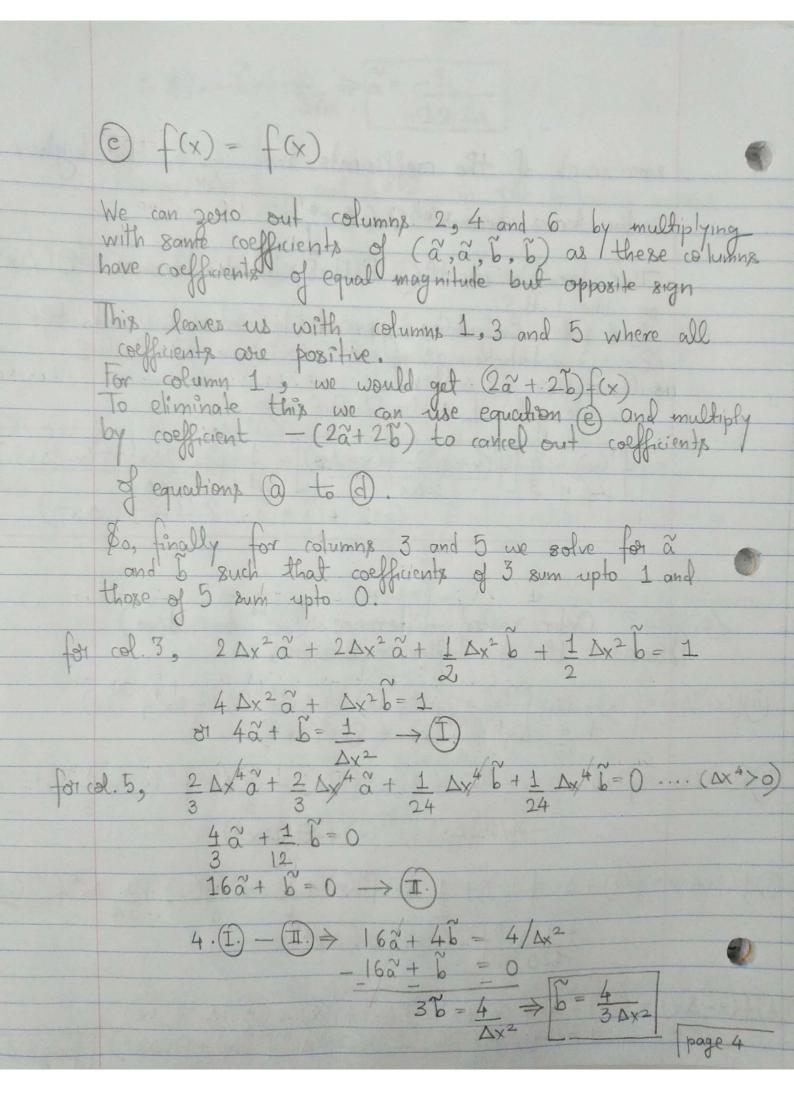
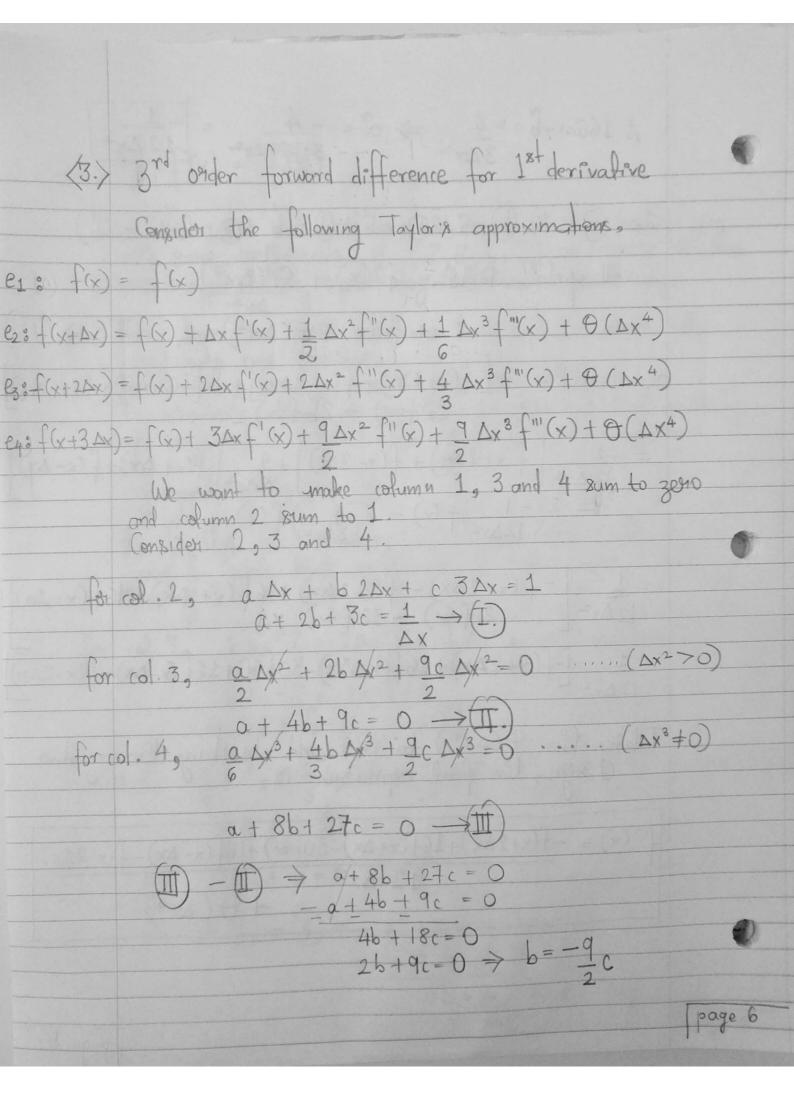
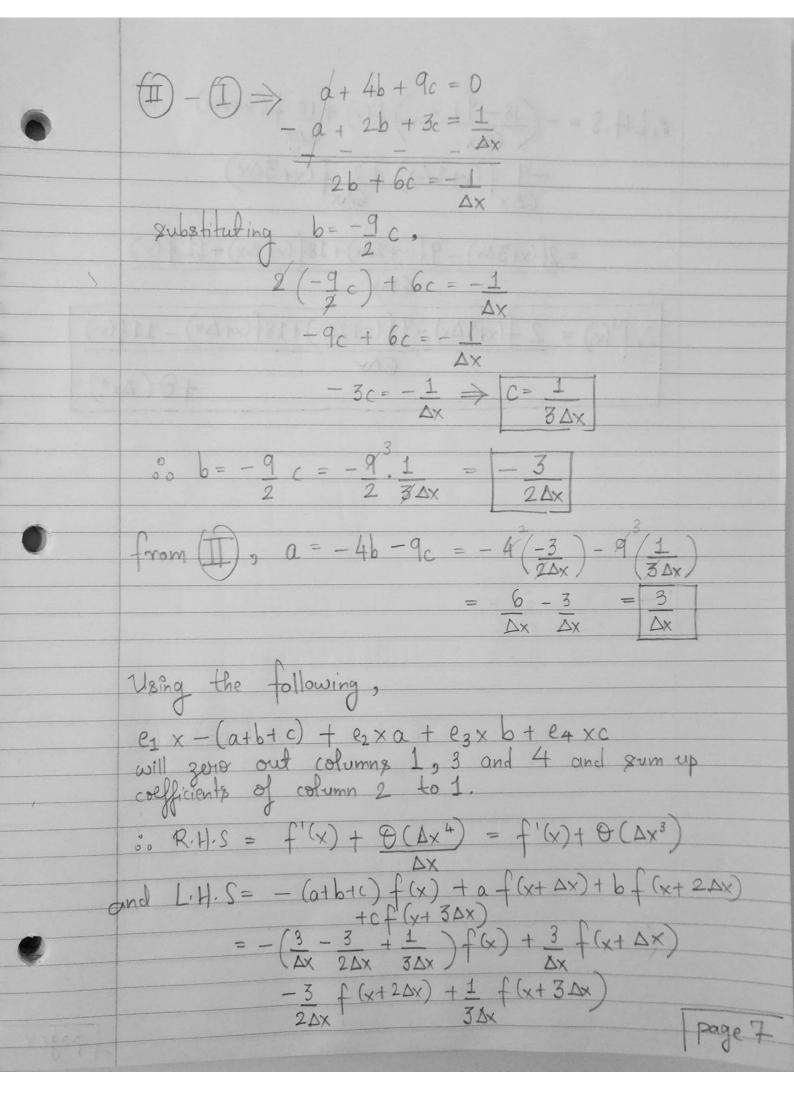


 $3.8 = -6 = -\frac{2}{3\Delta x} \Rightarrow \alpha = -\frac{1}{12\Delta x}$ Using $\tilde{a} = -\frac{1}{12\Delta x}$ and $\tilde{b} = \frac{2}{3\Delta x}$ in the following manner (**) @xa +(bx (-a) + (c) x b + (d) x (-b) will zero out columns 1,3 and 5 since coefficients will be equal and opposite and will also zero out column 4 and sum up column 2's coefficients to 1 as por above calculations Using there to evaluate the L.H.S., we have, $-H \cdot S = \alpha \cdot f(x + 2\Delta x) - \alpha \cdot f(x - 2\Delta x) + b \cdot f(x + \Delta x)$ $-b \cdot f(x - \Delta x)$ $\frac{-1}{12\Delta x} f(x+2\Delta x) - \left(-\frac{1}{12\Delta x} f(x-2\Delta x)\right) + \frac{2}{3\Delta x} f(x+\Delta x)$ $-\left(\frac{2}{3\Delta x} f(x - \Delta x)\right)$ $= \frac{1}{12} \left(f(x-2\Delta x) - f(x+2\Delta x) \right)$ $+\frac{2}{3} \times \left(\int (x + \Delta x) dx \right)$ $=\frac{-1}{3} \left[f(x+2Ax) - f(x-2Ax) \right]$ $+\frac{4}{3}\left[f(x+\Delta x)-f(x-\Delta x)\right]$ which proves the expression given! As for as the higher order terms, each of (a), (b), (c) However, when we multiply by coefficients as in (**)

since each of the coefficients has 1 , the higher order terms become O (1x4) This term is on the R.H.S, which becomes - O(Dx4) But, this negative sign can be absorbed into the Big - O notation giving O (Dx") on the Litis ° We get L.H.S=R.H.S ⇒ $f'(x) = \frac{4}{3} \left[f(x + \Delta x) - f(x - \Delta x) \right] - \frac{1}{3} \left[f(x + 2\Delta x) - f(x - 2\Delta x) \right]$ (2.) 4th Order central difference (2nd derivative)
Consider the 4 Taylor's expansions, $(0)f(x+2\Delta x) = f(x) + 2\Delta x f'(x) + 2\Delta x^{2} f''(x) + 4\Delta x^{3} f'''(x) + 2\Delta x^{4} f''(x) + 4\Delta x^{3} f'''(x) + 2\Delta x^{4} f''(x) + 4\Delta x^{5} f'''(x) + 4\Delta x^{5} f'''(x)$ (b) $f(x-2\Delta x) = f(x) - 2\Delta x f'(x) + 2\Delta x^2 f''(x) - \frac{4}{3}\Delta x^3 f'''(x) + \frac{2}{3}\Delta x^4 f''(x)$ -(4/15) DX 5 f(5) (X) + O (DX 6 εf(x+Δx) = f(x) + Δx f'(x) + 1 Δx2f"(x) + 1 Δx3f"(x) + 1 Δx4f(4)(x) + 1 Ax 5 ((5) (x) + O (Ax6) $(d)f(x-\Delta x) = f(x) - \Delta x f'(x) + (1/2) \Delta x^{2} f''(x) - (1/6) \Delta x^{3} f'''(x)$ $+ (1/24) \Delta x^{4} f^{(4)}(x) - (1/120) \Delta x^{5} f^{(5)}(x) + \Theta(\Delta x^{6})$ $+ (1/24) \Delta x^{4} f^{(4)}(x) - (1/120) \Delta x^{5} f^{(5)}(x) + \Theta(\Delta x^{6})$ $+ (1/24) \Delta x^{4} f^{(4)}(x) - (1/120) \Delta x^{5} f^{(5)}(x) + \Theta(\Delta x^{6})$ $+ (1/24) \Delta x^{4} f^{(4)}(x) - (1/120) \Delta x^{5} f^{(5)}(x) + \Theta(\Delta x^{6})$ $+ (1/24) \Delta x^{4} f^{(4)}(x) - (1/120) \Delta x^{5} f^{(5)}(x) + \Theta(\Delta x^{6})$ $+ (1/24) \Delta x^{4} f^{(4)}(x) - (1/120) \Delta x^{5} f^{(5)}(x) + \Theta(\Delta x^{6})$







** L.H.
$$S = -\left(\frac{18 - 9 + 2}{6\Delta x}\right) f(x) + \frac{18}{6\Delta x} f(x+\Delta x)$$

$$-\frac{9}{6\Delta x} f(x+2\Delta x) + \frac{2}{6\Delta x} f(x+3\Delta x)$$

$$= \frac{2}{6\Delta x} f(x+2\Delta x) - \frac{9}{6\Delta x} f(x+2\Delta x) + \frac{11}{6\Delta x} f(x)$$

$$= \frac{2}{6\Delta x} f(x+3\Delta x) - \frac{9}{6\Delta x} f(x+2\Delta x) + \frac{11}{6\Delta x} f(x)$$

$$= \frac{2}{6\Delta x} f(x+3\Delta x) - \frac{9}{6\Delta x} f(x+2\Delta x) + \frac{11}{6\Delta x} f(x)$$

$$= \frac{2}{6\Delta x} f(x+3\Delta x) - \frac{9}{6\Delta x} f(x+2\Delta x) + \frac{11}{6\Delta x} f(x)$$

$$= \frac{2}{6\Delta x} f(x+2\Delta x) + \frac{18}{6\Delta x} f(x+\Delta x) - \frac{11}{6\Delta x} f(x)$$

$$= \frac{2}{6\Delta x} f(x+2\Delta x) + \frac{18}{6\Delta x} f(x+\Delta x) - \frac{11}{6\Delta x} f(x)$$

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$$= \frac{2}{6\Delta x} f(x+2\Delta x) + \frac{18}{6\Delta x} f(x)$$

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$$= \frac{2}{6\Delta x} f(x$$