Табела 2.2: Фуриеови парови на некои елементарни функции

	$f(t), \quad t \in \mathbb{R}$	$\Longleftrightarrow$	$F(\omega) = \mathcal{F}\{f(t)\}, \quad \omega \in \mathbb{R}$
(1)	$\delta(t)$	$\Longleftrightarrow$	1
(2)	1	$\Longleftrightarrow$	$2\pi\,\delta(\omega)$
(3)	$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$	$\Longleftrightarrow$	$\dfrac{1}{j\omega} + \pi\,\delta(\omega)$
(4)	$p_\tau(t) = \begin{cases} 1, & \lvert\,t\,\rvert \leq \tau/2 \\ 0, & \lvert\,t\,\rvert > \tau/2 \end{cases}, \ \tau \in \mathbb{R}_+$	$\Longleftrightarrow$	$\tau\,\dfrac{\sin(\omega\tau/2)}{\omega\tau/2}$
(5)	$\dfrac{\sin(\Omega t)}{\pi t}, \ \Omega \in \mathbb{R}_+$	$\Longleftrightarrow$	$p_{2\Omega}(\omega) = \begin{cases} 1, & \lvert\,\omega\,\rvert < \Omega \\ 0, & \lvert\,\omega\,\rvert > \Omega \end{cases}$
(6)	$\operatorname{sgn}(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$	$\Longleftrightarrow$	$\dfrac{2}{j\omega}$
(7)	$\lvert\,t\,\rvert$	$\Longleftrightarrow$	$-\dfrac{2}{\omega^2}$
(8)	$\dfrac{1}{\pi t}$	$\Longleftrightarrow$	$-j\operatorname{sgn}(\omega)$
(9)	$e^{j\omega_0 t}, \ \omega_0 \in \mathbb{R}$	$\Longleftrightarrow$	$2\pi\,\delta(\omega - \omega_0)$
(10)	$\cos\omega_0 t, \ \omega_0 \in \mathbb{R}$	$\Longleftrightarrow$	$\pi\big(\delta(\omega + \omega_0) + \delta(\omega - \omega_0)\big)$
(11)	$\sin\omega_0 t, \ \omega_0 \in \mathbb{R}$	$\Longleftrightarrow$	$j\pi\big(\delta(\omega + \omega_0) - \delta(\omega - \omega_0)\big)$
(12)	$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT), \ T \in \mathbb{R}$	$\Longleftrightarrow$	$\dfrac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta\Big(\omega - \dfrac{2\pi n}{T}\Big)$

3.1.2 Properties

3.1.2.1 Properties of Fourier Transform

TABLE 3.1 Properties of Fourier Transform

	Operation	$f(t)$	$F(\omega)$
1.	Transform-direct	$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$
2.	Inverse transform	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$	$F(\omega)$
3.	Linearity	$af_1(t) + bf_2(t)$	$aF_1(\omega) + bF_2(\omega)$
4.	Symmetry	$F(t)$	$2\pi f(-\omega)$
5.	Time shifting	$f(t \pm t_o)$	$e^{\pm j\omega t_o}F(\omega)$
6.	Scaling	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
7.	Frequency shifting	$e^{\pm j\omega_o t}f(t)$	$F(\omega \mp \omega_o)$
8.	Modulation	$\begin{cases} f(t)\cos\omega_o t \\ f(t)\sin\omega_o t \end{cases}$	$\begin{matrix} \frac{1}{2}[F(\omega+\omega_o)+F(\omega-\omega_o)] \\ \frac{1}{2j}[F(\omega-\omega_o)-F(\omega+\omega_o)] \end{matrix}$
9.	Time differentiation	$\frac{d^n}{dt^n}f(t)$	$(j\omega)^n F(\omega)$
10.	Time convolution	$f(t)*h(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)\,d\tau$	$F(\omega)H(\omega)$
11.	Frequency convolution	$f(t)h(t)$	$\frac{1}{2\pi}F(\omega)*H(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\tau)H(\omega-\tau)\,d\tau$
12.	Autocorrelation	$f(t)\bigstar\, f^*(t) = \int_{-\infty}^{\infty} f(\tau)f^*(\tau-t)\,d\tau$	$F(\omega)F^*(\omega) = \lvert F(\omega)\rvert^2$
13.	Parseval's formula	$E = \int_{-\infty}^{\infty} \lvert f(t)\rvert^2\,dt$	$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2\,d\omega$
14.	Moments formula	$m_n = \int_{-\infty}^{\infty} t^n f(t)\,dt = \frac{F^{(n)}(0)}{(-j)^n}$ where	$F^{(n)}(0) = \left.\frac{d^n F(\omega)}{d\omega^n}\right\rvert_{\omega=0}$, $n = 0,1,2\cdots$
15.	Frequency differentiation	$\begin{cases} (-jt)f(t) \\ (-jt)^n f(t) \end{cases}$	$\begin{matrix} \frac{dF(\omega)}{d\omega} \\ \frac{d^nF(\omega)}{d\omega^n} \end{matrix}$
16.	Time reversal	$f(-t)$	$F(-\omega)$
17.	Conjugate function	$f^*(t)$	$F^*(-\omega)$
18.	Integral $(F(0)=0)$	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{1}{j\omega}F(\omega)$
19.	Integral $(F(0)\neq 0)$	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$