QUBE

QUantitative Bitcoin Exchange

Cryptocurrency Intelligent

Quantitative Investment Analysis Engine

White Paper

Table of Contents

Over the past two decades, the advancement of IT infrastructure, centralized data storage and the Internet ecosystem has encouraged traditional finance industry to phase out the use of manual bookkeeping and telegraphic transactions. The finance industry has now evolved into one of the main driving factors of the global economy and promoted the growth of data service providers such as Bloomberg. As blockchain, big data and artificial neural network brings cryptocurrency into the public eyes, the success of the future financial industry will definitely revolve around new investment opportunities from big data and the use of artificial intelligence models such as deep learning to perform market forecasting, generate risk alerts, as well as to assist other business decision making tasks.

1. Project Background & Positioning

There are now total 3781 types of cryptocurrencies all over the world, forming new blockchain ecology compositions every day. The cryptocurrency blockchain itself contains 17 types of data, such as issued amount, mining difficulty level, mining loss, circulation amount, block transaction data and miners' fee.

There are over 467 cryptocurrency trading platforms and 3781 cryptocurrencies. 1765 out of 3781 cryptocurrencies are listed on exchanges, forming 3183 transaction pairs and the number is still rapidly increasing. The exchanges will produce live transaction prices, orders and volume globally. Every second makes a difference. The average maximum difference of transaction price among different exchanges is 9.73%. There are 382 average long tail trading volume platforms. However, about 63.9% of total transactions are still carried out by OTC (over the counter) or through other means. Besides that, there are another 69 categories of relevant information, such as OTC data, non-trading data and futures derivatives data.

With the decentralized blockchain and anonymous technology model, cryptocurrencies are largely traded independently distributed exchanges where 98.9% of the conventional analysis data can be retrieved. Only 12.9% of the most critical data such as price and volume is available and real-time from 73 independent data resources. In addition, 98% of the exchanges currently only provide real-time transaction data (76.9% of the data is the current timestamp data) without historical data as investment decision-making reference.

Based on the statistics of the last year, it shows that an average of 17,875 pieces of news, 537,819 pieces of UGC information and 555,694 pieces of public sentiment information are generated from blockchain and cryptocurrencies per day, including market performance and investment's favor. To a certain extent, such uncertain information may make cryptocurrency prices fluctuate with an average of 5.5% on an hourly basis without stop in 24 hours.

The goal of QUBE is to retrieve numerous data from blockchain, exchanges and the complicated public sentiment information. After undergoing a series of structured processes on the average of 5.6017T data, it will be accumulated to be the world's unique structured data center of the blockchain market. We will use this as a basis to analyze the trend, find the insights the nature, and catch the opportunities, intelligently forecast the market and alert the risk. QUBE aims to create value for business decision making, form the industry foundation and standards with an open mindset, and provide intelligent quantitative analysis engine.

Based on the data on 1 Jan 2018, the global security market value is estimated to be 100 trillion USD. The total market value of blockchain cryptocurrency is 755.27 billion USD, which is only 0.76% of the market value of traditional securities market.

The quantified auxiliary transaction in the secondary market of traditional securities trading has accounted for 19.6%. QUBE aims to position and subdivide cryptocurrency market with great market potential.

QUBE serves:

- 30 million digital currency investor (10% VIP Paid users)
  - Plan to launch quantitative strategy robots based on currency units for individual investors
- 10 thousand professional financial institution(all of them are paying users)
- 500 professional digital asset fund (all of them are paying users)

2. Solutions & Models

QUBE intelligent quantitative analysis engine, the solution based on large-scale data processing and depth learning algorithm for data dimension reduction, noise removal, feature extraction, model training and so on, the optimization and normalization for neural network quantitative factors, to form the final output. Its core is to process the daily average 5.6017T (current) massive data, including blockchain data, various trading platform data, network information and public sentiment data and so on, finally to make model prediction and data quantification for the various types of game behavior of cryptocurrency.

2.1 Overview of Solution

1) Data acquisition

We get data through 276 global nodes, WebSocket and API interface, using Web crawlers access to prices, real-time orders and trading volume and other information of 1765 digital currencies pairs and 3183 transaction prices from 467 trading platform, covering over 95% of the currency species, more than 90% trading platform, to refresh the updated data with an average of 3.9 seconds.

We will grab information which affecting the blockchain market investment using the crawling technology and classify into news and UGC information. There are 6786 information sources in total and an average of 8.76 million unstructured data. We retrieve blockchain's hidden information by monitoring and mining the blockchain node data, group the data into 17 categories.

2）Data quantification

26 global data centers perform data processing, remove noise, input quantitative models, to form a central structured database of transaction data; through semantic analysis to process the original data of public sentiment processing, make word segmentation and tagging, depth learning quantified into a series of indicators of market bullish/bearish such as impact factors and emotional factors; with technology to process blockchain data, to form real-time data sources.

Through AI artificial intelligence to quantitative process the data, to make Quantitative processing around 983 dimensions, to form 28919 intermediate factor machine learning, which finally forms the output result of 397 kinds direct quantified data .As we all know, data modeling process and artificial intelligence training require a large amount of computation, we use blockchain technology to recruit partners to solve the problem of computing power with carrying out distributed computing, so as to promote the related technology landing time 87% higher than the traditional model ROI.

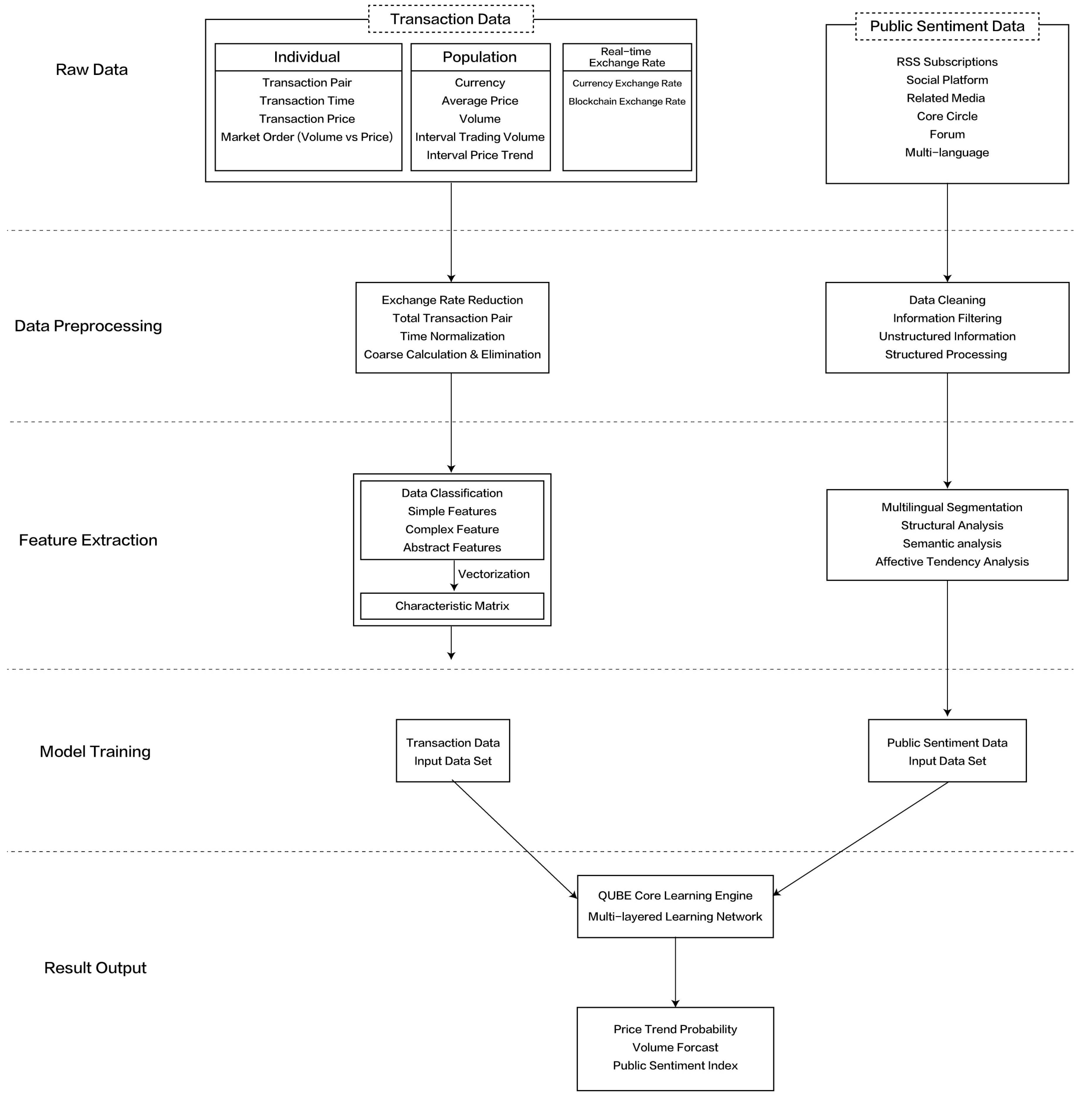
3） Intelligent Strategies

The daily average of 5.6017T data, including a lot of valuable information, through the machine learning and depth learning algorithm, on the dimension of the minimum timestamp for 1 minute to lead in training for iterative results, to optimize the best digital currency investment strategy. (For basic algorithms, see "Appendix: Open Engine Core Algorithm Documentation.")

The QUBE engine starting from July 2017, until now it has been running for five months with algorithm and model training, and after AI investment strategy has started, the same period earnings of bitcoin rise 27.9%. And the monthly increase the engine itself rises 11.6%. The goal of QUBE engine is to launch 60 kinds of intelligent quantitative strategy and productization in 2018.

2.2 Core Model

The core of the QUBE engine is a model based on depth learning algorithm, it designed to use massive historical data of various blockchain currencies, combined with automatically-captured daily trading data and relevant blockchain ecology data (such as various transaction information, public sentiment data, block data, currency exchange rate, etc.) as model input sample data, through a large number of operation training to optimize AI model iteratively, and finally to realize the dynamic risk rule of price changing of a block within a certain time or in a specific situation.



The original model data is mainly divided into transaction data and public sentiment data. Among them, the transaction data is quantitative data, including major trading platform transaction data, market data, OTC transaction data records, and real-time transaction rates of various currencies. Public sentiment data are qualitative data, including all kinds of news, consultation, announcement and user UGC information. The public sentiment data is finally quantified as an influential factor for a certain time period, and is also used as a data source for AI model together with the transaction data.

In accordance with the definition of market trends indicators, the market background is divided into several situations based on 27 factors such as bullish, bearish, shock; according to the currency issuance time, currency is divided into 36 kinds such as new currency, short-term, long-term; according to the currency price, the currency is divided into 45 types such as tiny, small, medium and large; according to public sentiment index, the market sentiment is divided into 13 types such as buy, sell, wait and see; according to the above methods, to extract and classify the entire blockchain market with N factors, that is, the N-dimensional market, if there are M categories in each dimension, the market is quantified as a matrix of NxM, to abstract the price, short-term trend, the mid and long-term trend of the NxM group, which named feature vectorization.

According to the above classification, the historical data at any time and the daily updated data are rapidly decomposed into descriptions of various features, it can be quickly incorporated into the final model as input data. Using the machine learning algorithm, to form an AI model to quickly interpret market and predict the trend. This is the core model of the engine.

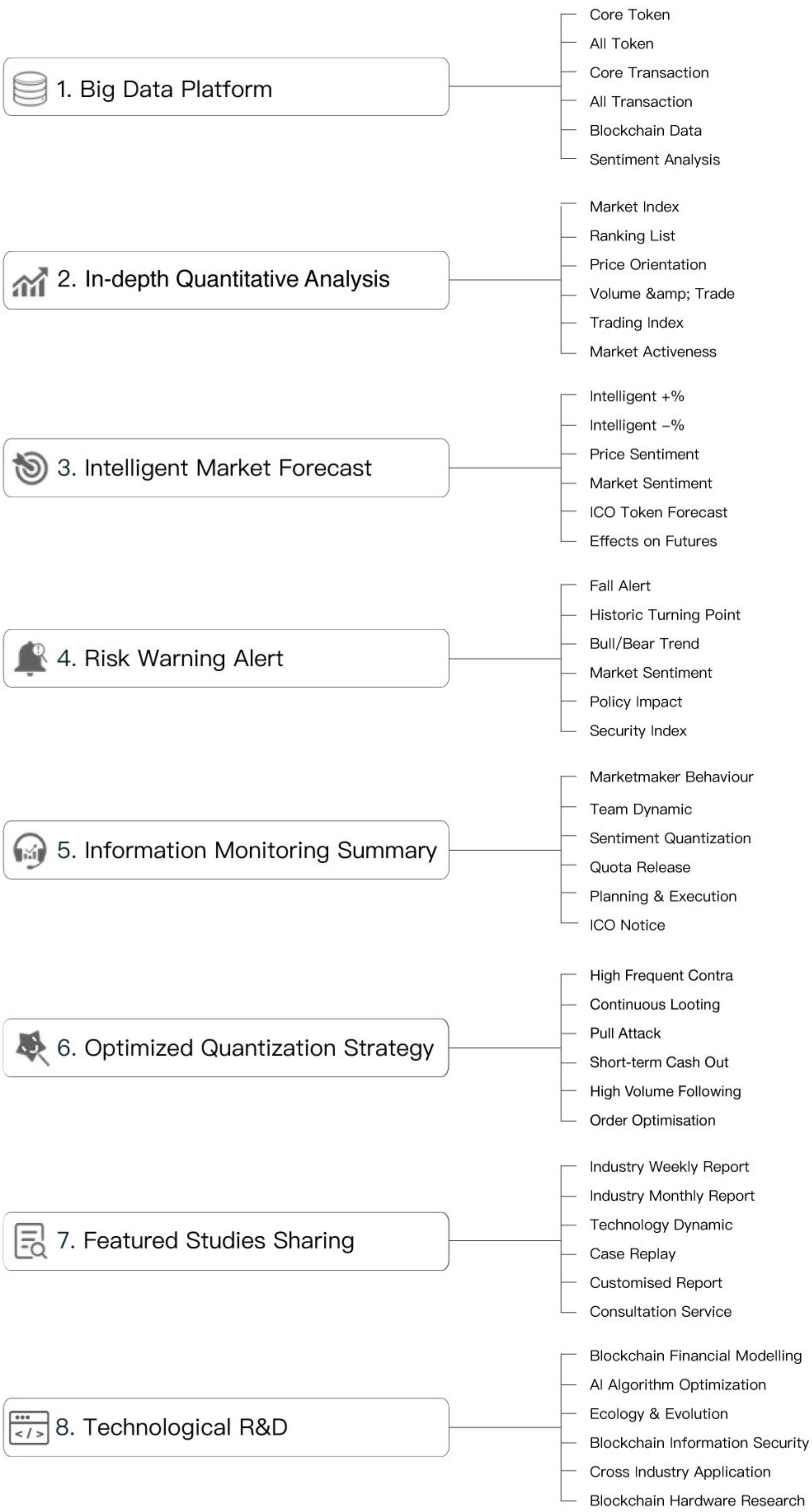
The model is a combination of supervised learning and unsupervised learning, the classification and input of the initial features to help the model to quickly interpret the market. With the increasing of data, and the features become more and more obvious, the model can automatically discover the hidden features in the market and also can automatically delete the features that continue to exceed the threshold of critical value. The final output of the AI model is the probability distribution of the price, volume, short-term trend and long-term trend of the market sentiment in the current period, and finally form the value output of direct quantified data of 397 categories.

2.3 Core Algorithm

The core algorithms of QUBE engine including public sentiment analysis, timing prediction, regression analysis and machine learning models, our team not only have sufficient technical reserves of artificial intelligence, but also has rich experience in quantitative financial modeling. The key of the artificial intelligence is the algorithm and the training of corresponding huge amount data, we public part of the engine core algorithms for industry feedback and verification. (See the part 6 appendix)

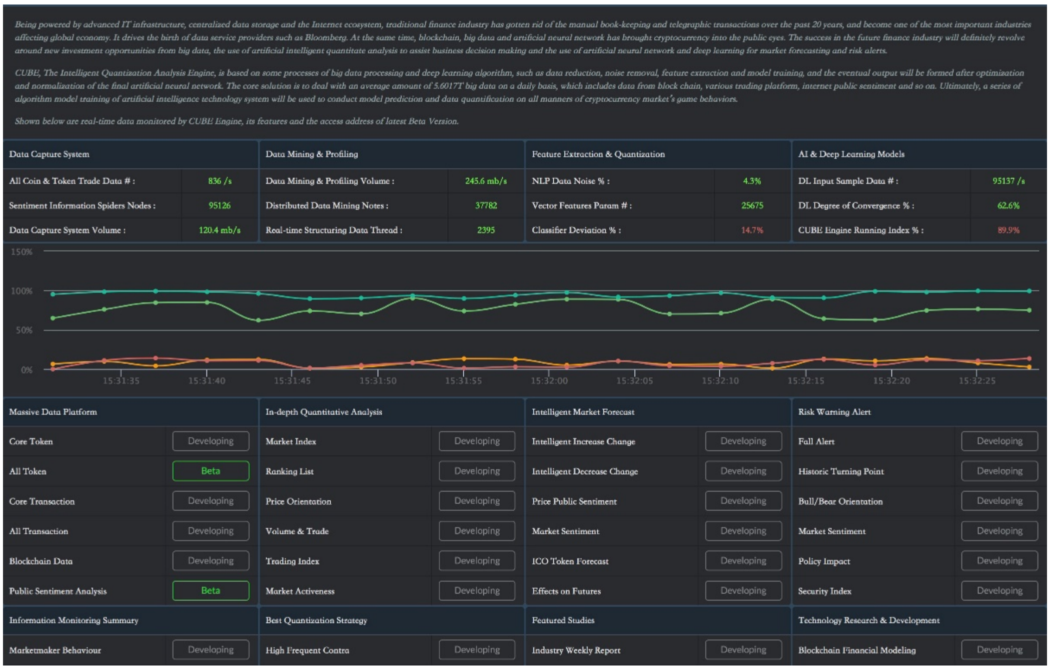
3. Product Structure & Plan

3.1 Product Structure Introduction



3.2 Product Structure and Planning

1) Showcase: Home Page (All function modules)

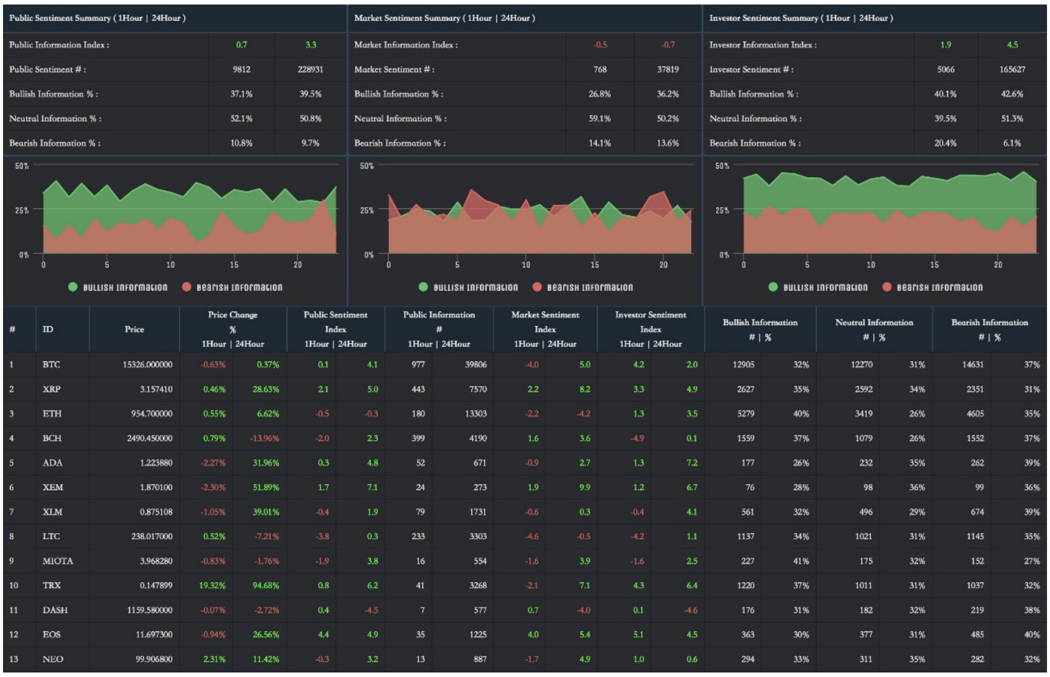


QUBE engine real-time updates the operating status indicators of the four key modules (the original data acquisition system, data processing and structured systems, feature extraction and quantification system, AI depth learning model system). QUBE engine will be gradually launched on schedule with 8 categories, 48 function modules.

2) Showcase - Function module: (Big data platform / All Tokens)

Currently, QUBE Beta V0.1 version has launched "All Token" function module, providing 376 trading platform prices, real-time pending orders, trading volume and other transaction data of 1385 currencies, every 3.6 minutes update and recycle, the data of which 176 coins is updated in 3.9 seconds.

3) Showcase: Function module (Public Sentiments quantitative indicators: [-10, +10])

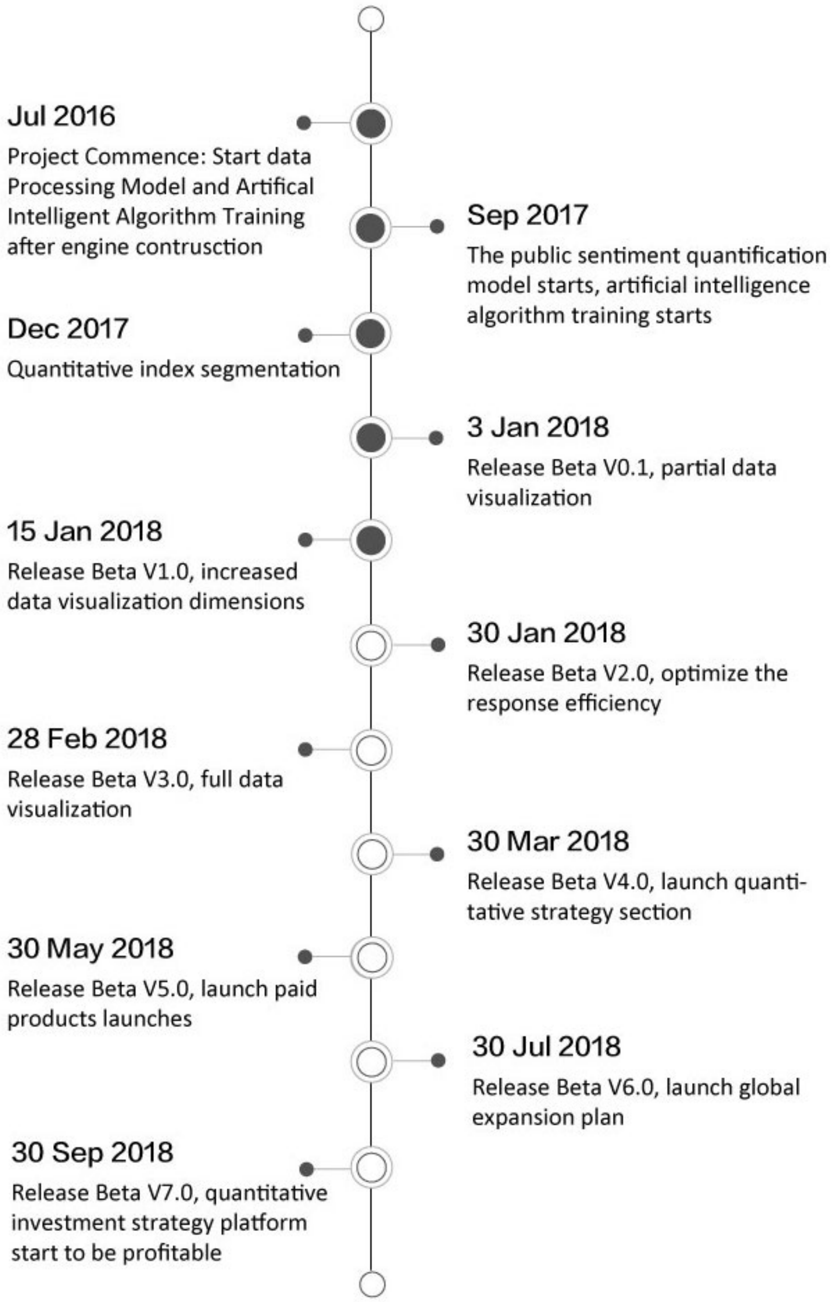


Public Sentiment Summary (1Hour | 24Hour)			Market Sentiment Summary (1Hour | 24Hour)			Investor Sentiment Summary (1Hour | 24Hour)		
Public Information Index :	0.7	3.3	Market Information Index :	-0.5	-0.7	Investor Information Index :	1.9	4.5
Public Sentiment # :	9812	228931	Market Sentiment # :	768	37819	Investor Sentiment # :	5066	165627
Bullish Information % :	37.1%	39.5%	Bullish Information % :	26.8%	36.2%	Bullish Information % :	40.1%	42.6%
Neutral Information % :	52.1%	50.8%	Neutral Information % :	59.1%	50.2%	Neutral Information % :	39.5%	51.3%
Bearish Information % :	10.8%	9.7%	Bearish Information % :	14.1%	13.6%	Bearish Information % :	20.4%	6.1%

#	ID	Price	Price Change % 1Hour | 24Hour		Public Sentiment Index 1Hour | 24Hour		Public Information # 1Hour | 24Hour		Market Sentiment Index 1Hour | 24Hour		Investor Sentiment Index 1Hour | 24Hour		Bullish Information # | %		Neutral Information # | %		Bearish Information # | %	
1	BTC	15326.000000	-0.63%	0.37%	0.1	4.1	977	39806	-4.0	5.0	4.2	2.0	12905	32%	12270	31%	14631	37%
2	XRP	3.157410	0.46%	28.63%	2.1	5.0	443	7570	2.2	8.2	3.3	4.9	2627	35%	2592	34%	2351	31%
3	ETH	954.700000	0.55%	6.62%	-0.5	-0.3	180	13303	-2.2	-4.2	1.3	3.5	5279	40%	3419	26%	4605	35%
4	BCH	2490.450000	0.79%	-13.96%	-2.0	2.3	399	4190	1.6	3.6	-4.9	0.1	1559	37%	1079	26%	1552	37%
5	ADA	1.223880	-2.27%	31.96%	0.3	4.8	52	671	-0.9	2.7	1.3	7.2	177	26%	232	35%	262	39%
6	XEM	1.870100	-2.30%	51.89%	1.7	7.1	24	273	1.9	9.9	1.2	6.7	76	28%	98	36%	99	36%
7	XLM	0.875108	-1.05%	39.01%	-0.4	1.9	79	1731	-0.6	0.3	-0.4	4.1	561	32%	496	29%	674	39%
8	LTC	238.017000	0.52%	-7.21%	-3.8	0.3	233	3303	-4.6	-0.5	-4.2	1.1	1137	34%	1021	31%	1145	35%
9	MIOTA	3.968280	-0.83%	-1.76%	-1.9	3.8	16	554	-1.6	3.9	-1.6	2.5	227	41%	175	32%	152	27%
10	TRX	0.147899	19.32%	94.68%	0.8	6.2	41	3268	-2.1	7.1	4.3	6.4	1220	37%	1011	31%	1037	32%
11	DASH	1159.580000	-0.07%	-2.72%	0.4	-4.5	7	577	0.7	-4.0	0.1	-4.6	176	31%	182	32%	219	38%
12	EOS	11.697300	-0.94%	26.56%	4.4	4.9	35	1225	4.0	5.4	5.1	4.5	363	30%	377	31%	485	40%
13	NEO	99.906800	2.31%	11.42%	-0.3	3.2	13	887	-1.7	4.9	1.0	0.6	294	33%	311	35%	282	32%

QUBE Beta Version V0.1 has launched a public sentiment analysis module for public sentiment function, QUBE crawls the world's a total of 6786 information sources, an average of 8.76 million unstructured data per day. Through the semantic analysis to identify quantification intelligently, finally generate three major quantitative index (network public sentiment index, market bullish index and bearish index, and investment sentiment index), from -10 to +10, has supported 1 hour / 24 hours dimension, the data updated within 5 minutes. QUBE Beta V0.1 has supported information quantification index for Chinese and English, and the information quantification index of the new language with each 5-month cycle.

3.3 Product Development Plan



Global expansion plan:

- 30 Jan 2018: launch Chinese and English versions, including Chinese and English scripts quantitative analysis of public sentiment.
- 30 Feb 30 2018: launch Japan and South Korea version, 5 months later will support Japanese and Korean quantitative analysis of public sentiment.
- By 30 May 2018: more than 30 languages are supported globally.
- After 30 Jan 2018: every 5 months to support one language script quantitative analysis of public sentiment

QUBE only official information website: http://www.qube.vip, welcome to test t QUBE engine Beta version.

4. Early Investment & Team

4.1 Early Institutional Investment

INBlockchain Inc. （Li Xiao Lai）

Alpha Key Capital Inc. （Travis Chaw）

4.2 Consultant Team

Li Xiaolai

The early investor of EOS, SIA, ZCash and yunbi.com

Zhao Dong

Co-found of Moji Weather; Renown investor of blockchain and cryptocurrency; Angel investor of Internet Startup Companies

Shuai Chu

Founder of Qtum; Half-way Doctor of University of Chinese Academy of Sciences; Worked in Alibaba Group; Rich experiences in blockchain technology development and management

Huang Minqiang

Founder and CEO of GXS; MBA of Hong Kong Finance And Economics College; 10 years' experience in Data Exchange, Internet Finance and Blockchain Industry

David Vorick

The co-founder of SIA, Bitcoin Core's contributor

4.3 Founding Team Member

Michael Chen - Founder & CEO

Immense knowledge and rich experiences in finance data and quantitative analysis; 10 years' experience in data mining and analysis in North America; analyze cryptocurrency trend since 2013; successfully develop and operate several internet projects

Zhou Zhuming - CTO (Engineering)

Worked as Product Director in Microsoft headquarters; more than 15 years' experience in data mining analysis; lead commercial data products; sophisticated in quantitative research

Ethan Loh - Chief Quantitative Analysis Scientist

Master of Finance Engineering in Massachusetts Institute of Technology; Expert in Financial Quantitative Analysis; Worked in Standard Chartered Bank and N2N Connect Bhd

Xu Qian - Quantitative Analysis Scientist

Doctor of Finance Engineering in Singapore National University; Data analysis scientists in green energy and economics; in charge of research & development of multiple financial data projects in Credit Suisse Bank of Switzerland

Cao Bin - Product Technology Lead

Doctor of in Tongji University, specific in big data mining analysis; Worked as Data architect at Honeywell; 18 years' experience in data quantification; in charge of research & development of multiple financial data projects

Liang Peng - Public Sentiment Lead

Bachelor and master of Optoelectronic Technology Science in Tsinghua University; worked as core technician in Google's public sentiment analysis team; expert in public sentiment technology; participated in the R&D of multiple public sentiment analysis project

Wang Wei - Artificial Intelligence Technology Lead

Bachelor and master of Computer Science in Tsinghua University; Worked as AI Core Engineer in Beijing Perfect World Network Technology Co. Ltd; AI Technical Consultant at a few Silicon Valley's Project Teams; Senior AI project contributor at Open Source Community

Shawn Xu - Quantitative investment Lead

Master of Computer Science at University of Stamford; Quantitative Investment Analysis; an delver in blockchain technology; Worked at a few blockchain companies and in charge of data technology related work

Liu Jianing - Blockchain Lead

Nanjing University; Senior Expert in blockchain technology; Worked as team lead at several internet security firm; Sophisticated in distributed system, cryptocurrency and encrypted algorithm

Vicky Tian - Project Operation Lead

Singapore Management University & University of Manchester Business School, specified in finance & information management, business innovation, blockchain, equities trading and investment portfolio; Worked as operation lead at several multi-national finance institutions

4.4 Technical Operation Consultant

Azmi Suhaimi - Data Analysis Consultant

Doctor in University of Michigan, specified in artificial intelligence, mathematical model analysis and data study and deep learning of Bloomberg

Johnny Goh Chia Min - Risk Control Consultant

Studied business management in Singapore Management University; worked as at Credit Suisse Bank of Switzerland, in charge of Asia Pacific listed derivatives line control, regulatory oversight and control, sophisticated in OTC trades and settlements

David Chau - Marketing Consultant (International Market)

Rich experience in the financial industry and the marketing field in Southeast Asia and European market

Gary Lim - Legal Consultant

Singapore Lee & Lee law firm's core member of the legal consultant team, in charge of companies' legal risk consultation services

5. Token Assets & Release Plan

We've created QUBE Tokens ("QUBE"), which is based on the Ethereum ERC 20 token standard.

QUBE Token is the official cryptocurrency to make purchase on QUBE, the Cryptocurrency Intelligent Investment Analysis Engine's related services.

The QUBE Foundation (hereinafter referred to as "QUBE Foundation") will build a third party integrated information service platform for cryptocurrency's private offering. QUBE Foundation will charge third party service provider a certain amount of service charge accordingly, the revenue will be distributed to the holders of QUBE Tokens.

5.1 Token Release Plan

Total Token Issued will be fixed at1 billion which will be allocated to Targeted Offering (30%), QUBE Foundation (30%), Business Development (20%), QUBE Team (15%) and Community Award (5%).

	First Round	**Second Round**	**Third Round**
	Targeted Cornerstone Investors	**Targeted Institutional Investors**	**Targeted Investors**
Share	30%		
Description	Locked for 6 months	Locked for 3 months before releasing 1/3 monthly	Not locked From 150 ETH
Market $	1 ETH = 10000 QUBE		
Start/End	Start: 9 Jan 2018; End: 16 Jan 2018		
Total	30000 ETH		

- <u>Targeted Offering:</u> 300,000,000 QUBE Tokens, which is 30% of the total amount. Unsold tokens will go to QUBE Foundation.
- <u>QUBE Foundation:</u> 300,000,000 QUBE Tokens, which is 30% of the total amount. The tokens will be locked for half a year and released on a yearly basis. The use of QUBE assets must follow the principles of openness and transparency. And their use must follow independent accounts and digital asset wallet that determines based on the

budget. QUBE Foundation will publicize the use of assets on our official website for governance.

- Business Development: 200,000,000 QUBE Tokens, which is 20% of the total amount. It will be used for QUBE's business development and strategic cooperation.
- QUBE Team: 150,000,000 QUBE Tokens, which is 15% of the total amount. 5% of QUBE Tokens belong to QUBE Team will be released within 6 months while 10% of that will be released during next 6 months.
- Community Rewards: 50,000,000 QUBE Tokens, which is 5% of the total amount. QUBE will issue no more than 5% QUBE Tokens of the total amount to our community members. The QUBE Tokens can be used to purchase our paid services.

5.2 Token Repurchase Plan

QUBE will use part of profit from platform sales to repurchase the QUBE tokens in the secondary market. The repurchased QUBE Tokens will be burned until its reaches 20% of the total issued amount. We will ensure the openness and transparency of the whole process, and users can refer to the QUBE blockchain browser.

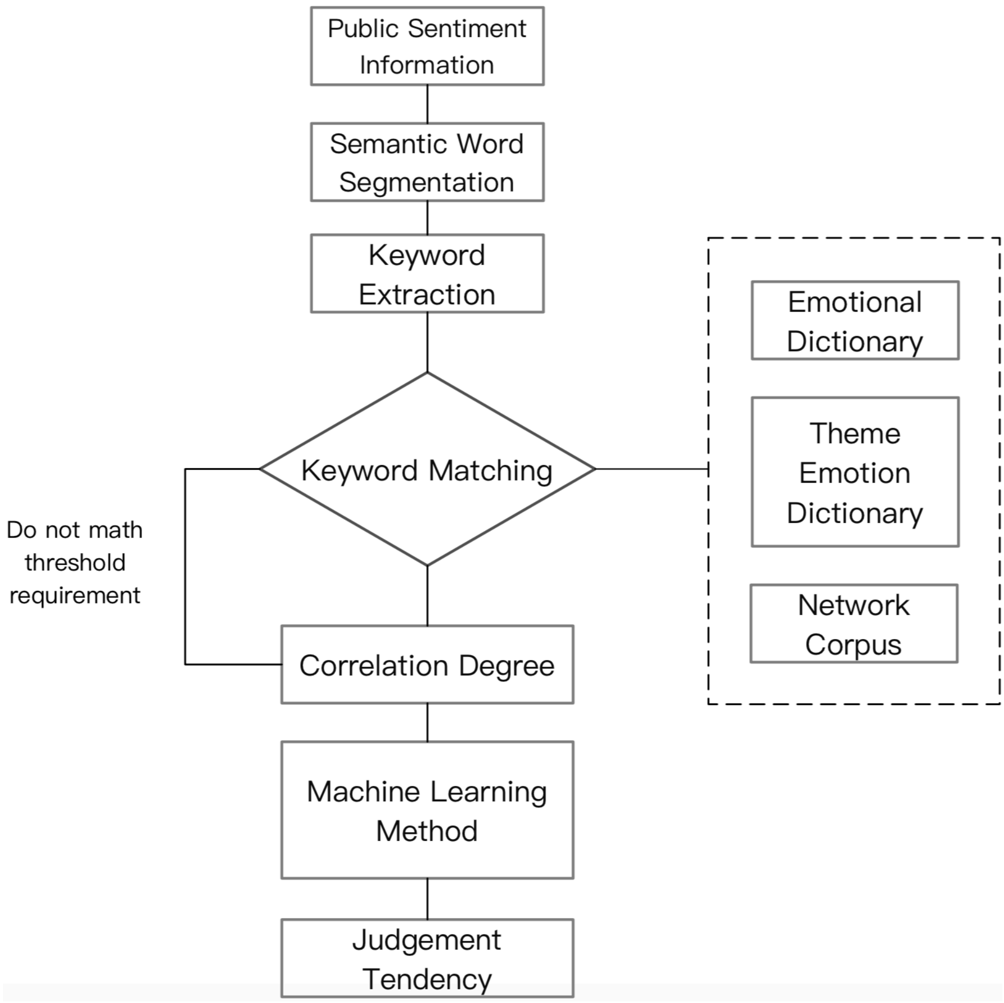
5.3 Rules of Online Trading

QUBE Token will be listed on the exchange before 30 Jan 2018.

6. Appendix： Engine's Open Core Algorithm Document

The core algorithm of QUBE engine includes public sentiment analysis, time sequence forecast, regression analysis and machine learning model. Our team has sufficient technical skills on artificial intelligence and rich experience in quantitative financial modeling. The key to artificial intelligence is the algorithm and the corresponding large amount of data training, and we expose a small number of basic algorithms that don't involve the core.

6.1 Public Sentiment Quantification Model



We will apply "feature extractions" methodology to the main content of public sentiment such as title, published time, author, detailed information and comments. After undergoing word segmentation, the features of the content will be converted into a feature set shown as below:

$$\mathrm{T} = \left(\text{Keyword 1、Keyword 2、Keyword n} \ldots,\ t_n\right)$$

Calculate the similarity between public sentiments by normalizing the contents to feature vectors. Computing function:

Similar function - Using the angle between vectors to represent the similarity, which is angel cosine:

$$\cos(\mathrm{A}, \mathrm{B}) = \frac{\sum A \times B}{\sqrt{\sum A^2}\sqrt{\sum B^2}}$$

Distance Function - Using distance between vectors to represent similarity. The smaller the distance, the higher similarity:

$$\mathrm{dist(A,B)} = \sqrt{\sum |A-B|^2}$$

A and B are two feature vectors of two public sentiment contents. Eventually, the public sentiment factors are formed based on features' distributions by using statistics.

6.2 Feature Extraction Algorithm

Use and support vector machine algorithm to generalize error bounds:

$$\mathrm{R(w)} \le R_{emp}(\mathrm{w}) + \Phi(\mathrm{n/h})$$

Among this equation, $\mathrm{R(w)}$ represents real risk, $R_{emp}(\mathrm{w})$ represents experience risk while $\Phi(\mathrm{n/h})$ represents confidence risk.

Find out the optimal set of vector factors by using Logistic algorithm. The value after mapping is considered to be the probability when y=1:

$$h_\theta(x) = \mathrm{g}(\theta^T\mathrm{x}) = \frac{1}{1+e^{-\theta^T x}} \quad , \quad \mathrm{g(z)} = \frac{1}{1+e^{-z}} \begin{cases} \mathrm{P}(y=1 \mid x;\theta) = h_\theta(\mathrm{x}) \\ \mathrm{P}(y=0 \mid x;\theta) = 1 - h_\theta(\mathrm{x}) \end{cases}$$

The ultimate goal is to make the model training data features $\theta^T x \gg 0$. The hyperplane formula $\mathrm{f(x)} = w^T\mathrm{x} + \mathrm{b}$ represents：

$$\begin{cases} \mathrm{f(x)} > 0 & \mathrm{y} = 1 \\ \mathrm{f(x)} < 0 & \mathrm{y} = -1 \end{cases}$$

Bring the data point x into the classification function to judge its category, and the classification problem is transformed to find the golden cut. Provide a training sample$\left(x^{(i)}, y^{(i)}\right)$, X represents a feature and Y represents the result label. i represents No. i sample:

$$\hat{\gamma}^{(i)} = y^{(i)}\left(w^T x^{(i)} + b\right) \qquad\qquad \hat{\gamma} = \min_{i=1,\ldots,\mathrm{m}} \hat{\gamma}^{(i)}$$

Add constraints to W and B, determine the only w and B, and describe the normalized distance from the geometric interval.

$$x = x^{(i)} - \gamma^{(i)} \frac{w}{||w||} \quad w^T x + b = 0 \text{且} \; w^T w = ||w||^2$$

$$\gamma^{(i)} = \frac{w^T x^{(i)} + \mathrm{b}}{||w||} = (\frac{w}{||w||})^T x^{(i)} + \frac{b}{||w||}$$

$$\gamma = \frac{w^T x + b}{||w||} = \frac{f(x)}{||w||}$$

$$\mathrm{y}(w^T\mathrm{x} + \mathrm{b}) = 1,$$

$$y(w^T x + b) > 1$$

6.3 Time series prediction algorithm

In the time dimension, the regression moving average model (ARMA) will conduct price forecasting based on Autoregressive model（AR）and Moving Average Model（MA）.

$$x_t = \phi x_{t-1} + \epsilon_t$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p} + \epsilon_t.$$

$$\begin{aligned}
E_t(x_{t+1}) &= E_t(\phi x_t + \epsilon_{t+1}) = \phi x_t \\
E_t(x_{t+2}) &= E_t(\phi^2 x_t + \phi \epsilon_{t+1} + \epsilon_{t+2}) = \phi^2 x_t \\
\ldots &= \ldots \\
E_t(x_{t+k}) &= E_t(\phi^k x_t + \phi^{k-1} \epsilon_{t+1} + \ldots + \epsilon_{t+k}) = \phi^k x_t
\end{aligned}$$

$$\begin{aligned}
\mathrm{Var}_t(x_{t+1}) &= \mathrm{Var}_t(\phi x_t + \epsilon_{t+1}) = \sigma_\epsilon^2 \\
\mathrm{Var}_t(x_{t+2}) &= \mathrm{Var}_t(\phi^2 x_t + \phi \epsilon_{t+1} + \epsilon_{t+2}) = (1 + \phi^2)\sigma_\epsilon^2 \\
\ldots &= \ldots \\
\mathrm{Var}_t(x_{t+k}) &= \mathrm{Var}_t(\phi^k x_t + \phi^{k-1} \epsilon_{t+1} + \ldots + \epsilon_{t+k}) = \sum_{j=0}^{k-1} \phi^{2j} \sigma_\epsilon^2
\end{aligned}$$

$$E_t(x_{t+k}) \to 0$$

$$\mathrm{Var}_t(x_{t+k}) \to \sigma_\epsilon^2 / (1 - \phi^2)$$

$$x_t = \epsilon_t + \theta \epsilon_{t-1}$$

$$x_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}.$$

$$\begin{aligned}
E_t(x_{t+1}) &= E_t(\epsilon_{t+1} + \theta \epsilon_t) = \theta \epsilon_t \\
E_t(x_{t+2}) &= E_t(\epsilon_{t+2} + \theta \epsilon_{t+1}) = 0 \\
\ldots &= \ldots \\
E_t(x_{t+k}) &= E_t(\epsilon_{t+k} + \theta \epsilon_{t+k-1}) = 0
\end{aligned}$$

$$\begin{aligned}
\mathrm{Var}_t(x_{t+1}) &= \mathrm{Var}_t(\epsilon_{t+1} + \theta \epsilon_t) = \sigma_\epsilon^2 \\
\mathrm{Var}_t(x_{t+2}) &= \mathrm{Var}_t(\epsilon_{t+2} + \theta \epsilon_{t+1}) = (1 + \theta^2)\sigma_\epsilon^2 \\
\ldots &= \ldots \\
\mathrm{Var}_t(x_{t+k}) &= \mathrm{Var}_t(\epsilon_{t+k} + \theta \epsilon_{t+k-1}) = (1 + \theta^2)\sigma_\epsilon^2
\end{aligned}$$

$$
\begin{aligned}
E_t(x_{t+1}) &= E_t(\sum_{j=0}^{q}\theta_j\epsilon_{t+1-j}) = \sum_{j=1}^{q}\theta_j\epsilon_{t+1-j} \\
E_t(x_{t+2}) &= E_t(\sum_{j=0}^{q}\theta_j\epsilon_{t+2-j}) = \sum_{j=2}^{q}\theta_j\epsilon_{t+2-j} \\
\ldots &= \ldots \\
E_t(x_{t+k}) &= E_t(\sum_{j=0}^{q}\theta_j\epsilon_{t+k-j}) = \sum_{j=k}^{q}\theta_j\epsilon_{t+k-j} \quad \text{for} \quad k \le q \\
E_t(x_{t+k}) &= E_t(\sum_{j=0}^{q}\theta_j\epsilon_{t+k-j}) = 0 \quad \text{for} \quad k > q
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Var}_t(x_{t+1}) &= \mathrm{Var}_t(\sum_{j=0}^{q}\theta_j\epsilon_{t+1-j}) = \sigma_\epsilon^2 \\
\mathrm{Var}_t(x_{t+2}) &= \mathrm{Var}_t(\sum_{j=0}^{q}\theta_j\epsilon_{t+2-j}) = 1 + \theta_1^2\sigma_\epsilon^2 \\
\ldots &= \ldots \\
\mathrm{Var}_t(x_{t+k}) &= \mathrm{Var}_t(\sum_{j=0}^{q}\theta_j\epsilon_{t+k-j}) = \sum_{j=0}^{k}\theta_j^2\sigma_\epsilon^2 \quad \forall \quad k > 0
\end{aligned}
$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q}.$$

6.4 Machine Learning Algorithm

$$F(P'_{j+1}) = \sum_{v \in L(P'_{j+1})} \hat{p}_v F(\hat{q}_v) = \sum_{i=1}^{w_{j+1}} \hat{p}_i F_i,$$

$$F(P_{j+1}) = \sum_{i=1}^{w_{j+1}} \hat{p}_i F(\hat{q}_i).$$

$$F(P_{j+1}) - F(P'_{j+1}) = \sum_{i=1}^{w_{j+1}} \hat{p}_i (F(\hat{q}_i) - F_i).$$

$$
\begin{aligned}
F(P_{j+1}) - F(P'_{j+1}) &\le \frac{\gamma_{j+1}}{3} \sum_{i=2}^{w_{j+1}-1} \hat{p}_i \inf_{x \in I_i} F(x) + (\hat{p}_1 + \hat{p}_{w_{j+1}}) \frac{\gamma_{j+1}}{6} F(P_j) \\
&\le \left(\frac{\gamma_{j+1}}{3} + \frac{\gamma_{j+1}}{6} \right) F(P_j).
\end{aligned}
$$

$$
\begin{aligned}
D_{P_0}(P_0^{'} || P_1) &= \sum_x P_0(x) \log \frac{P_0^{'}(x)}{P_1(x)} = \sum_{x^1, x^2, \ldots, x^n} P_0(x^1, x^2, \ldots, x^n) \sum_i \log \frac{P_0(x^i)}{P_1(x^i)} \\
&= \sum_i P_0(x^i) \log \frac{P_0(x^i)}{P_1(x^i)} = D(P_0^{'} || P_1) = D(P_0 || P_1) - D(P_0 || P_0^{'})
\end{aligned}
$$

$$P(z|u) = \frac{\sum_{(y, v_{u,y})} P(z|u, y, v_{u,y})}{\sum_{z'} \sum_{(y, v_{u,y})} P(z'|u, y, v_{u,y})} = \frac{\sum_{(y, v_{u,y})} P(z|u, y, v_{u,y})}{|\{(u, y, v_{u,y})\}|}.$$