Average consensus in networks of multi-agents with both switching topology and coupling time-delay

Christophoros Bekos*
* undergraduate students in Aristotle university of Thessalonki (auth), email : mpekchri@auth.gr

Abstract— **This article sums up the most used methods for solving the average consensus problem in directed networks of agents with both time-delay and switching topology. The stability analysis is performed by employing a linear matrix inequality method .Particularly, several feasible linear matrix inequalities are established to determine the maximal allowable upper bound of time-varying communication delays. Numerical results are given with the matlab code to reproduce them.**

I. INTRODUCTION

Cooperative control of multiple agents poses significant theoretical and practical challenges. First, the research objective is to develop a system of subsystems rather than a single system. Second, the communication bandwidth and connectivity of the team are often limited, and the information exchange among agents may be unreliable. It is also difficult to decide what to communicate and when and with whom the communication takes place. Third, arbitration between team goals and individual goals needs to be negotiated. Fourth, the computational resources of each individual agent will always be limited.

A centralized coordination scheme relies on the assumption that each member of the team has the ability to communicate to a central location or share information via a fully connected network. As a result, the centralized scheme does not scale well with the number of vehicles. The centralized scheme may result in a catastrophic failure of the overall system due to its single point of failure. Also, real-world communication topologies are usually not fully connected. In many cases, they depend on the relative positions of other agents and on other environmental factors and are therefore dynamically changing in time.

To understand the fundamental issues inherent in all cooperative control problems, we offer the following, intuitively appealing, fundamental axiom:

Shared information is a necessary condition for cooperation. Due to the introduction of switching topologies and time-varying delays, the present methods in [1] and [2] do not apply. In this article, we present a linear matrix inequality method to deal with this problem , proposed in [3]. The linear matrix inequality method has been extensively used in delay system. However, few people extend this approach to consensus problems. Here, with the help of the linear matrix inequality approach we will prove that the group of dynamic agents can reach average consensus asymptotically for appropriate time-varying delays if the network topology is connected.

II. NOTATIONS AND PRELIMINARIES

Let G = (V, E, A) be a weighted undirected graph of order n (n $\geq$ 2) with the set of nodes V = { v_1 , ... , v_n }, set of edges E $\subseteq$ V $\times$ V, and a symmetric weighted adjacency matrix A = $[a_{ij}]$ with nonnegative adjacency elements a_{ij}. The node indexes belong to a finite index set I = {1,2,...,n}. An edge of G is denoted by e_{ij} = (v_i,v_j). The adjacency elements associated with the edges of the graph are positive, i.e., $e_{ij} \in$ E if and only if $a_{ij} > 0$. Moreover, we assume a_{ii} = 0 for all i $\in$ I.The set of neighbors of node v_i is denoted by N_i = { v_j $\in$ V: (v_i ,v_j) $\in$ E}. An undirected graph is called connected if any two distinct nodes of the graph can be connected via a path that follows the edges of the graph. Let $x_i \in$ R denote the value of node v_i. We refer to G_x = (G, x) with x = { x_1 , ... , x_n }^{T} as a network (or algebraic graph) with value x $\in R^n$ and topology (or information flow) G.The value of a node might represent physical quantities such as attitude, position, temperature, voltage, and so on.

III. PROBLEM FORMULATION

Suppose each node of a graph is a dynamic integrator agent with dynamics

$$\dot{x_i} = u_i, \ \ i \in I \ \ (1)$$

We say a state feedback u_i = k_i(x_{j1},...,x_{jm_i}) is a protocol (A) with topology G , if the cluster J_i = { v_{j1} , ... , v_{jm_i} } of nodes with $j1$,...,jm_i $\in$ I satisfies the property :

$$J_i \subseteq v_i \cup N_i \ .$$

Under protocol (A), system (1) reduces to

$$\dot{x} = k_i(x_{j1}, ..., x_{jm_i}), \ \ i \in I \ \ (2)$$

Let a = x(0) be the initial state of system (2). We say system (2) achieves X-consensus asymptotically if for any a $\in R^n$, $x_i(t) \rightarrow$ X(a) as t $\rightarrow \infty$ for each i $\in$ I, where X : $R^n \rightarrow$ R be a function of n variables $x_1, ..., x_n$.

Particularly, when X(x) = Ave(x(0)) = $\frac{1}{n}\sum_{i=1}^{n}(x_i(0))$ we say system (2) achieves average consensus asymptotically. Solving the average consensus problem is a typical example of distributed computation of a linear function X(a) = Ave(a) using a network of dynamic systems. This is a more challenging task than reaching a consensus with initial state a, since an extra condition $\lim_{t\rightarrow\infty} x_i(t)$ = Ave(a), i $\in$ I has to be satisfied, which relates the final state of the system to the initial state a .

IV. METHODOLOGY

A. Proposed methodologies

Here we will discuss 3 different proposed methodologies the average consensus problem when communication is affected by time-delays . Let τ_{ij} denote the time delay for information communicated from agent j to agent i.The first consensus protocol is :

$$u_i(t) = \sum_{v_j \in N_i} a_{ij} * [x_j(t-\tau_{ij}) - x_i(t-\tau_{ij})] \quad (P1)$$

This protocol has been proposed in [2] and it has been proved that in the simplest case where $\tau_i j = \tau$ and the network topology is fixed and undirected, average consensus is achieved if and only if $\tau \in [0, \frac{\pi}{2\lambda_{max}(L)}]$, where L is the graph Laplacian matrix of topology G. A second consensus protocol has been established in [1], as :

$$u_i(t) = \sum_{v_j \in N_i} a_{ij} * [x_j(t-\tau_{ij}) - x_i(t)] \quad (P2)$$

That is, communication delays only affect the information state that is being transmitted.However, for a switching topology, the case where communication delays are different and time-varying is examined in [3] using the protocol (P1).

B. Protocol P1 for solving average consensus problem with both switching topologies and multiple time-varying delays

Using the protocol (P1) we have :

$$\dot{x_i}(t) = \sum_{v_j \in N_i} a_{ij} * [x_j(t-\tau_{ij}) - x_i(t-\tau_{ij})]$$

where $\tau_{ij}(t)$, i,j $\in$ I, are time-varying communication delays and satisfy $\tau_{ij}(t) = \tau_{ji}(t)$. This protocol can be written in matrix form as :

$$\dot{x(t)} = -\sum_{k=1}^{r} L_k * x(t-\tau_k(t)))$$

where r $\leqslant \frac{n(n-1}{2}$, $\tau_k(\cdot) \in \{\tau_{ij}(\cdot) : i,j = 1...n\}$, for k=1,...,r and $L_k = [l_{ij}]$ is the matrix defined by :

$$L_{kij} = \begin{cases} -a_{ij} & \text{j} \neq \text{i} , \tau_k(\cdot) = \tau_{ij}(\cdot) \\ 0 & \text{j} \neq \text{i} , \tau_k(\cdot) \neq \tau_{ij}(\cdot) \\ \sum_{j=1}^{n} l_{kij} & \text{j = i} \end{cases}$$

Based on the assumptions $\tau_{ij}(\cdot) = \tau_{ji}(\cdot)$ and A is symmetric, it is easy to see that L_k is symmetric and $\sum_{k=1}^{r} L_k$ = L = $[l_{ij}]$, where L is the graph Laplacian induced by the information flow G and is defined by :

$$l_{ij} = \begin{cases} \sum_{j=1, j\neq i}^{n} a_{ij} & \text{j = i} \\ -a_{ij} & \text{j} \neq \text{i} \end{cases}$$

In a network of mobile agents, it is not hard to imagine that some of the existing communication links can fail simply due to the existence of an obstacle between two agents.The opposite situation can arise where new links between nearby agents are created because the agents come to an effective range of detection with respect to each other. We are interested in investigating such a problem: for a network with switching topology, whether it is still possible to reach a consensus or not. In this case, the following hybrid system is considered:

$$\dot{x(t)} = -\sum_{k=1}^{r} L_{ks} * x(t-\tau_k(t)))$$

Here L_{ks} is the symmetric matrix as defined before(L_k) , but s = $\sigma(t)$ means that the symmetric array changes over time , each time new edges(connections between two agents) are added or erased from the graph .
That being said , reader may refer to [3] for a mathematical prove that protocol P1 guarantees consensus in networks of multi-agents with both switching topology and coupling time-delay , under specific assumptions .

V. NUMERICAL RESULTS

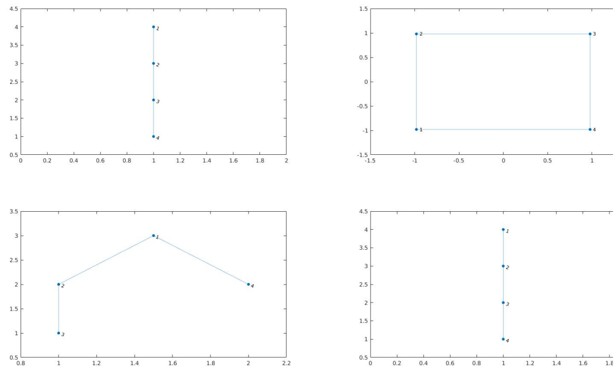
In order to ensure the reader that protocol P1 guarantees consensus we propose numerical results . One can implement P1 using matlab's toolbox LMI . However , in order to present a more compact and comprehensible solution we created our own scripts ,where function average_consensus_test has a prominent role. Function's inputs are :

1) N : total number of agents.
2) x0 : a vector with the initial states of all agents.
3) A : matrix A has an amount of sub-matrices , one for each time iteration . Each sub-matrix of A refers to the adjacency matrix of the agents, at a specific moment (iteration) .
4) iters : total number of iterations required for our simulation
5) Ts : sampling time in seconds
6) times : matrix times has an amount of sub-matrices , one for each time iteration . The expression times(i,j,k) returns the delay time required for communication between agent i and agent j , at simulation's iteration k.
7) titl : is a string that will be displayed in the figure that the function creates

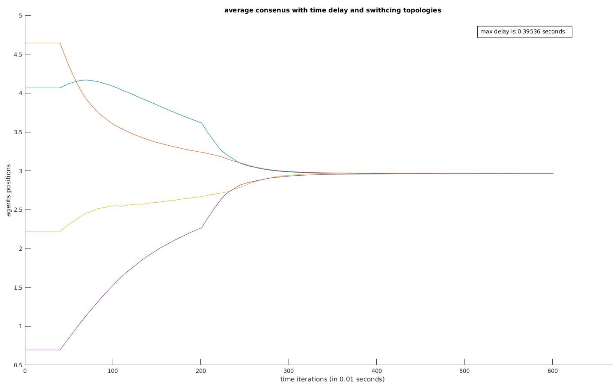
Matrix A and matrix times are size of N $\times$ N $\times$ iters . This function returns the final positions of all agents and also plots theirs positions through iterations. Note that it is constructed in order to simulate agent's positions in 1-D , but is not difficult to be generalized for more dimensions .
Since we have clarified the input and output arguments of average_consensus_is now easy to understand script final. Here we use average_consensus_in order to simulate a situation with both time delays and switching topology . Below we list our results.

A. Switching topology and random time delays between agents among iterations

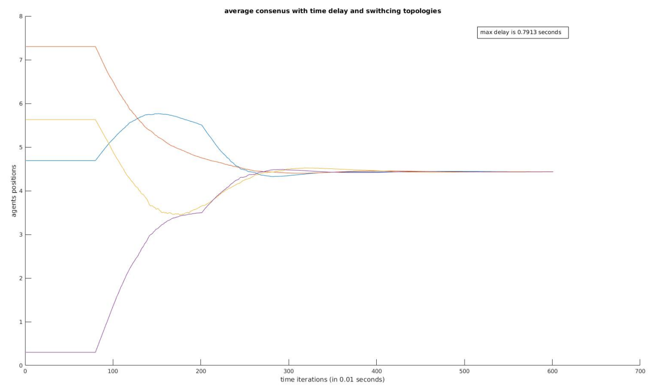
The different topologies used among time iterations can be seen in the figure bellow :



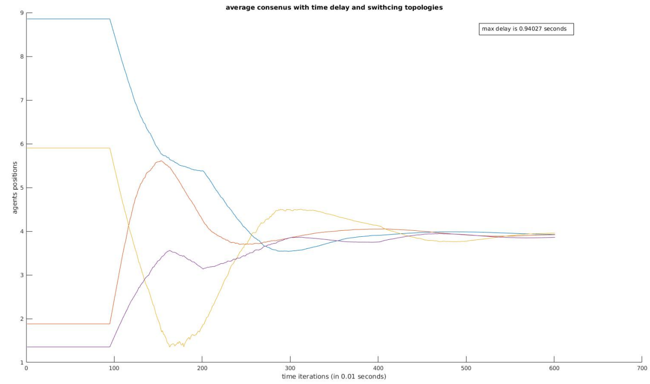
Bellow there are the simulation results for N = 4 , and different maximum time delay choos



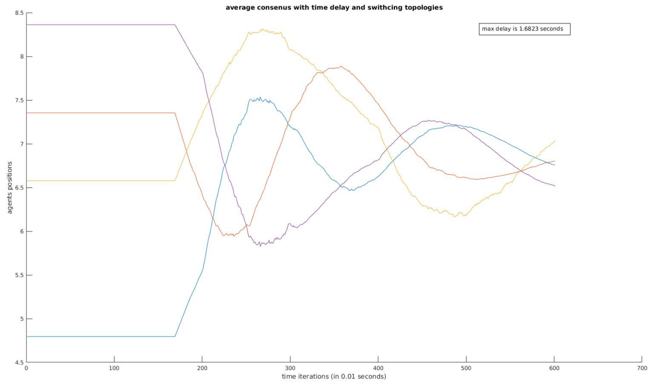
maximum delay = 0,395 seconds



maximum delay = 0,791 seconds



maximum delay = 0,940 seconds



maximum delay = 1,682 seconds

One can realize the significance of the effect of time delays on the final result . Matlab's script named final.m can be used to reproduce the results .

VI. CONCLUSIONS

This article is a project within the framework of the Intelligent Robot Systems course, of Aristotle University of Thessalonki, in year 2017/2018 . The instructor of the course was Mr. Bechlioulis Charalambos and project's task was to investigate , understand and simulate the most famous techniques used for solving average consensus problem . Based on the above one can understand that communication between agents is a a necessary but not sufficient condition for reaching a consensus result. In real life situations , an agent may be able to communicate with only a few others , so a centralized solution must be avoided .On the contrary , using the protocol P1 agents are able to reach a consensus under specific assumptions , even if there is some small delay in their communication . The goal is reached under switching topology , provided that the graph G , that represents agent's network topology , is connected . Simulations were created using only Matlab's scripts , but one could use Matlab's toolbox LMI [4].

VII. FURTHER READING

For a prove of P1 protocol using LMI-based method refer to [3] , for a prove based on a Lyapunov-Krasovskii function refer to [5] . Graph theory , Consensus algorithms, simulations and implementations in multi-vehicle control can be found in [6].A propose of two consensus algorithms, WHCA and LBHCA based on Vicsek's model, can be found in [7] , while [8] is a complete book about consensus problems which includes detailed analyzes of different aspects of the problem and rich literature.

REFERENCES

[1] L. Moreau, Stability of multiagent systems with time-dependent communication links, IEEE Trans. Automat. Control 50 (2005) 169182.

[2] R. Olfati-Saber, R.M. Murray, Consensus problems in networks of agents with switching topology and time-delays, IEEE Trans. Automat. Control 49 (2004) 15201533.

[3] Yuan Gong Sun, Long Wang, Guangming Xie, Average consensus in networks of dynamic agents with switching topologies and multiple time-varying delays, In Systems and Control Letters, Volume 57, Issue 2, 2008, Pages 175-183, ISSN 0167-6911

[4] LMI control toolbox user's guide

[5] Tiecheng ZHANG, Hui YU , Average Consensus in Networks of Multi-Agent with Multiple Time-Varying Delays , Scientific Research , January 21, 2010

[6] Wei Ren and Randal W. Beard ,Distributed Consensus in Multi-vehicle Cooperative Control Theory and Applications

[7] Qin Li, Zhong-Ping Jiang, "Global analysis of multi-agent systems based on Vicseks model", Decision and Control 2007 46th IEEE Conference on, pp. 2943-2948, 2007, ISSN 0191-2216.

[8] Cheng-Lin Liu, Fei Liu ,Consensus Problem of Delayed Linear Multi-agent Systems Analysis and Design