Homework 1

ISyE 6420

Fall 2023

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Fall23 HW1.1. Chad is taking a Bayesian Analysis course. He believes he will get an A with probability 0.7. At the end of semester he will get a car as a present form his rich uncle depending on his class performance. For getting an A in the course he will get a car with probability 0.9, for anything less than A, he will get a car with probability of 0.1. If Chad gets a car, he would travel to Cocoa Beach with probability 0.7, or stay on campus with probability 0.3. If he does not get a car, these two probabilities are 0.2 and 0.8, respectively.

After the semester was over you learn that Chad is in Cocoa Beach. What is the probability that he got a car?

Fall23 HW1.2. Carpal tunnel syndrome is the most common entrapment neuropathy. The cause of this syndrome is hard to determine, but it can include trauma, repetitive maneuvers, certain diseases, and pregnancy.

Three commonly used tests for carpal tunnel syndrome are Tinel's sign, Phalen's test, and the nerve conduction velocity test. Tinel's sign and Phalen's test are both highly sensitive (0.98 and 0.92, respectively) and specific (0.91 and 0.88, respectively). The sensitivity and specificity of the nerve conduction velocity test are 0.93 and 0.87, respectively.[2]

Assume that the tests are conditionally independent.

Calculate the sensitivity and specificity of a combined test if combining is done

(a) in a serial manner;[3]

(b) in a parallel manner.[4]

(c) Find Positive Predictive Value (PPV) for tests in (a) and (b) if the prevalence of carpal tunnel syndrome in the general population is approximately 50 cases per 1000 subjects.

Fall23 HW1.3. A student answers a multiple choice examination with two questions that have four possible answers each. Suppose that the probability that the student knows the answer to a question is 0.70 and the probability that the student guesses is 0.30. If the student guesses, the probability of guessing the correct answer is 0.25. The questions are

[2]For definitions of Sensitivity, Specificity, and PPV, consult Chapter 4: Sensitivity, Specificity, and Relatives, in the textbook at `http://statbook.gatech.edu`.

[3]Tests are combined in a serial manner if the combination is declared positive when all tests are positive.

[4]Tests are combined in a parallel manner if the combination is declared positive when at least one test is positive.

independent, that is, knowing the answer on one question is not influenced by the other question.

(a) What is the probability that the both questions will be answered correctly?

(b) If answered correctly, what is the probability that the student really knew the correct answer to both questions?

(c) How would you generalize the above from 2 to n questions, that is, what are answers to (a) and (b) if the test has n independent questions? What happens to probabilities in (a) and (b) if $n \to \infty$.