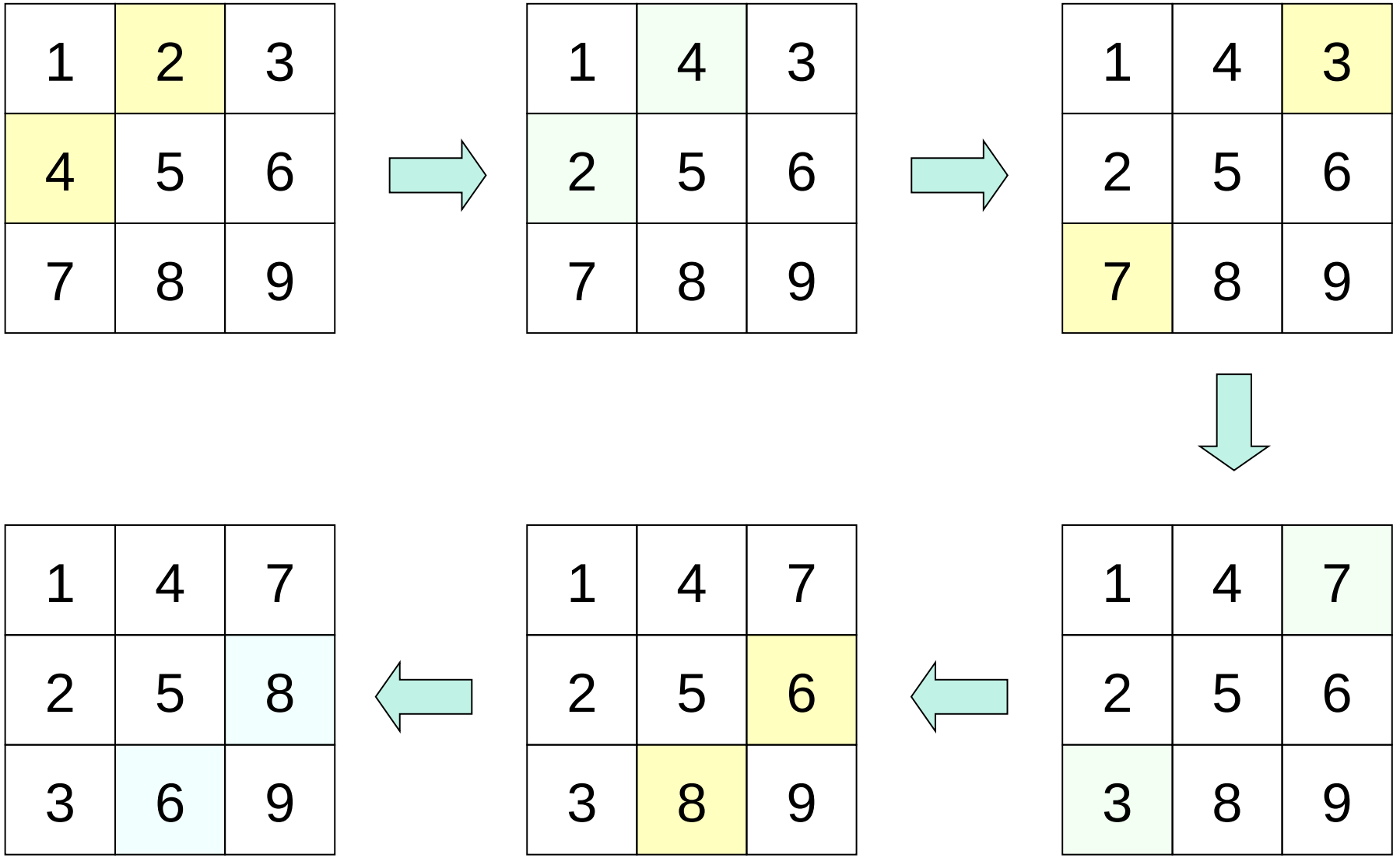
  - In second loop set diagonal elements to one

```
for (int dx=0; dx<rows; ++dx)
  { dmat[dx][dx]=1; }
```

Think-Pair-Share: Write a program to transpose a square matrix

- Think (Individually): Write the pseudo-code in your notebook; See example on next slide.
- Pair (with your neighbour): Write the C++ code.
- Share (with class): Compare your solution with others.

Transpose a square matrix – example
Write program using Think-Pair-Share



Transpose a square matrix - code

- Don’t double transpose back to square one!

No need to transpose diagonal or below

```
for (int rx = 0; rx < rows; ++rx) {
  for (int cx = rx+1; cx < cols; ++cx) {
    float tmp = fmat[rx][cx];
    fmat[rx][cx] = fmat[cx][rx];
    fmat[cx][rx] = tmp;
  }
}
```

Swap [rx] [cx] with [cx] [rx]

Matrix vector multiplication – example
Write program using Think-Pair-Share

a00	a01	a02
a10	a11	a12

x0
x1
x2

=

a00*x0 +a01*x1 +a02*x2	y0
a10*x0 +a11*x1 +a12*x2	y1

- Think: Write the pseudo-code in your notebook.
- Pair: Write the C++ code.
- Share: Compare your solution with others.

Matrix-vector multiplication - code

```
float A[rows][cols];
float x[cols], y[rows];
// fill up A and x
for (int rx=0; rx < rows; ++rx) {
  y[rx] = 0;
  for (int cx=0; cx < cols; ++cx) {
    y[rx] += A[rx][cx] * x[cx];
  }
}
```

Matrix-matrix multiplication

```
float amat[lsize][msize], bmat[msize][nsize],
  cmat[lsize][nsize];
for (int crx=0; crx<lsize; ++crx) {
  for (int ccx=0; ccx<nsize; ++ccx) {
    cmat[crx][ccx] = 0;
    for (int abx=0; abx<msize; ++abx) {
      cmat[crx][ccx] +=
         amat[crx][abx] * bmat[abx][ccx];
    }
  }
}
```

- Like matrix-vector multiplication
- But x has many columns
- Triple nested loop
- Time required is `lsize*msize*nsize`

Gaussian elimination method

Consider two equations: $2x + 4y = 8$ and $4x + 5y = 1$

They can be represented as:

$$[2\ 4]\ [x] = [8]$$

$$[4\ 5]\ [y] = [1]$$

Gauss elimination method involves transformations (by using multiplication and linear combinations) to reduce the coefficient matrix to upper triangular form:

$$[1\ 2]\ [x] = [4]$$

$$[0\ 1]\ [y] = [5]$$

Then solving by back-substitution to get $y = 5$, $x = -6$.

Gaussian elimination representation

The previous coefficient matrix can be stored as:

A[0][0] = 2, A[0][1] = 4, A[1][0] = 4, A[1][1] = 5.

A general system of n equations can be written as

$$a[][]\ x[] = b[]$$

With the coefficient matrix in upper triangular form, we have the following system, using which, back substitution can be applied

$$x[0] + a[0][1]\ x[1] + \ldots + a[0][n-1]\ x[n-1] = b[0]$$
$$x[1] + \ldots + a[1][n-1]\ x[n-1] = b[1]$$
$$\ldots$$
$$x[n-1] = b[n-1]$$

Gaussian elimination pseudo-code

- Read matrices A[][] and B[]
- For each row
  - Divide the row by the coeff on the diagonal
  - Recalculate all the coeffs in that row
  - Normalize (recalculate) the RHS of that row
- Replace subsequent rows, by subtracting the appropriate portion of the ith equation from it
- Do back-substitution starting from the last row
  - Sum up the ith row using the values of x[] already determined
- Output the results – values of x[i]

Gaussian elimination program

- Work with your neighbour to write the C++ code

- Compare with: demo11-gauss.cpp

Handling large inputs

- Tedious to type the input values one at a time!
- Solution: I/O Redirection …

- When we execute our program, OS assigns it three standard files - stdin, stdout, stderr
- By default, OS ‘connects’ these to devices:
  - stdin to keyboard, stdout and stderr to monitor

- ./a.out < input.txt → Redirects stdin from a file
- ./a.out > output.txt → Redirects stdout to a file