Tutorial 1.3. Introduction to Statistical Quantities in Wind Engineering

Description: Wind data (measured or simulated) in wind engineering is usually recorded as a time series. Typical quantities measured are velocity (certain components) at a reference height or pressure measured at locations of interest along the structure. Evaluating the statistical quantities of these time series is a crucial task. In this tutorial a time series is generated and analyzed. Various statistical quantities, which are introduced during the lecture, are calculated for a generated signal. Tools for extreme values statistics are also addressed with computations demonstrated for the generated signal. Some additional exercises are proposed for individual studies.

Students are advised to complete the proposed excercises

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Reference: G. Coles, Stuart. (2001). An introduction to statistical modeling of extreme values. Springer. 10.1007/978-1-4471-3675-0.

Contents:

1. Generating a time series as a superposition of constant, cosine and random signals
2. Introduction of some common statistical tools in python
3. Interquartile range and box plots
4. Probability Distribution Function (PDF)
5. Fast Fourier Transform (FFT)
6. Extreme Value Statistics
7. Block Maxima (BM)
8. Peak Over Threshold (POT)
9. Generalized Extreme Values (GEV)

In [1]:

```
# import python modules
import numpy as np
import scipy
from matplotlib import pyplot as plt

# import own modules
import custom_utilities as c_utils
```

Creating the time instances as an array

The start time, end time and the number of time steps are specified here for generating the time series.

In [2]:

```
# start time
start_time = 0.0
# end time
end_time = 10.0
# steps
n_steps = 10000
# time step
delta_time = end_time / (n_steps-1)
# time series
# generate grid size vector (array) 1D
time_series = np.arange(start_time, end_time + delta_time, delta_time)
```

Generating signals in time domain (from herein referred to as a certain series (of values)).

Three signals are created.

1. A Harmonic (cosine) signal with given amplitude and frequency
2. A constant signal with given amplitude
3. A random signal with specified distribution and given properties

1. Cosine signal with given amplitude and frequency

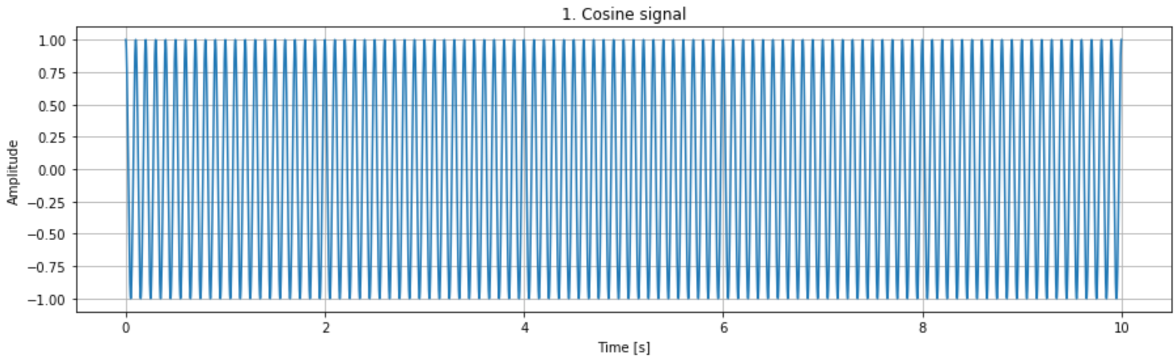
In [3]:

```
# frequency of the cosine
cos_freq = 10
# amplitude of the cosine
cos_ampl = 1
# series of the cosine
cos_series = cos_ampl * np.cos(2*np.pi * cos_freq * time_series)
```

Let us look at the plot to see how the signal looks like

In [4]:

```
plt.figure(num=1, figsize=(15, 4))
plt.plot(time_series, cos_series)
plt.ylabel('Amplitude')
plt.xlabel('Time [s]')
plt.title('1. Cosine signal')
plt.grid(True)
```



Exercise 1: Try different frequencies

Try different frequencies for the harmonic function. What is the relation between frequency and time period? What do you observe in the plots?

2. Constant signal with given amplitude

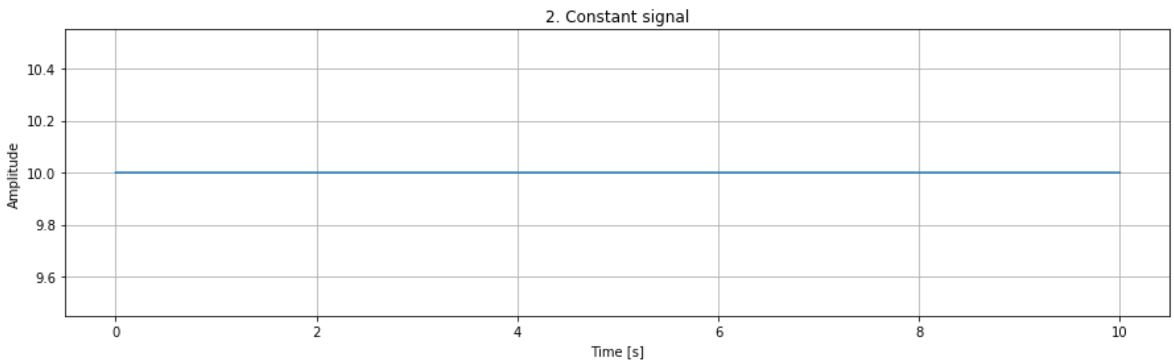
In [5]:

```
# amplitude of the constant
const_ampl = 10
# series of the constant
const_series = const_ampl * np.ones(len(time_series))
```

Let us look at the plot to see how the signals look like

In [6]:

```
plt.figure(num=2, figsize=(15, 4))
plt.plot(time_series, const_series)
plt.ylabel('Amplitude')
plt.xlabel('Time [s]')
plt.title('2. Constant signal')
plt.grid(True)
```



3. Random signal with specified distribution and given properties

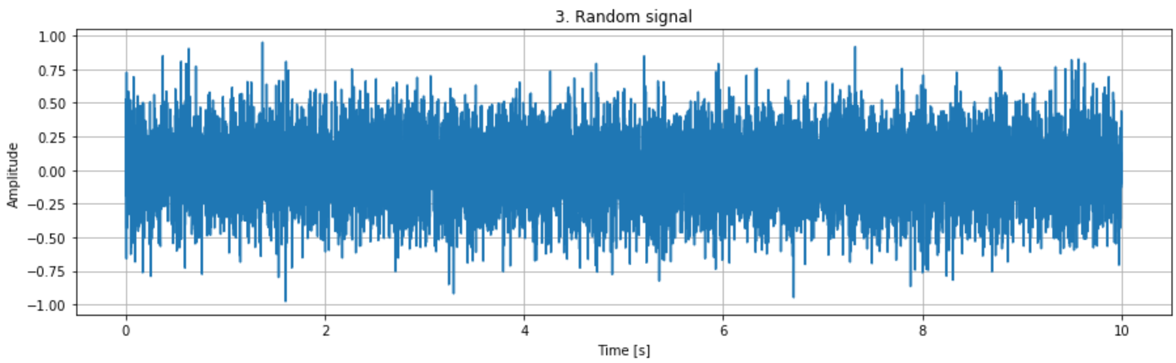
In [7]:

```
# random signal
# assuming nomarl distribution
# with given mean m = 0 and standard deviation std = 0.25
rand_m = 0.0
rand_std = 0.25
# series of the random
rand_series = np.random.normal(rand_m, rand_std, len(time_series))
```

Let us look at the plot to see how the signal looks like

In [8]:

```
plt.figure(num=3, figsize=(15, 4))
plt.plot(time_series, rand_series)
plt.ylabel('Amplitude')
plt.xlabel('Time [s]')
plt.title('3. Random signal')
plt.grid(True)
```



Exercise 2 : Different distributions and parameters for random signal

Instead of the normal (https://docs.scipy.org/doc/numpy/reference/generated/numpy.random.normal.html) distribution for the random signal try lognormal (https://docs.scipy.org/doc/numpy/reference/generated/numpy.random.lognormal.html), beta (https://docs.scipy.org/doc/numpy/reference/generated/numpy.random.beta.html), standard normal (https://docs.scipy.org/doc/numpy/reference/generated/numpy.random.randn.html) and uniform (https://docs.scipy.org/doc/numpy/reference/generated/numpy.random.uniform.html) distribution

In [9]:

```
#rand_series = np.random.lognormal(0, 0.25, len(time_series))
#rand_series = np.random.beta(1, 0.25, len(time_series))
#rand_series = np.random.rand(len(time_series))
#rand_series = np.random.uniform(0,1,len(time_series))
```

4. Generic signal - for example a superposition of the above ones

A general signal (here) is represented as a superposition of the above three - constant, cosine and random signals

Superposed signal

The above three signals are superposed with corresponding weights

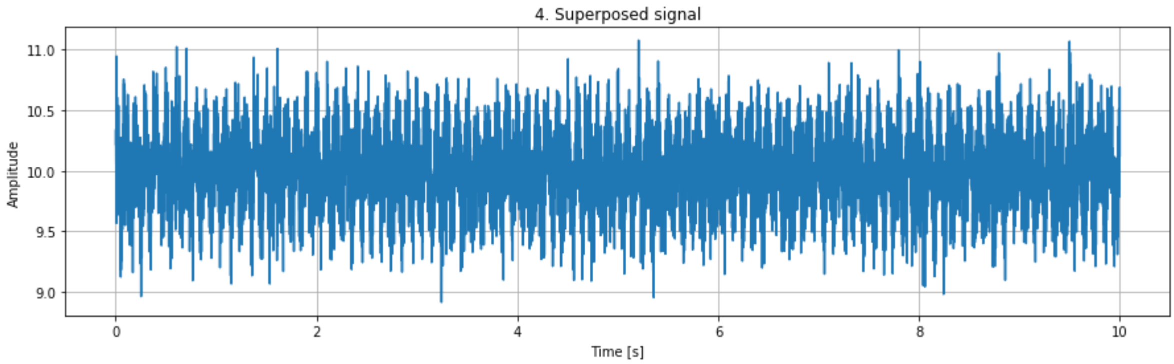
In [10]:

```
# coefs -> weighting factors for the respective series of signals
coef_signal1 = 1
coef_signal2 = 0.25
coef_signal3 = 1
superposed_series = coef_signal1 * const_series + coef_signal2 * cos_series + coef_signal3
```

Let us look at the plot to see how the signal look like

In [11]:

```
plt.figure(num=4, figsize=(15, 4))
plt.plot(time_series, superposed_series)
plt.ylabel('Amplitude')
plt.xlabel('Time [s]')
plt.title('4. Superposed signal')
plt.grid(True)
```



Exercise 3: Different weights for superposition

Try different weights for the superposition. What do you observe in the plots?

Try different frequencies for the cosine function and observe the difference in the superposed signal.

Check Point 1: Discussion

Discuss among groups the observations and outcomes from exercise 1-3.

1.1 Statistical tools and quantities used to evaluate the signal

The following statistical quantities are computed for the given signal.

1. Mean (Arithmetic)

2. Root Mean Square (RMS)
3. Median
4. Standard deviation
5. Skewness

Recall from the lecture the definitions of these quantities. These quantites can be computed using the inbuilt functions of numpy [mean (arithmetic) (https://docs.scipy.org/doc/numpy/reference/generated/numpy.mean.html)](https://docs.scipy.org/doc/numpy/reference/generated/numpy.mean.html), [median (https://docs.scipy.org/doc/numpy/reference/generated/numpy.median.html)](https://docs.scipy.org/doc/numpy/reference/generated/numpy.median.html), [standard deviation (https://docs.scipy.org/doc/numpy/reference/generated/numpy.std.html#numpy.std)](https://docs.scipy.org/doc/numpy/reference/generated/numpy.std.html#numpy.std) and [skewness (https://docs.scipy.org/doc/scipy-0.15.1/reference/generated/scipy.stats.skew.html)](https://docs.scipy.org/doc/scipy-0.15.1/reference/generated/scipy.stats.skew.html)

1. Cosine signal with given amplitude and frequency

In [12]:

```
# computing statistical quantitites (scalar values) and "converting" to an array for later
cos_series_m = np.mean(cos_series) * np.ones(len(time_series))
cos_series_std = np.std(cos_series) * np.ones(len(time_series))
cos_series_rms = np.sqrt(np.mean(np.square(cos_series)))  * np.ones(len(time_series))

# printing statistical quantitites (scalar values) to the console
print('Mean: ', np.mean(cos_series))
print('STD: ', np.std(cos_series))
print('RMS: ', np.sqrt(np.mean(np.square(cos_series))))
print('Median: ', np.median(cos_series))
print('Skewness: ',(np.mean(cos_series) - np.median(cos_series))/np.std(cos_series))
```

```
Mean:  9.999999999999467e-05
STD:  0.7071421285710532
RMS:  0.7071421356417675
Median:  0.00015709533381615863
Skewness:  -8.074095929136411e-05
```

2. Constant signal with given amplitude

In [13]:

```
const_series_m = np.mean(const_series) * np.ones(len(time_series))
const_series_std = np.std(const_series) * np.ones(len(time_series))
const_series_rms = np.sqrt(np.mean(np.square(const_series))) * np.ones(len(time_series))

print('Mean: ', np.mean(const_series))
print('STD: ', np.std(const_series))
print('RMS: ', np.sqrt(np.mean(np.square(const_series))))
print('Median: ', np.median(const_series))
print('Skewness: ', (np.mean(const_series) - np.median(const_series))/np.std(const_series))
```

```
Mean:  10.0
STD:  0.0
RMS:  10.0
Median:  10.0
Skewness:  nan

C:\Users\ga39med\Anaconda3\lib\site-packages\ipykernel_launcher.py:9: Runtim
eWarning: invalid value encountered in double_scalars
  if __name__ == '__main__':
```

3. Random signal with specified distribution and given properties

In [14]:

```
rand_series_m = np.mean(rand_series) * np.ones(len(time_series))
rand_series_std = np.std(rand_series) * np.ones(len(time_series))
rand_series_rms = np.sqrt(np.mean(np.square(rand_series))) * np.ones(len(time_series))

print('Mean: ', np.mean(rand_series))
print('STD: ', np.std(rand_series))
print('RMS: ', np.sqrt(np.mean(np.square(rand_series))))
print('Median: ', np.median(rand_series))
print('Skewness: ', (np.mean(rand_series) - np.median(rand_series))/np.std(rand_series))
```

```
Mean:  -0.0009363110791706012
STD:  0.2518335786910148
RMS:  0.2518353192758325
Median:  -0.0013484126305386408
Skewness:  0.0016364043012455634
```

Superposed signal

In [15]:

```
superposed_series_m = np.mean(superposed_series) * np.ones(len(time_series))
superposed_series_std = np.std(superposed_series) * np.ones(len(time_series))
superposed_series_rms = np.sqrt(np.mean(np.square(superposed_series))) * np.ones(len(time_s

print('Mean: ', np.mean(superposed_series))
print('STD: ', np.std(superposed_series))
print('RMS: ', np.sqrt(np.mean(np.square(superposed_series))))
print('Median: ', np.median(superposed_series))
print('Skewness: ', (np.mean(superposed_series) - np.median(superposed_series))/np.std(supe
```

```
Mean:  9.99908868892083
STD:  0.3095645494930983
RMS:  10.00387948843884
Median:  9.999686383134103
Skewness:  -0.0019307579445104687
```

What do the mean, median, mode, RMS, standard deviation and skewness represent?

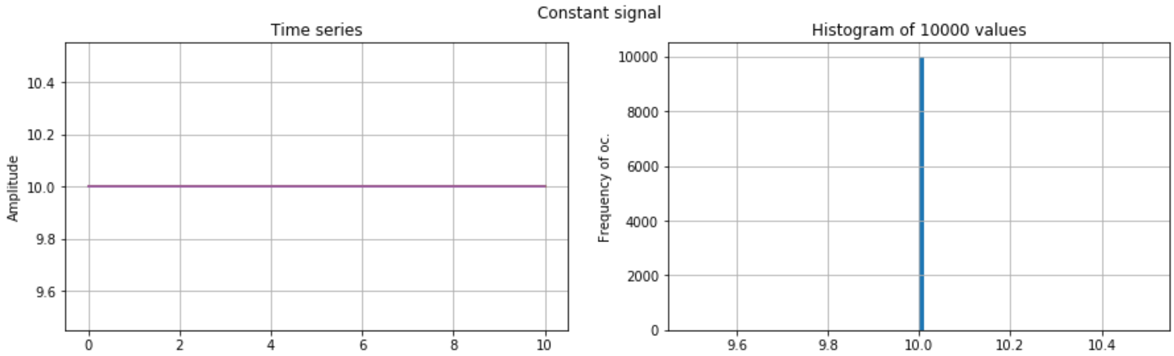
Histogram of the signals

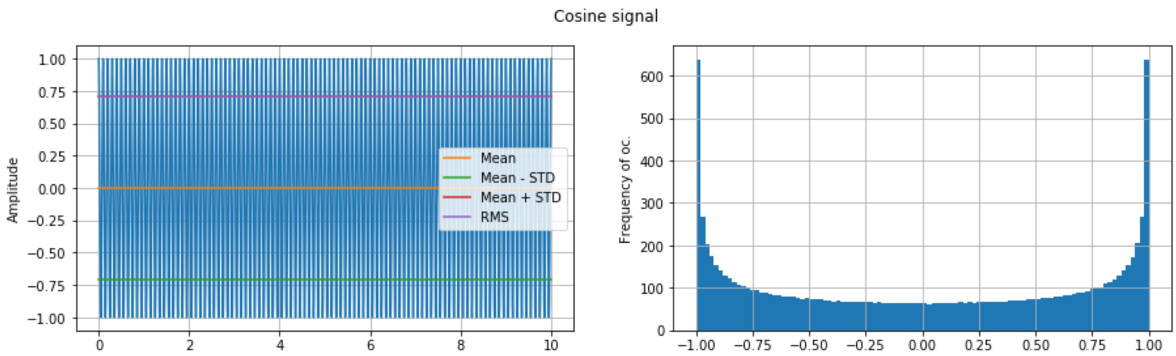
The variation of each signal with time and their histograms are plotted.

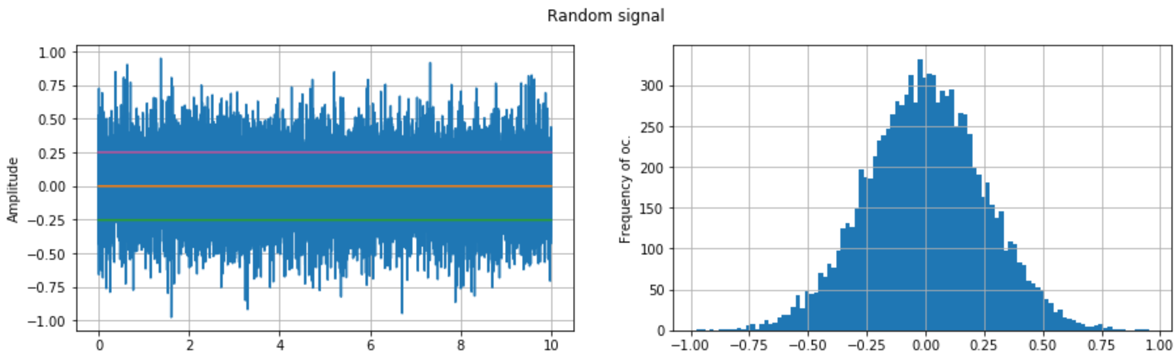
In [16]:

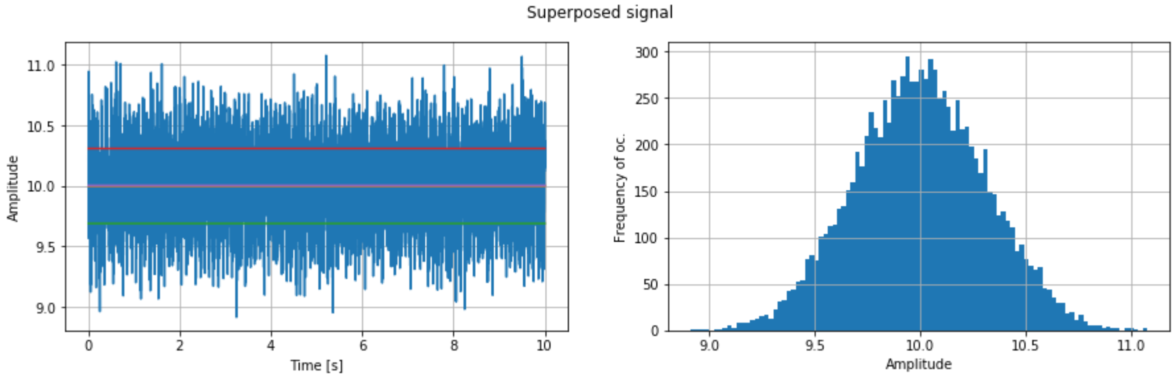
```
# const
plt.figure(num=5, figsize=(15, 4))
plt.suptitle('Constant signal')
plt.subplot(1, 2, 1)
plt.plot(time_series, const_series,
         time_series, const_series_m,
         time_series, const_series_m - const_series_std,
         time_series, const_series_m + const_series_std,
         time_series, const_series_rms)
plt.ylabel('Amplitude')
plt.title('Time series')
plt.grid(True)

bins = 100
plt.subplot(1, 2, 2)
plt.hist(const_series, bins)
plt.title('Histogram of ' + str(n_steps) +' values')
plt.ylabel('Frequency of oc.')
plt.grid(True)

# cos
plt.figure(num=6, figsize=(15, 4))
plt.suptitle('Cosine signal')
plt.subplot(1, 2, 1)
plt.plot(time_series, cos_series)
plt.plot(time_series, cos_series_m, label = 'Mean')
plt.plot(time_series, cos_series_m - cos_series_std, label = 'Mean - STD')
plt.plot(time_series, cos_series_m + cos_series_std,label = 'Mean + STD')
plt.plot(time_series, cos_series_rms, label = 'RMS')
plt.ylabel('Amplitude')
plt.legend()
plt.grid(True)

plt.subplot(1, 2, 2)
plt.hist(cos_series, bins)
plt.ylabel('Frequency of oc.')
plt.grid(True)


# rand
plt.figure(num=7, figsize=(15, 4))
plt.suptitle('Random signal')
plt.subplot(1, 2, 1)
plt.plot(time_series, rand_series,
         time_series, rand_series_m,
         time_series, rand_series_m - rand_series_std,
         time_series, rand_series_m + rand_series_std,
         time_series, rand_series_rms)
plt.ylabel('Amplitude')
plt.grid(True)

plt.subplot(1, 2, 2)
plt.hist(rand_series, bins)
plt.ylabel('Frequency of oc.')
plt.grid(True)

# superposed
```

```
plt.figure(num=8, figsize=(15, 4))
plt.suptitle('Superposed signal')
plt.subplot(1, 2, 1)
plt.plot(time_series, superposed_series,
         time_series, superposed_series_m,
         time_series, superposed_series_m -  superposed_series_std,
         time_series, superposed_series_m +  superposed_series_std,
         time_series, superposed_series_rms)
plt.ylabel('Amplitude')
plt.xlabel('Time [s]')
plt.grid(True)

plt.subplot(1, 2, 2)
plt.hist(superposed_series, bins)
plt.ylabel('Frequency of oc.')
plt.xlabel('Amplitude')
plt.grid(True)
```









Interquartile range and percentile

The [interquartile range (IQR) (https://en.wikipedia.org/wiki/Interquartile_range)](https://en.wikipedia.org/wiki/Interquartile_range) , also called the midspread or middle 50%, or technically H-spread, is a measure of statistical dispersion. This is computed as the difference between 75th and 25th percentiles, or between upper and lower quartiles. In statistics of extreme values the interquartile range is also considered along with standard deviation as a measure of the dispersion. The [percentile (https://en.wikipedia.org/wiki/Percentile)](https://en.wikipedia.org/wiki/Percentile) is a measure used in statistics indicating the value below which a given percentage of observations in a group of observations fall. These quantites can be computed using the inbuilt functions of numpy [interquartile range (IQR) (https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.iqr.html)](https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.iqr.html) [percentile (https://docs.scipy.org/doc/numpy/reference/generated/numpy.percentile.html)](https://docs.scipy.org/doc/numpy/reference/generated/numpy.percentile.html)

In [17]:

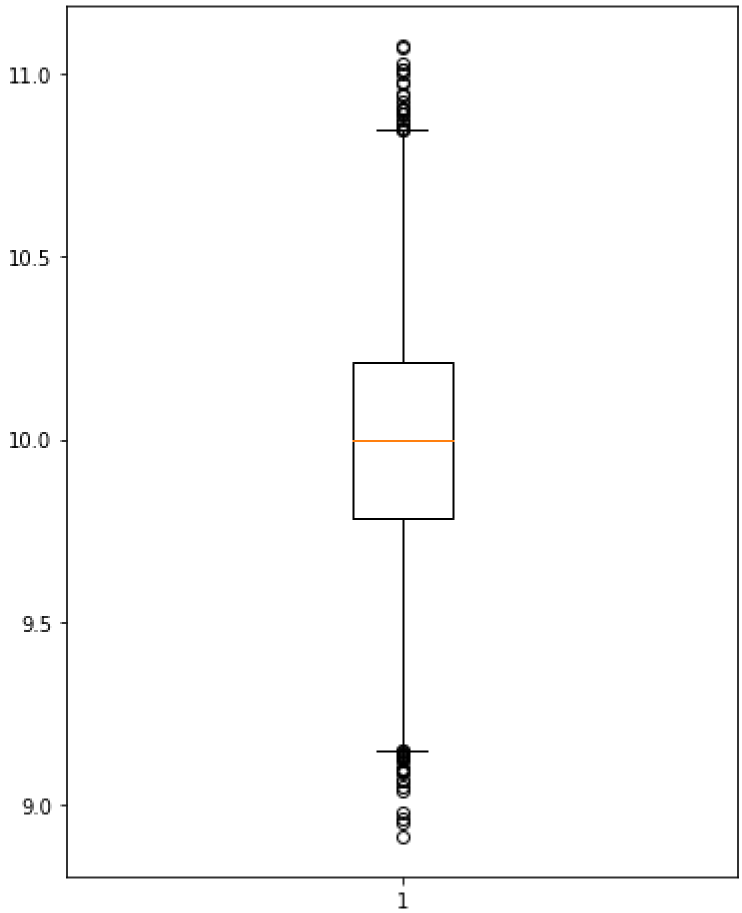
```
iqr = scipy.stats.iqr(superposed_series)
q75, q25 = np.percentile(superposed_series, [75 ,25])
print('Interquartile range = ',iqr, 'Interquantile range computed = ', q75-q25)
```

```
Interquartile range =  0.4240170705134183 Interquantile range computed =  0.
4240170705134183
```

The [boxplots (https://en.wikipedia.org/wiki/Box_plot)](https://en.wikipedia.org/wiki/Box_plot) can be obtained from the interquartile range to identify possible outliers. The box indicate the middle quartile and the lines extending indicating the variability outside the lower and upper quartiles. The in built python function [boxplots (https://matplotlib.org/api/_as_gen/matplotlib.pyplot.boxplot.html)](https://matplotlib.org/api/_as_gen/matplotlib.pyplot.boxplot.html) can be used for plotting.

In [18]:

```
plt.figure(num=9, figsize=(6, 8))
plt.boxplot(superposed_series)
plt.show()
```



Probability Distribution Function (PDF) and Cumulative Distribution Function (CDF)

The PDF and CDF of the signals are derived and are plotted later. Recall from the lecture the definitions of PDF, CDF of a continuous random variables.

Tip: Have a look at the get_pdf function in the "custom_utilities.py" for details

In [19]:

```
# const
[const_pdf_x, const_pdf_y] = c_utils.get_pdf(const_series,'Constant')
# the 'Constant' is used for obtaining pdf of a constant signal.
# check the implimentation for details

# cos
[cos_pdf_x, cos_pdf_y] = c_utils.get_pdf(cos_series)

# rand
[rand_pdf_x, rand_pdf_y] = c_utils.get_pdf(rand_series)

# superposed
[superposed_pdf_x, superposed_pdf_y] = c_utils.get_pdf(superposed_series)
```

Converting to Frequency domain - Fast Fourier Transform (FFT)

FFT computes the frequency contents of the given signal. Recall from the lecture the basic definitions and procedure for FFT.

Tip: Have a look at the get_fft function in the "custom_utilities.py" for details

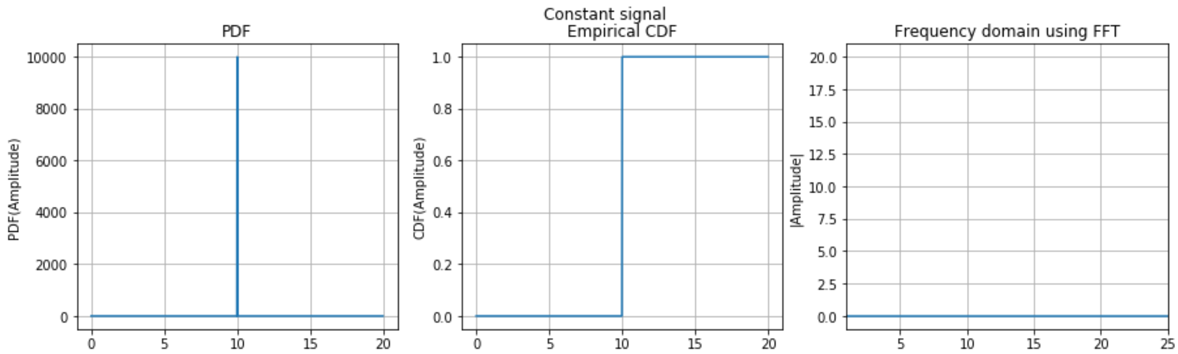
In [20]:

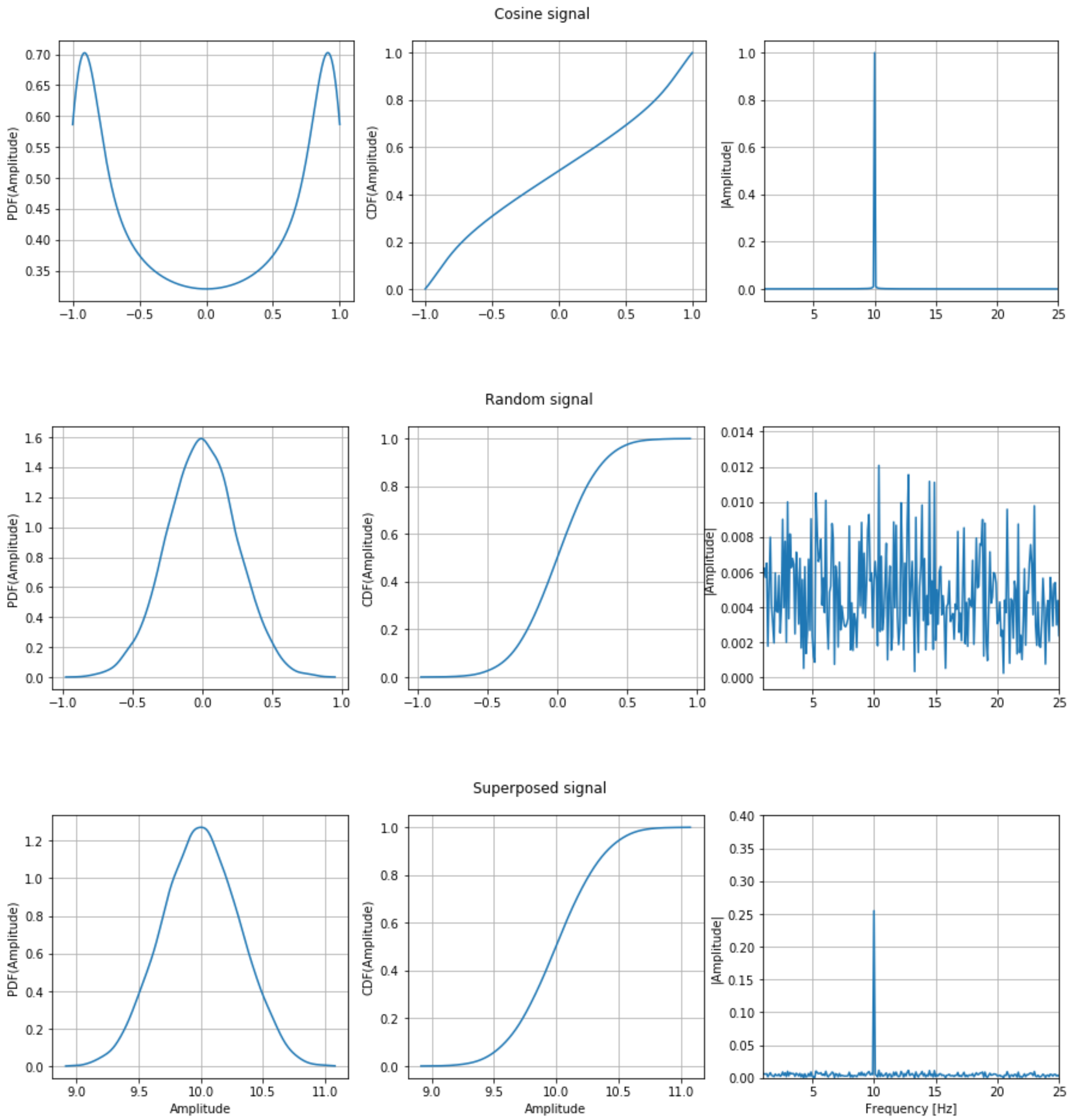
```
# sampling frequency the same in this case for all time series
sampling_freq = 1/delta_time

# const
[const_freq_half, const_series_fft] = c_utils.get_fft(const_series, sampling_freq)

# cos
[cos_freq_half, cos_series_fft] = c_utils.get_fft(cos_series, sampling_freq)

# rand
[rand_freq_half, rand_series_fft] = c_utils.get_fft(rand_series, sampling_freq)

# superposed
[superposed_freq_half, superposed_series_fft] = c_utils.get_fft(superposed_series, sampling
```

In [21]:

```
# pdf, cdf and frequency domain
plt.rcParams["figure.figsize"] = (15,4)

# const
plt.figure(num=10)
plt.suptitle('Constant signal')

plt.subplot(1,3,1)
plt.plot(const_pdf_x, const_pdf_y)
plt.xlabel(' ')
plt.ylabel('PDF(Amplitude)')
plt.title('PDF')
plt.grid(True)

const_ecdf = c_utils.get_ecdf(const_pdf_x, const_pdf_y)

plt.subplot(1,3,2)
plt.plot(const_pdf_x, const_ecdf)
plt.ylabel('CDF(Amplitude)')
plt.title('Empirical CDF')
plt.grid(True)

plt.subplot(1,3,3)
plt.plot(const_freq_half, const_series_fft)
plt.xlim([1, 25])
plt.ylabel('|Amplitude|')
plt.title('Frequency domain using FFT')
plt.grid(True)
plt.show()

# cos
plt.figure(num=11)
plt.suptitle('Cosine signal')

plt.subplot(1,3,1)
plt.plot(cos_pdf_x, cos_pdf_y)
plt.xlabel(' ')
plt.ylabel('PDF(Amplitude)')
plt.grid(True)

cos_ecdf = c_utils.get_ecdf(cos_pdf_x, cos_pdf_y)

plt.subplot(1,3,2)
plt.plot(cos_pdf_x, cos_ecdf)
plt.ylabel('CDF(Amplitude)')
plt.grid(True)

plt.subplot(1,3,3)
plt.plot(cos_freq_half, cos_series_fft)
plt.xlim([1, 25])
plt.ylabel('|Amplitude|')
plt.grid(True)
plt.show()

# rand
plt.figure(num=12)
plt.suptitle('Random signal')
```

```
plt.subplot(1,3,1)
plt.plot(rand_pdf_x, rand_pdf_y)
plt.xlabel(' ')
plt.ylabel('PDF(Amplitude)')
plt.grid(True)

rand_ecdf = c_utils.get_ecdf(rand_pdf_x, rand_pdf_y)

plt.subplot(1,3,2)
plt.plot(rand_pdf_x, rand_ecdf)
plt.ylabel('CDF(Amplitude)')
plt.grid(True)

plt.subplot(1,3,3)
plt.plot(rand_freq_half, rand_series_fft)
plt.xlim([1, 25])
plt.ylabel('|Amplitude|')
plt.grid(True)

plt.show()

# superposed
plt.figure(num=13)
plt.suptitle('Superposed signal')
plt.subplot(1,3,1)
plt.plot(superposed_pdf_x, superposed_pdf_y)
plt.xlabel(' ')
plt.ylabel('PDF(Amplitude)')
plt.xlabel('Amplitude')
plt.grid(True)

superposed_ecdf = c_utils.get_ecdf(superposed_pdf_x, superposed_pdf_y)

plt.subplot(1,3,2)
plt.plot(superposed_pdf_x, superposed_ecdf)
plt.ylabel('CDF(Amplitude)')
plt.xlabel('Amplitude')
plt.grid(True)

plt.subplot(1,3,3)
plt.plot(superposed_freq_half, superposed_series_fft)
plt.ylim([0, 0.4])
plt.xlim([1, 25])
plt.xlabel('Frequency [Hz]')
plt.ylabel('|Amplitude|')
plt.grid(True)

plt.show()
```





PDF follows the normalized hystograms. Observe the predominant frequency in the superimposed signal.

Excercise 4: Try two or more harmonic function

Try two or more cosine functions and superimpose them. What difference do you observe? What do you observe in the FFT plots?

Check Point 2: Discussion

Discuss among groups the uses of various statistical quantities and their significance.

1.2 Extreme value statistics

Two methods for extreme value analysis of (time) series are presented: Block Maxima (BM) and Peak Over Threshold (POT). Recall from the lecture the basic definitions and differences between these two

parameters.

The extreme value statistics of the generated superimposed signal generated earlier will be computed in this section.

In [22]:

```
# here give the value for given_series
# as you have 4 series at hand already generated, you could
# choose one of const_series, cos_series, rand_series, superposed_series
given_series = rand_series
```

Block Maxima (BM)

Recall from the lecture the basics of block maxima. A window size is chosen in computation of BM. The signal will be divided into the given window size. the extrema is extracted at each of the window. The mean, std and other statistics will be computed for the extrema.

In [23]:

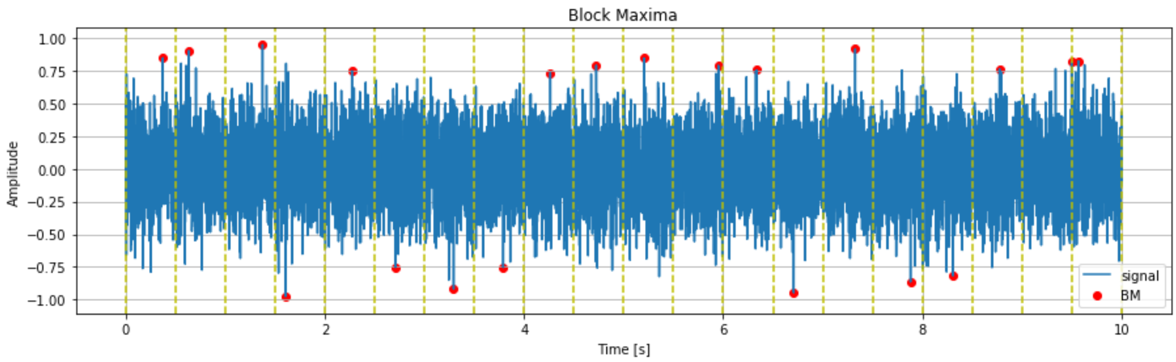
```
block_size = np.round(len(given_series)/20) #  /20 -> 5% parent size for around
# 0.2% from parent distribution to be in tails
[bm_index, bm_extreme_values] = c_utils.get_bm(given_series, block_size)
[bm_pdf_x, bm_pdf_y] = c_utils.get_pdf(bm_extreme_values)
```

Here the block size is chosen in such a way that each block will be 5% of the complete signal.

Tip: Have a look at the get_bm function in the "custom_utilities.py" for details

In [24]:

```
# plotting the initial time series and selected signal series - as a line plot
plt.figure(num=14, figsize=(15, 4))
plt.plot(time_series, given_series, label  ='signal')

# plotting the extracted bm - as a scatter plot with round red markers
plt.scatter(time_series[bm_index], given_series[bm_index], marker = 'o', color = 'r', label
plt.ylabel('Amplitude')
plt.title('Block Maxima')
plt.xlabel('Time [s]')

# add a verticle yellow dashed line to mark the separation between blocks used for extracti
for idx in np.arange(len(bm_index)):
    plt.axvline(x=time_series[np.int(block_size * idx)], color='y', linestyle='--')
plt.axvline(x=time_series[-1], color='y', linestyle='--')

plt.legend()
plt.grid(True)
```



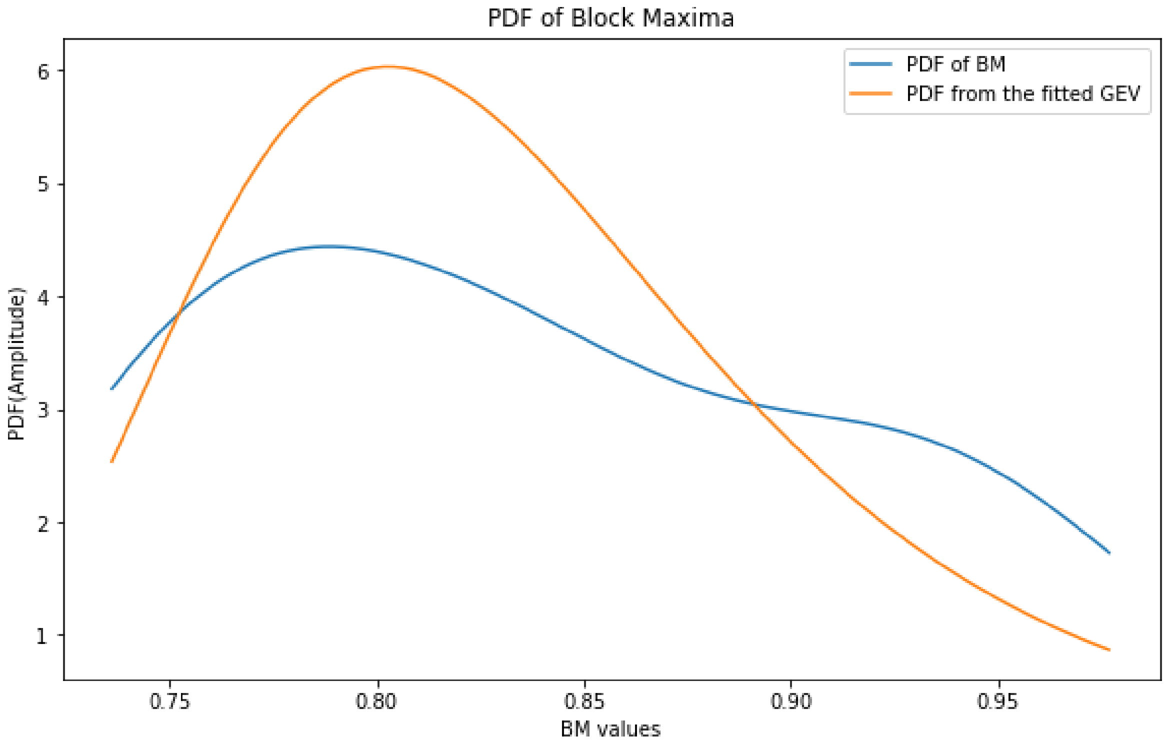
Note: Deciding the block size in computation of BM can be critical. If the size of blocks is too big (resulting in very few blocks - too few extracted maxima), it may lead to large variance. If the block size is too small (resulting in too many blocks - too many extracted maxima) this may lead to large bias. The block size is decided as a trade off between the bias and the variance.

The Generalized Extreme Value distribution (GEV)

The generalized extreme value (GEV) distribution function is a best fit to block maxima of data. Recall the details of the [extreme value distributions (GEV) (https://en.wikipedia.org/wiki/Generalized_extreme_value_distribution)](https://en.wikipedia.org/wiki/Generalized_extreme_value_distribution). The scipy implementation of [GEV (https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.genextreme.html)](https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.genextreme.html) is used.

In [25]:

```
# importing additional necessary modules
from scipy.stats import genextreme as gev

# getting the fitting parameters shape, location and scale for the bm_extreme_values based
bm_shape, bm_loc, bm_scale = gev.fit(bm_extreme_values)

bm_pdf_x2 = np.linspace(np.min(bm_extreme_values), np.max(bm_extreme_values), 100)
bm_pdf_y2 = gev.pdf(bm_pdf_x2, bm_shape, bm_loc, bm_scale)

plt.figure(num=15, figsize=(10, 6))

# PDF calculated using the get_pdf from custom_function_utilities
plt.plot(bm_pdf_x, bm_pdf_y, label = 'PDF of BM')
# PDF generated as a fitted curve using generalized extreme distribution
plt.plot(bm_pdf_x2, bm_pdf_y2, label = 'PDF from the fitted GEV')
plt.xlabel('BM values')
plt.ylabel('PDF(Amplitude)')
plt.title('PDF of Block Maxima')
plt.legend()

plt.show()
```



Note: What do the parameters of GEV distribution indicate? How to classify the given extreme to be Gumbel (type I GEV), Frêchet (type II GEV) or Weibull (type III) distribution based on these parameters? Discuss

Peak Over Threshold (POT)

Recall from the lecture the basics of peak over threshold. Every value exceeding this predefined threshold is considered an extrema. The threshold has to be decided in the beginning of the analysis, its choice is usually a function of the mean and the standard deviation of the signal.

In [26]:

```
series_m = np.mean(given_series)
series_std = np.std(given_series)

threshold_param = 2.5
threshold_value = series_m + threshold_param * series_std # for around 0.25% from parent

# distribution to be in tails
# here end_time means values extracted after the whole given_series is available
[pot_endtime_index, pot_endtime_extreme_values] = c_utils.get_pot(given_series, threshold_v
[pot_endtime_pdf_x, pot_endtime_pdf_y] = c_utils.get_pdf(pot_endtime_extreme_values)

print("POT: Threshold value: ", threshold_value)
```

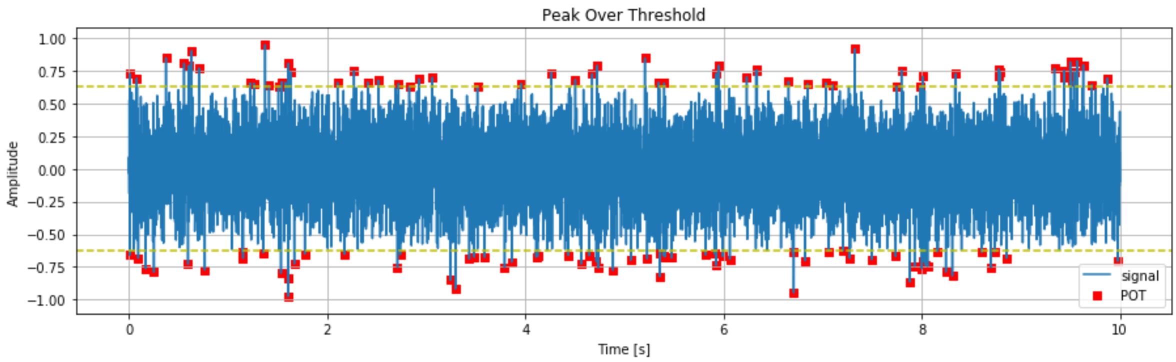
```
POT: Threshold value:  0.6286476356483665
```

Here, the threshold is set as mean plus 3 times standard deviation. For a normal distributed random variable this interval $\mu \pm \sigma$ is found to have 99.75% density inside.

Tip: Have a look at the get_pot function in the "custom_utilities.py" for details

In [27]:

```
plt.figure(num=16, figsize=(15, 4))
# plotting the initial time series and selected signal series - as a line plot
# for this case the whole series is available
# it represents a signal being made available at the end of a mearsuremen or simulation
plt.plot(time_series, given_series, label = 'signal')

# plotting the extracted pot - as a scatter plot with round red markers
plt.scatter(time_series[pot_endtime_index], given_series[pot_endtime_index], marker ='s', c
plt.ylabel('Amplitude')
plt.title('Peak Over Threshold')

# add a horizontal yellow dashed line to mark the the two trehsholds (upper and lower) used
plt.axhline(y=threshold_value, color='y', linestyle='--')
plt.axhline(y=-threshold_value, color='y', linestyle='--')

plt.xlabel('Time [s]')
plt.legend()
plt.grid(True)
```



Runtime evaluation of POT

In many situations the whole data is not available in the begining of the analysis. The POT need to be evaluated on the fly.

The mean and standard deviation used for defining the threshold criteria are also updated as new data becomes available. Here, the runtime behavior is replicated by considering the new values as soon as these become available.

Tip: Have a look at the get_pot_runtime function in the "custom_utilities.py" for details

In [28]:

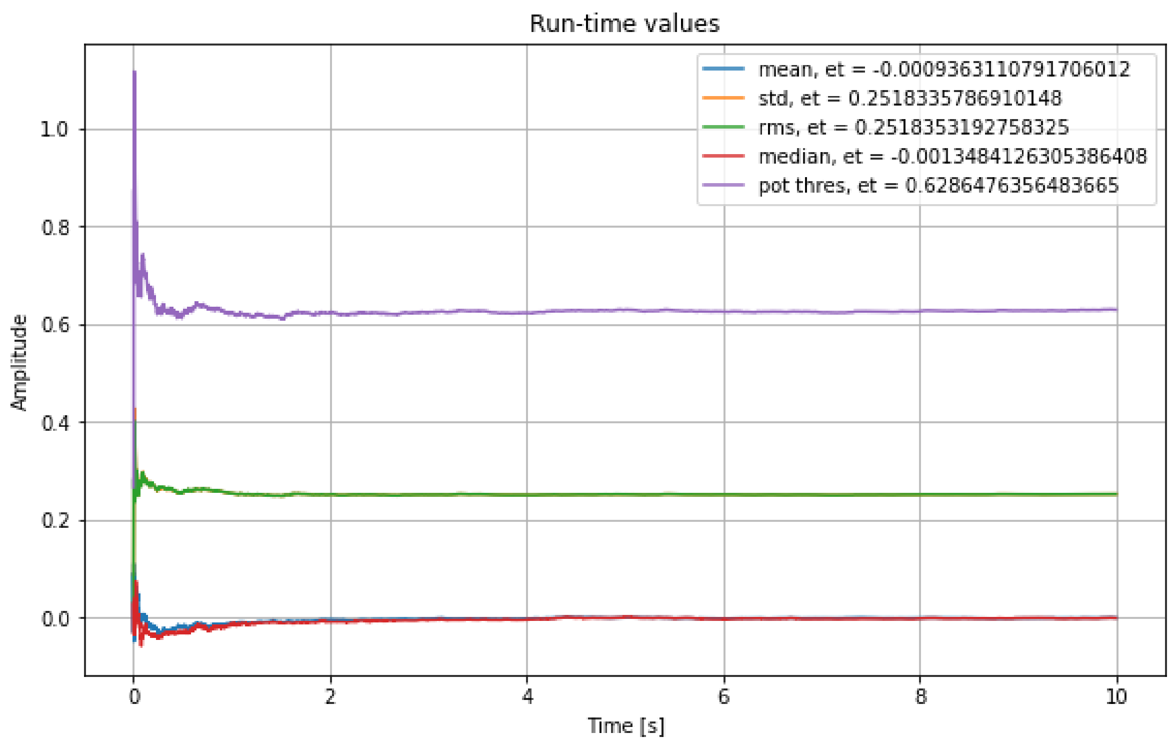
```
[res_m, res_rms, res_std, res_med, res_skew,
 res_thres, pot_runtime_index, pot_runtime_extreme_values] = \
                                c_utils.get_pot_runtime(given_series, threshold_param)
[pot_runtime_pdf_x, pot_runtime_pdf_y] = c_utils.get_pdf(pot_runtime_extreme_values)
```

```
C:\Users\ga39med\LRZ Sync+Share\StructWindEngHiWi\Finalized\Ex01WindClimateA
BL\custom_utilities.py:225: RuntimeWarning: divide by zero encountered in do
uble_scalars
  standarddev = np.sqrt((part1 - 2* meannew * part2 + meannew*meannew* i)/(i
-1))

Elapsed time for get_pot_runtime function evaluation:  8.576890526093074 s
```

Let us plot the run-time statistical quantities and look how they evolve with time

In [29]:

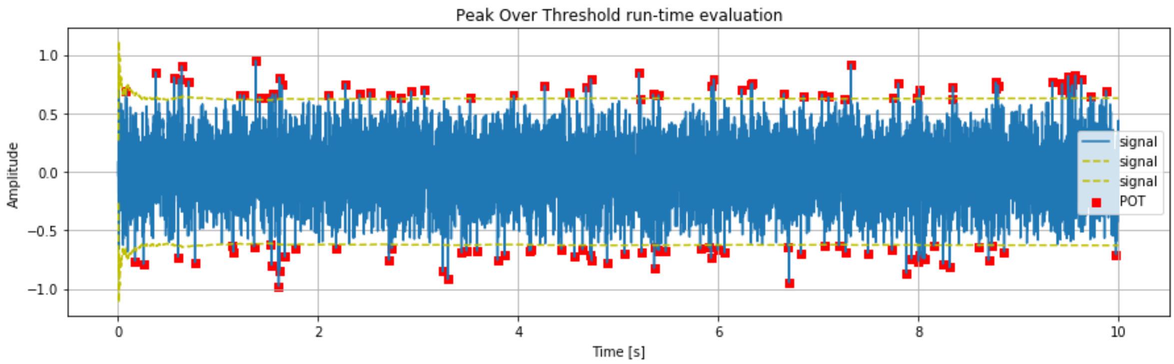
```
plt.figure(num=17, figsize=(10, 6))
plt.plot(time_series, res_m,
         time_series, res_std,
         time_series, res_rms,
         time_series, res_med,
         time_series, res_thres)
plt.legend(['mean, et = '+ str(np.mean(given_series)),
            'std, et = ' + str(np.std(given_series)),
            'rms, et = ' + str(np.sqrt(np.mean(np.square(given_series)))),
            'median, et = '+ str(np.median(given_series)),
            'pot thres, et = '+ str(threshold_value)])
plt.ylabel('Amplitude')
plt.title('Run-time values')
plt.xlabel('Time [s]')
plt.grid(True)
```



Let us plot the run-time evaluation of POT

In [30]:

```
plt.figure(num=18, figsize=(15, 4))
# plotting the initial time series and selected signal series - as a line plot
# for this case the whole series is not available, but it is being made available one time
# it represents a signal being made available in run-time
plt.plot(time_series, given_series, label = 'signal')

# plotting the extracted pot - as a scatter plot with round red markers
plt.scatter(time_series[pot_runtime_index], given_series[pot_runtime_index], marker ='s', c
plt.ylabel('Amplitude')
plt.title('Peak Over Threshold run-time evaluation')

# add a horizontal yellow dashed line to mark the the two trehsholds (upper and lower) used
# note that these are not totally straight lines from the beginning until the end, but vary
plt.plot(time_series, res_m + threshold_param * res_std, label = 'signal', color='y', lines
plt.plot(time_series, -res_m - threshold_param * res_std, label = 'signal', color='y', line

plt.xlabel('Time [s]')
plt.legend()
plt.grid(True)
```

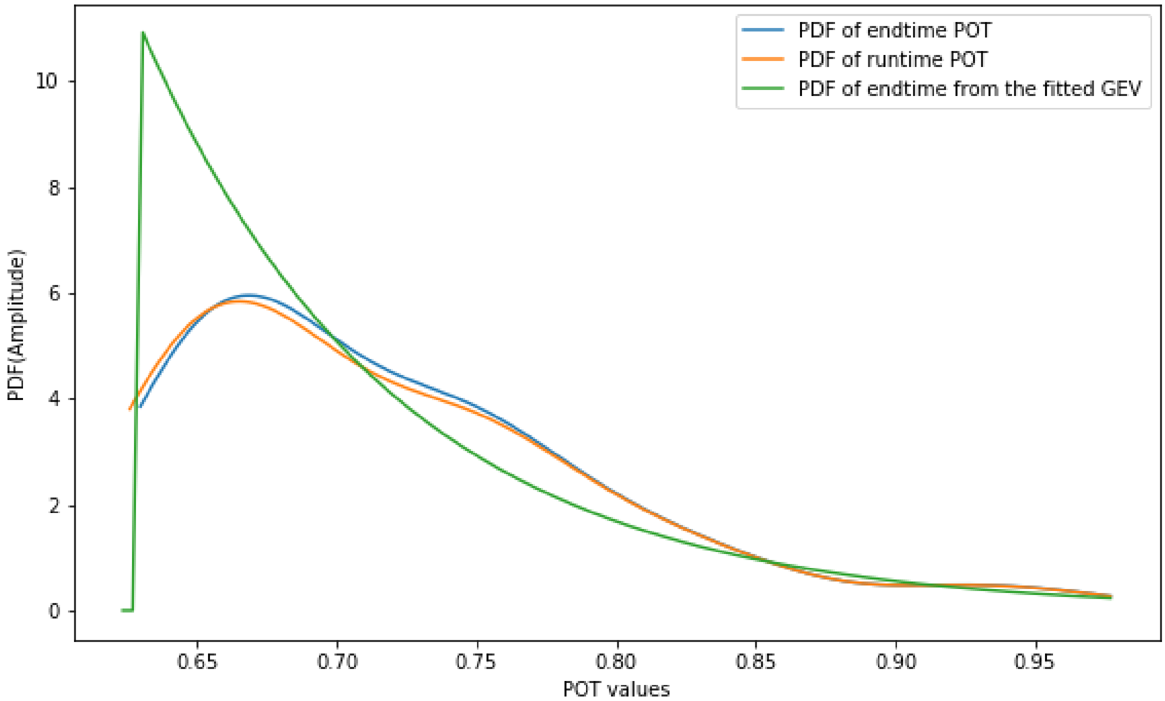


The Generalized Pareto distribution (GP)

The generalized Pareto (GP) distribution function is a best fit to peak over threshold data. Recall the details of the generalized Pareto (GP) distribution (https://en.wikipedia.org/wiki/Generalized_Pareto_distribution). The scipy implementation of GP (https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.genpareto.html) can be used.

In [31]:

```
# importing additional necessary modules
from scipy.stats import genpareto as gp

# getting the fitting parameters shape, location and scale for the bm_extreme_values based
pot_shape, pot_loc, pot_scale = gp.fit(pot_endtime_extreme_values, 0 , loc = threshold_valu

pot_endtime_pdf_x2 = np.linspace(0.99 * np.min(pot_endtime_extreme_values), np.max(pot_endt
pot_endtime_pdf_y2 = gp.pdf(pot_endtime_pdf_x2, pot_shape, pot_loc, pot_scale)

plt.figure(num=19, figsize=(10, 6))

# PDF calculated using the get_pdf from custom_function_utilities
# for endtime and runtime
plt.plot(pot_endtime_pdf_x, pot_endtime_pdf_y, label = 'PDF of endtime POT')
plt.plot(pot_runtime_pdf_x, pot_runtime_pdf_y, label = 'PDF of runtime POT')

# PDF generated as a fitted curve using generalized extreme distribution
plt.plot(pot_endtime_pdf_x2, pot_endtime_pdf_y2, label = 'PDF of endtime from the fitted GE

plt.xlabel('POT values')
plt.ylabel('PDF(Amplitude)')
plt.legend()
plt.show()
```



Excercise 5: Observe the changes with varying the number of blocks and threshold value

Change **'block_size'** and **'threshold_param'**. Observe the difference, comment and discuss.

Check Point 3: Discussion

Discuss among groups the observations and outcomes regarding extreme value statistics.

Assignment: Compute the extreme value statistics for the given signal

Two data sets 'given_data1.dat' and 'given_data2.dat' are provided which contains the time domain data of bending moment. Compute the statistical quantities of the given data. Plot the corresponding functions. Compute the BM and POT for these given data sets. Make necessary changes and adaptation in block 22 from the follwing snippet in block 32. (use 'Ctrl' + '/' to uncomment multiple lines)

In [32]:

```
# file_name = 'given_data1.dat'  # has 5350 values for each column
# time_series = np.loadtxt(file_name, skiprows=0, usecols = (0,)) # in [s]
# bending_moment_series = np.loadtxt(file_name, skiprows=0, usecols = (1,)) # in [kNm]

# file_name = 'given_data2.dat' # has 53491 values for each column
# time_series = (np.loadtxt(file_name, skiprows=0, usecols = (0,))-1000)/100  # shift 1000
# bending_moment_series = np.loadtxt(file_name, skiprows=0, usecols = (1,))/1000 # to get [

# # assign the bending moment series to the given series
# given_series = bending_moment_series
```