More DAGs; models with interactions

ISTA 410 / INFO 510: Bayesian Modeling and Inference

U. of Arizona School of Information

March 29, 2021

Outline

Previously:

- Multiple regression
- Total vs. direct causal effect
- Causal DAGs and various forms of confounding

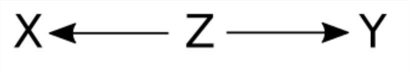
Today:

- More DAGs
- The backdoor criterion
- Intro to interactions

The backdoor criterion

The "fork" path

The *fork* is the form most students learn as the sole definition of "confounding" in introductory classes: X and Y are confounded by their common cause, Z:



A statistical association exists between X and Y because they are both influenced by Z.

Example: X is ice cream sales; Y is drowning deaths; Z is temperature

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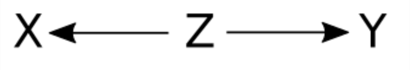


Conditional independence:

- DAG property means: conditional on Z, X and Y are independent.
- So, condition/stratify/control on Z to block the path and estimate effect of X on Y

The "chain" path

The *chain* is a similar-looking form, where Z sits in the middle of a causal path:

$$X \longrightarrow Z \longrightarrow Y$$

Typical case: Z is an effect of X that mediates the effect on Y

Example: X is pesticide application; Z is the pest population; Y is crop yield.

Controlling for Z blocks information flow along the path.

The “collider” path

The third form is the *collider* or inverted fork, and it behaves quite differently:

$$X \longrightarrow Z \longleftarrow Y$$

In contrast to the fork or chain, information flows through the collider only when it *is* observed / controlled; controlling *unblocks* the path.

The explaining-away effect

This property of colliders is responsible for a sometimes counterintuitive effect:

- "explaining away": observing one of the common causes
- Berkson's paradox: conditioning on a variable can introduce a spurious association

They're really the same effect; explaining away common in AI/ML; Berkson's paradox in statistics

Explaining away: the burglar alarm

Causality

Information Theory, Inference, and Learning Algorithms

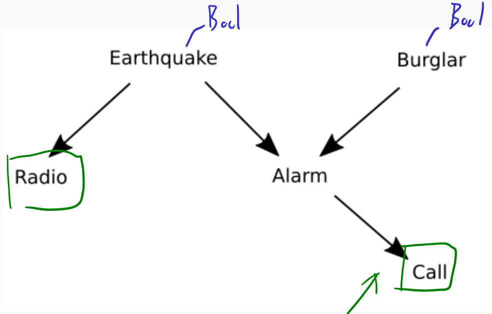
From Pearl by way of Mackay:

> *Fred lives in Los Angeles and commutes 60 miles to work. Whilst at work, he receives a phone-call from his neighbour saying that Fred's burglar alarm is ringing. What is the probability that there was a burglar in his house today? While driving home to investigate, Fred hears on the radio that there was a small earthquake that day near his home. 'Oh', he says, feeling relieved, 'it was probably the earthquake that set off the alarm'.*

What is the causal reasoning here?

Explaining away: the burglar alarm

A DAG for the burglar alarm problem, showing the collider:



Conditioning on descendant has a similar effect to conditioning on the collider itself

The alarm sits at a collider.

d-separation

A (possibly undirected) path p through a DAG G is said to be *d-separated* or *blocked* by a set of nodes Z if:

1. p contains a chain $X_i \to M \to X_j$ or fork $X_i \leftarrow M \to X_j$ such that $M \in Z$; or,
2. p contains a collider $X_i \to M \leftarrow X_j$ such that $M \notin Z$ and no descendent of M is in Z.

(Why the descendant property? Look back at the burglar alarm.)

The d-separation (blocking) definition for paths leads to another definition, for sets of variables.

1. at least one chain or fork is included in Z;

or, there is a collider not in Z.

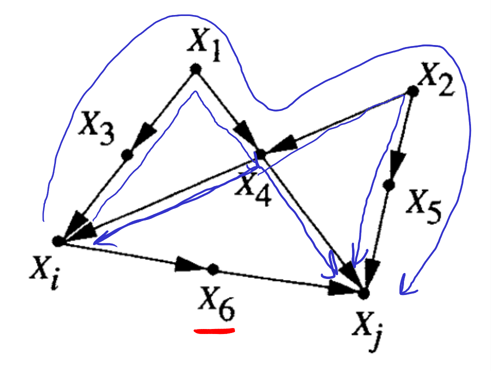
The backdoor criterion

A related definition:

Definiton
A set of variables Z satisfies the backdoor criterion with respect to an ordered pair of variables (X_i, X_j) in G if:

1. *no node in Z is a descendent of X_i; and,* — don't block chains $X_i \to X_j$
2. *Z blocks every path from X_i to X_j that contains an arrow into X_i.* — do block non-causal paths

To estimate the causal effect of X on Y, condition on a set of variables satisfying the backdoor criterion with respect to (X, Y).

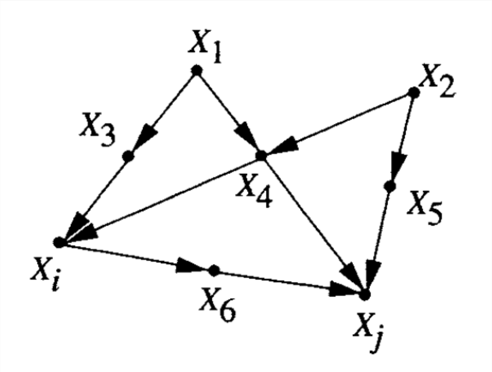


Which variables satisfy the backdoor criterion?

What do you condition on to estimate causal effect of X_i on X_j?

$$X_i \leftarrow X_4 \rightarrow X_j$$

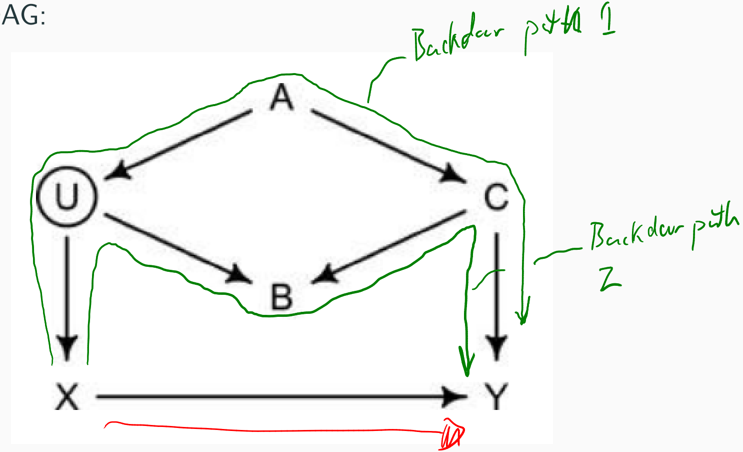
Group exercise



Which variables satisfy the backdoor criterion?

- $\{X_3, X_4\}$ or $\{X_4, X_5\}$
- Not $\{X_4\}$ (doesn't block every backdoor path), nor $\{X_6\}$ (descendent of X_i)

Another DAG:



We want to estimate $X \rightarrow Y$. What should we condition on? (U is unobserved; we can't use it.)

Fake data simulation

To demonstrate the effect, let's use a fake data simulation:

$$
\begin{aligned}
A &\sim \mathrm{Normal}(0, 1) \\
U &= A + \mathrm{noise} \\
C &= -2A + \mathrm{noise} \\
B &= -2C + 3U + \mathrm{noise} \\
X &= U + \mathrm{noise} \\
Y &= 1.5X + C + \mathrm{noise}
\end{aligned}
$$

In all cases, $\mathrm{noise} \sim \mathrm{Normal}(0, 0.1)$

What do we seek?

Before we run any regressions, what *should* we see?

- All paths $X \rightarrow Y$ except the direct one are non-causal (backdoor)
- We want to estimate the direct (causal) effect
- We know $Y = 1.5X + \text{other effects}$
- An unconfounded estimate of $\hat{Y} = \beta_x X + \text{others}$ should have $\beta_x \approx 1.5$

Let's see some regression results

- Regression including only X:

	mean	sd	hdi_3%	hdi_97%
alpha	0.070	0.031	0.011	0.125
beta_x	-0.465	0.033	-0.525	-0.403
sigma	0.302	0.023	0.261	0.342

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- Condition on A:

	mean	sd	hdi_3%	hdi_97%
alpha	0.027	0.013	0.006	0.054
beta_x	1.461	0.093	1.290	1.636
beta_a	-1.948	0.093	-2.124	-1.782
sigma	0.126	0.009	0.109	0.145

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- Condition on A and B:

	mean	sd	hdi_3%	hdi_97%
alpha	0.020	0.011	-0.002	0.040
beta_x	1.893	0.110	1.687	2.107
beta_a	-1.044	0.190	-1.428	-0.710
beta_b	-0.192	0.036	-0.263	-0.129
sigma	0.112	0.008	0.097	0.127

Let's see some regression results

- Using X alone: badly confounded
- Using X, A: good estimate of $X \to Y$
- Using X, A, B: confounded again

What other options do we have?

More variables

- Condition on C:

	mean	sd	hdi_3%	hdi_97%
alpha	0.018	0.009	0.002	0.034
beta_x	1.539	0.065	1.410	1.650
beta_c	1.017	0.033	0.957	1.076
sigma	0.087	0.006	0.075	0.098

More variables

- Condition on C:

	mean	sd	hdi_3%	hdi_97%
alpha	0.018	0.009	0.002	0.034
beta_x	1.539	0.065	1.410	1.650
beta_c	1.017	0.033	0.957	1.076
sigma	0.087	0.006	0.075	0.098

- Use everything:

	mean	sd	hdi_3%	hdi_97%
alpha	0.019	0.009	0.003	0.036
beta_x	1.559	0.100	1.372	1.756
beta_a	-0.233	0.186	-0.593	0.102
beta_b	-0.002	0.038	-0.076	0.067
beta_c	0.905	0.121	0.686	1.138
sigma	0.087	0.007	0.075	0.099

Using everything works because the collider path is blocked at C; but note precision

Unobserved variables in a DAG

The previous example had an unobserved variable, U:

- If a variable is unobserved, then we can't stratify/adjust for it in the regression
- ...but that doesn't mean we are off the hook for thinking about it!
- Unobserved variables can confound estimates
- Unobserved variables can form colliders

Sometimes this means there is no way to make the estimates that you want!

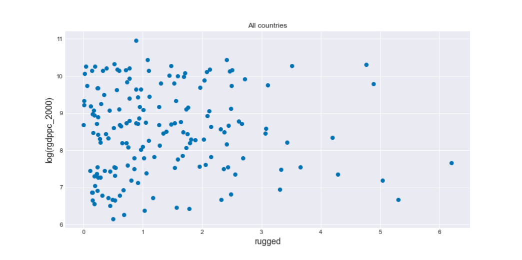
Modeling interactions

Terrain "ruggedness" and economy

(Example from Statistical Rethinking Ch7.) What is the relationship between the geographic terrain in a nation and its economy?

Data: observations on many countries

- Outcome: log GDP (as of 2000, when data was collected)
- Predictor: terrain "ruggedness" index



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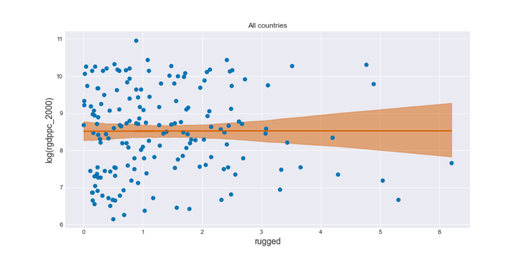
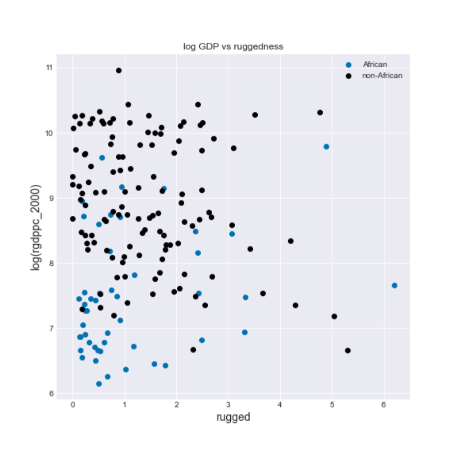


Terrain ”ruggedness” and economy

Closer examination of the data reveals an interesting phenomenon: the relationship is different for countries in Africa.



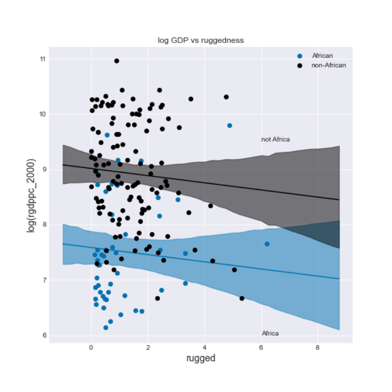
A simple approach that won't work

A simple approach that's not quite good enough: add an indicator variable for African countries, and do a bivariate regression:

$$\begin{aligned}
\log GDP &\sim \mathrm{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_R R + \beta_A A \\
\beta_R &\sim \mathrm{Normal}(0, 1) \\
\beta_A &\sim \mathrm{Normal}(0, 1) \\
\sigma &\sim \mathrm{HalfCauchy}(5)
\end{aligned}$$

A simple approach that won't work

Problem:



Allows for a shift, but not a change in slopes.

See this also with the fox problem from last week.

Allowing interactions

To add interactions:

$$
\begin{aligned}
\log GDP &\sim \mathrm{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_R R + \beta_A A + \beta_{AR} AR \\
\beta_R &\sim \mathrm{Normal}(0, 1) \\
\beta_A &\sim \mathrm{Normal}(0, 1) \\
\beta_{AR} &\sim \mathrm{Normal}(0, 1) \\
\sigma &\sim \mathrm{HalfCauchy}(5)
\end{aligned}
$$

So we have a third slope, for the *product* of R and AR.

Why is this the approach?

Where this comes from: just model the slope β_R as being itself a linear function of A:

$$\begin{aligned}
\log GDP &\sim \mathrm{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \gamma_i R + \beta_A A \\
\gamma_i &= \beta_R + \beta_{AR} A \\
\beta_R &\sim \mathrm{Normal}(0, 1) \\
\beta_A &\sim \mathrm{Normal}(0, 1) \\
\beta_{AR} &\sim \mathrm{Normal}(0, 1) \\
\sigma &\sim \mathrm{HalfCauchy}(5)
\end{aligned}$$

The result

Result from the interaction model:

Here, we can see the different slope. Ruggedness has a positive association with GDP for African nations, negative for others.

Summary

Today:

- More DAGs
- More confounding
- Intro to interactions

Next time:

- More interactions