Homework #1: Transport PDE

January 9, 2019

1 Nonhomogeneous with Constant Coefficients

Given the following PDE for the function $u(x,t)$ as well as initial value $g(x)$ at $t=0$

$$\begin{aligned} u_t + bu_x &= e^t \\ u(x,0) &= g(x) \end{aligned}$$

derive the solution for $u(x,t)$ in terms of the function g. To obtain full credit, you should not just propose a solution and show that it solves the PDE nor should you simply plug this into the more general formula we derived in class (nor simply rewrite that derivation verbatim for that same more generalized form) for the nonhomogenous transport PDE. Instead, you should directly develop the solution for this specific PDE using the same principles from class. For partial credit, make sure you at least show how the PDE can be interpreted as a special directional derivative and how the solution of u evolves along this direction.

2 Homogeneous with Time-Varying Coefficient

Using the exact same principles, but noting that the directional derivatives no longer occur along fixed/constant directions (and therefore no longer give rise to straight line characteristic trajectories), derive the solution $u(x,t)$ for the following PDE in terms of its initial condition g (only partial credit will be given if a solution is proposed and shown to satisfy the PDE rather than derived using the principles discussed in class).

$$\begin{aligned} u_t - tu_x &= 0 \\ u(x,0) &= g(x) \end{aligned}$$

3 Two Space Dimensions

Write out the solution $u(x,y,t)$ for the following PDE and initial condition. You may utilize the formulas (and their higher dimensional generalizations) derived in class rather than deriving the solution from scratch.

$$\begin{aligned} 3u_t + 6u_x - 9u_y &= x\cos t \\ u(x,y,0) &= y^2 e^x \end{aligned}$$