Lossless Compression of Uniform Binary Sources with Coded Side-Information

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***Abstract*—This paper studies code design for lossless compression with coded side-information for uniform binary sources with symmetric correlation. We propose LDPC codes for binning and convolutional codes for coding the side-information, and we present two strategies to design the codes. We further investigate the rate gaps due to the imperfections of the practical binning and rate-distortion codes.**

I. INTRODUCTION

Suppose that two discrete sources X and Y are correlated with joint probability distribution $P_{XY}(x, y)$ and marginal probability distributions $P_X(x)$ and $P_Y(y)$, respectively. Encoder E_X encodes X at a rate of R_X and has no knowledge about Y. Similarly, encoder E_Y encodes Y at a rate of R_Y and has no knowledge about X. At the receiver side, the combined decoder D tries to recover X as $\hat{X}$ perfectly, while Y only serves as side information and is not reconstructed. This setting is known as lossless source coding with coded side-information, and is depicted in Fig. 1.

The coded side-information problem is a special problem within the area of distributed source coding or distributed compression. Among those, a very important one is the Slepian-Wolf problem: two sources are compressed separately and both are to be reconstructed lossless at the destination [1]. The achievable rate region for this problem is well known [1], [2], and it has found lots of attention and various applications in areas like sensor networks and video coding [3]–[5]. Codes for the case of uncoded side-information have been proposed in [6].

As opposed to the Slepian-Wolf problem, the coded side-information problem has attracted less attention and no practical coding schemes have been suggested so far. Ahlswede and Körner introduced this problem in [7] and characterised the rate region in terms of an auxiliary random variable. Marco and Effros revisited the problem for the case of decomposable source distributions and provided some computable achievable rate pairs [8], [9]. For the same type of source distributions, Land, Weidmann and Vellambi presented a refined proof technique and obtained further computable rate points [10]. For the case of binary sources, Gu, Kötter, Effros and Ho showed that a binary auxiliary random variable is sufficient to characterise the rate region [11], allowing for easy computation of the rate region.

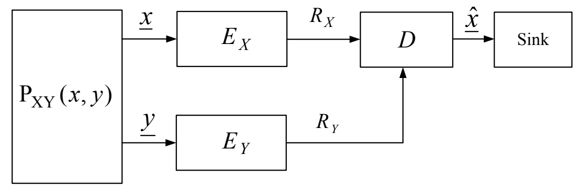

Fig. 1. Lossless compression of X with coded side information Y.

Though the achievable rate region for the coded side information problem has been well studied in literature, no practical codes have been developed yet. The aim of the present work is to design a source coding scheme for this problem for the case of binary uniform sources with symmetric correlation.

With reference to Fig. 1, encoder E_X requires a binning code and encoder E_Y a rate-distortion code. In our work we propose to use low-density parity check (LDPC) codes and message-passing decoding [12], [13] for the binning part, and convolutional codes and Viterbi decoding [14] under Hamming distortion for the rate-distortion part. We investigate the effects of using different LDPC and convolutional codes, and we analyse the limits due to practical issues like suboptimal decoding of the LDPC codes, finite-length codes and finite-memory convolutional codes.

This paper is organized as follows. In Section II, we present the system setting for the coded side-information problem, the bound of the achievable rate region, and we motivate our chosen code construction. Section III deals with the rate-distortion code for the side information, where we use convolutional codes under Hamming distortion. Section IV addresses the use of LDPC codes for the binning part, their design and their performance within the overall setting. Numerical results of the overall system and practical performance limits are presented in Section V. Finally, we summarise our work in Section VI.

II. SYSTEM MODEL AND RATE REGION

Consider the compression problem shown in Fig. 1. The two binary memoryless sources X and Y, $X, Y \in \mathbb{F}_2$, where $\mathbb{F}_2 := \{0, 1\}$, are assumed to be individually uniformly distributed

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and to have the joint distribution

$$P_{XY}(x,y) = \begin{cases} \frac{1}{2}\varepsilon_{SRC} & \text{for } x \neq y, \\ \frac{1}{2}(1-\varepsilon_{SRC}) & \text{for } x = y, \end{cases}$$

with parameter $\varepsilon_{SRC} \in [0,1]$. Note that

$$\varepsilon_{SRC} = P(Y \neq X),$$

and that Y may be interpreted as the output of a binary symmetric channel (BSC) with crossover probability ε_{SRC} and input X.

The source sequence $\underline{x}$ of length n, $\underline{x} \in \mathbb{F}_2^n$, produced by source X, is encoded at a rate of R_X using a binning code. The source sequence $\underline{y}$ of length n, $\underline{y} \in \mathbb{F}_2^n$, produced by source Y, is encoded at a rate of R_Y using a rate-distortion code. At the destination side, the combined decoder D tries to recover X as $\hat{X}$ lossless, i.e. such that $P(X \neq \hat{X}) \to 0$ for $n \to \infty$.

Necessary conditions to recover X without loss are [1], [2]

$$R_X \geq H(X \mid Y)$$
$$R_X + R_Y \geq H(X)$$

If $R_Y > H(Y)$, then $R_X = H(X \mid Y)$ suffices to describe X. On the other hand, if $R_Y = 0$, then $R_X = H(X)$ is necessary to describe X.

A single-letter characterization of the achievable rate region for this problem was determined by Ahlswede and Körner [7]. A rate pair (R_X, R_Y) is achievable if and only if there exists a random variable U such that X, Y and U form the Markov chain $X - Y - U$ and

$$R_X \geq H(X \mid U)$$
$$R_Y \geq I(Y;U)$$

for some joint probability mass function

$$P_{XYU}(x,y,u) = P_{XY}(x,y)P_{U|Y}(u \mid y).$$

A binary U is sufficient if both X and Y are binary [11].

To motivate our following practical code construction, we revisit some properties of the information-theoretic code used in the achievability part of the coding theorem [2], [7]. Assume a rate pair (R_X, R_Y) achieved by the distribution $P_{U|Y}(u|y)$, and define the distribution $P_U(u) = \sum_y P_{U|Y}(u|y)P_Y(y)$.

Consider first the side channel. From $P_U(u)$, we construct a random code of rate R_Y. Encoding is performed by matching the sequences from source Y to codewords of this code by typicality. In our case, Y and U are both uniform and binary, and $P_{U|Y}(u|y)$ corresponds to a BSC [11]. Therefore, a linear binary rate-distortion code is appropriate, and typical-set matching may be replicated by minimising the Hamming distance. Neglecting the memory introduced into the codewords by the linearity of the code, a good code will achieve an average symbol-wise mutual information $\bar{I}(Y_i;U_i)$ as close as possible to R_Y, and thus minimises the average Hamming distortion between source sequences and codewords.

Consider now the main channel. For the source X, the encoder performs simple binning, and the decoder recovers the original source sequence by searching within the corresponding bin for the source sequence that is jointly typical with the transmitted rate-distortion codeword. In our case, binning can be accomplished by computing the syndrome with respect to a linear binary code. The decoder operation corresponds then to searching for a codeword within the coset for this syndrome, where again due to the symmetrical statistical structure, typicality corresponds to minimising Hamming distance.

These considerations for the binning code and the rate-distortion code lead to the practical constructions presented in the following two sections.

III. Compression using Convolutional Codes

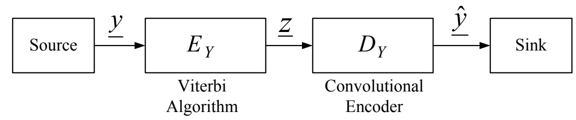

Fig. 2. Binary source compression using a convolutional code.

We are interested in lossy compression of the source Y. A block representation of a lossy compression system is depicted in Fig. 2, where the source encoder E_Y maps the source sequence $\underline{y} \in \mathbb{F}_2^n$ of length n to $\underline{z} \in \mathbb{F}_2^m$ of length m at the compression rate

$$R = \frac{m}{n}.$$

The source decoder D_Y maps the compressed sequence $\underline{z}$ to the reconstruction sequence $\hat{\underline{y}} \in \mathbb{F}_2^n$ of length n. This rate-distortion code introduces a distortion between Y and $\hat{Y}$.

Motivated by the discussion in the previous section, we use Hamming distortion to assess the quality of the rate-distortion code. Let y_i be the i^{th} element of the source vector $\underline{y}$ and let $\hat{y}_i$ be the corresponding reconstructed sample. The Hamming distance between them is defined as

$$d_H(y_i, \hat{y}_i) = \begin{cases} 0 & \text{if } y_i = \hat{y}_i, \\ 1 & \text{if } y_i \neq \hat{y}_i. \end{cases} \tag{1}$$

The average element-wise distortion between the vectors $\underline{y}$ and $\hat{\underline{y}}$ is then given by

$$d_H(\underline{y}, \hat{\underline{y}}) = \frac{1}{n}\sum_{i=1}^{n} d_H(y_i, \hat{y}_i). \tag{2}$$

The average distortion between the random source sequence $\underline{Y}$ and the random reconstruction sequence $\hat{\underline{Y}}$ is then given by the expected value

$$D = E[d_H(\underline{Y}, \hat{\underline{Y}})] = \frac{1}{n}\sum_{i=1}^{n} E[d_H(Y_i, \hat{Y}_i)],$$

where $E[\cdot]$ denotes expectation. Assuming that $E[d_H(Y_i, \hat{Y}_i)]$ does not depend on the index i, we have

$$D = P(\hat{Y} \neq Y).$$

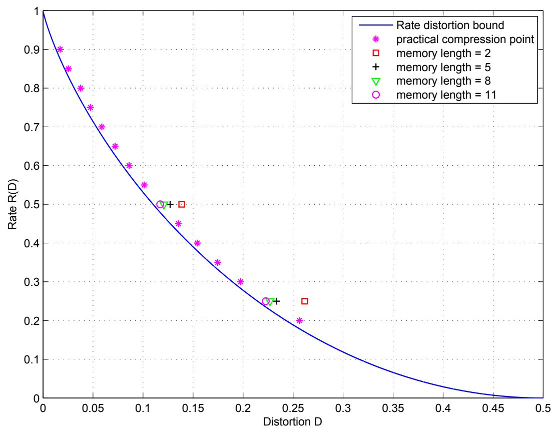

Fig. 3. Lossy compression with convolutional codes.

TABLE I
CONVOLUTIONAL CODES.

Code	Generator Polynomials
C1	$(257, 233, 323, 271, 357)$
C2	$(4767, 5723, 6265, 7455)$
C3	$(4767, 5723, 6265)$
C4	$(4335, 5723)$
C5	$(561, 753)$

From [2], for lossy compression of a binary uniform source, the minimum rate required for achieving a given Hamming distortion D is

$$R(D) = \begin{cases} 1 - H_b(D) & \text{for } \ 0 \le D \le \frac{1}{2}, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $H_b(p) = -p \log_2 p - (1-p) \log_2 (1-p)$ denotes the binary entropy function. This rate-distortion function is shown in Fig. 3. Our target is to construct practical coding schemes that achieve rate-distortion pairs as close as possible to this lower bound.

The task of encoder E_Y is to map a source vector $\underline{y}$ to the codeword with the smallest Hamming distance to this source vector. In this work, we consider convolutional codes and hence we use the Viterbi algorithm for this quantisation step.

The performance of the system, as shown in Fig. 2, depends on the codebook and hence on the choice of the convolutional code. We consider convolutional codes with maximum free distance profile given in Table III of [15] and Table I, Table II and Table III of [16].

As long as the length n of the source vector is significantly larger than the constraint length of the convolutional code, the achieved rate-distortion pair will be nearly independent of the length n. For the following experimental investigation we use block length $n = 10^4$.

First, we investigate the influence of the memory length of the convolutional code. For this experiment we consider codes of rate $\frac{1}{2}$ and $\frac{1}{4}$ listed in Table I and Table III of [16], respectively. These codes have memory 2, 5, 8 and 11. The results in Fig. 3 (markers for rate $\frac{1}{2}$ and $\frac{1}{4}$) show that convolutional codes with higher memory achieve rate-distortion pairs closer to the rate-distortion bound.

TABLE II
RATE-DISTORTION CODES AND PUNCTURING PATTERNS.

Code	Rate R	D	Puncturing Pattern
C1	0.20	0.25628	1
C2	0.25	0.22268	1
	0.30	0.19752	1 1 1 1 0 1
C3	0.35	0.17463	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1
	0.40	0.15414	1 1 1 0 1 1
C2	0.45	0.13537	1 1 0 0 1 0 1 0 1
C4	0.50	0.11718	1
	0.55	0.10109	1 1 1 1 1 1 1 1 1 0 1
	0.60	0.08603	1 1 1 1 0 1
	0.65	0.07202	1 1 1 0 1 1 0 1 1 0 1 1 1
	0.70	0.05882	1 1 0 1 1 0 1
	0.75	0.04743	1 0 1
	0.80	0.03764	1 1 0 1 0 1 0 1
	0.85	0.02533	1 1 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
C5	0.90	0.01711	1 0 1 0 1 0 1 0 1

Next, we are interested in achieving multiple rates along the rate-distortion bound. For this purpose we select five parent codes, C1, C2, C3, C4, and C5, with generator polynomials as given in Table I in octal notation. In addition, we perform puncturing where the puncturing patterns are listed in Table II: a 0 in the puncturing pattern means that the corresponding bit is punctured and a 1 indicates that it is not punctured. The achieved rate-distortion pairs are shown in Fig. 3. We see that for all rates we are able to operate very close to the rate-distortion bound with these punctured convolutional codes.

IV. BINNING USING LDPC CODES

To design a practical source coding scheme for the coded side information problem, as shown in Fig. 1, we use a binning approach to encode source X. Encoder E_X maps the source sequence $\underline{x}$, $\underline{x} \in \mathbb{F}_2^n$, of length n to syndrome $\underline{s}$, $\underline{s} \in \mathbb{F}_2^k$, of length k and sends it to decoder D_X. The encoding rate is

$$R_X = \frac{k}{n}$$

where $k < n$. For encoding of the side information Y, we assume the lossy compression scheme discussed in the previous section using convolutional codes and the Viterbi algorithm. (The choice of Hamming distortion will be discussed later on.) Encoder E_Y encodes $\underline{y}$ to $\underline{z}$ at rate R_Y. To recover $\underline{x}$ as $\underline{\hat{x}}$, the combined decoder D operates in two steps:

Step1: Decoder D_Y recovers $\underline{y}$ as $\underline{\hat{y}}$.

Step2: Decoder D_X considers $\underline{\hat{y}}$ as a noisy (and memoryless) observation of $\underline{x}$. Using this and the syndrome $\underline{s}$, it recovers $\underline{x}$ as $\underline{\hat{x}}$.

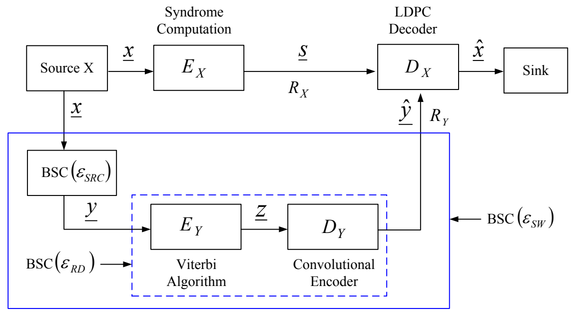

Fig. 4. Model for the design of the binning code.

The corresponding model is shown in Fig. 4.

Now, we need a code such that the decoder at the destination side can use the syndrome $\underline{s}$ and the noisy observation $\hat{\underline{y}}$ to estimate $\hat{\underline{x}}$ without loss. This problem corresponds to the Slepian-Wolf problem where one source is available at the decoder, corresponding to a corner point of the rate region. For such problems LDPC have been shown to be a good choice [3], [5], [6]. The channel for which the LDPC code is to be designed is investigated in the following.

The Hamming distortion between Y and $\hat{Y}$ introduced by the rate-distortion code is given by D. We now neglect the memory introduced by the convolutional code. (This assumption has been justified by experiments.) Then we may model the channel between Y and $\hat{Y}$ by a BSC with crossover probability

$$\varepsilon_{RD} := P(\hat{Y} \neq Y) = D, \tag{4}$$

where $0 \leq \varepsilon_{RD} \leq \frac{1}{2}$.

After replacing the rate-distortion code with the BSC model, see Fig. 4, the channel between X and $\hat{Y}$ consists of two cascaded BSCs with crossover probabilities ε_{SRC} and ε_{RD}. Thus the crossover probability of the overall channel results as

$$\varepsilon_{SW} := \varepsilon_{SRC}(1-\varepsilon_{RD}) + \varepsilon_{RD}(1-\varepsilon_{SRC}). \tag{5}$$

Note that

$$\varepsilon_{SW} = P(\hat{Y} \neq X)$$

where $0 \leq \varepsilon_{SW} \leq \frac{1}{2}$.

By these considerations, the LDPC code for binning simply needs to be designed for a BSC with crossover probability ε_{SW}, as shown in Fig. 4. The required rate R_X can thus be written as

$$R_X \geq H(X) - I(X;\hat{Y}) = H(X \mid \hat{Y}) = H_b(\varepsilon_{SW}), \tag{6}$$

where ε_{SW} is given by (5). For a given $D = \varepsilon_{RD}$, the required rate R_Y is given by the rate-distortion bound in (3):

$$R_Y \geq 1 - H_b(\varepsilon_{RD}) = 1 - H_b(D). \tag{7}$$

Using (6) and (7) we get the rate pairs for our practical setting, and the boundary of the rate region for the case where we have equality in both relations.

We now discuss the choice of Hamming distortion for the rate-distortion code in our setting. The reconstruction symbol $\hat{Y}$ of the rate-distortion code can be identified with the auxiliary random variable U discussed in Section II. Using (4) and (5), we can write the conditional entropy from (6) as

$$\begin{aligned} H(X \mid \hat{Y}) &= H_b(\varepsilon_{SW}) \\ &= H_b\Big(\varepsilon_{SRC}(1-D) + D(1-\varepsilon_{SRC})\Big). \end{aligned}$$

Note that the binary entropy function is strictly concave and monotonically increasing for $\varepsilon_{SW} \in [0, \frac{1}{2}]$ [2]. Further, for a given ε_{SRC}, ε_{SW} is minimum if $\varepsilon_{RD} = D$ is minimum. Therefore, minimising the Hamming distortion D means minimising ε_{SW}, and thus leads to the minimum value of $H(X \mid \hat{Y})$. This again allows for the minimum R_X, and therefore moves the rate-pair (R_X, R_Y) closer to the boundary of the achievable rate region.

As an example, the rate-pair bounds for $\varepsilon_{SRC} \in \{0.01, 0.045, 0.1\}$ are depicted in Fig. 5. These rate pair points are achievable only if both the binning code and the rate-distortion code are perfect.

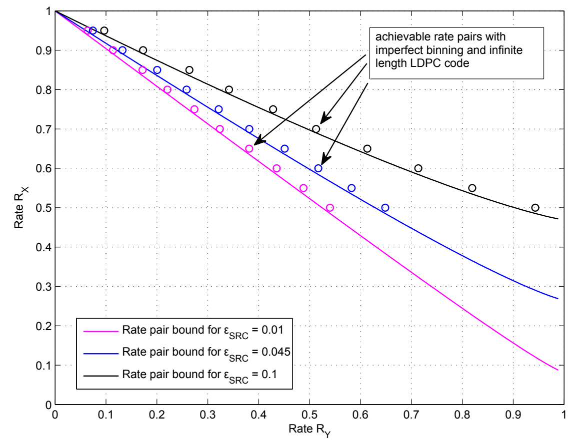

Fig. 5. Rate pair bounds and achievable rate pairs for infinite-length BSC-optimised LDPC codes and perfect rate-distortion codes.

Consider now an infinite-length LDPC code under iterative decoding with decoding threshold ε_{ThLDPC} for a BSC, and assume that a perfect rate-distortion code with distortion $D = \varepsilon_{RD}$ is available. The source is assume to have a correlation of ε_{SRC}. Then using (5), we can calculate ε_{RD} as

$$\varepsilon_{RD} = \frac{\varepsilon_{SW} - \varepsilon_{SRC}}{1 - 2\varepsilon_{SRC}} = \frac{\varepsilon_{ThLDPC} - \varepsilon_{SRC}}{1 - 2\varepsilon_{SRC}} \tag{8}$$

Thus we obtain achievable rate pairs as a function of ε_{ThLDPC}.

Fig. 5 shows such rate pairs for a set of LDPC codes of different rates that have been optimised for the BSC. Due to the gap between the rate of the LDPC does and the capacity of the corresponding BSC, binning with these codes is not optimal. Therefore, there is a gap between the boundary of the rate region and the actual rate pairs. Note that perfect rate-distortion codes have been assumed for this investigation, and the imperfectness is only due to the LDPC codes.

Consider now the case of practical rate distortion codes and finite-length LDPC codes. For practical rate-distortion codes, the actual rate needs to be higher than the theoretical rate required to achieve a certain distortion D, as depicted in Fig. 3. Assume that we want reconstruct X with a certain probability of error, $P(X \neq \hat{X}) \leq \eta$. Then for a given joint distribution $P_{XY}(x, y)$ with correlation ε_{SRC}, we may design the two codes for the desired system performance according to the following two methods. Note that we consider finite-length LDPC codes.

In *Method 1* we first select the LDPC code and then adapt the rate-distortion code:

1. Fix the binning rate R_X.
2. Select an LDPC code of rate $R_{LDPC} = 1 - R_X$.
3. Determine the maximum crossover probability ε_{Th} for the LDPC code such that $P(X \neq \hat{X}) \leq \eta$.
4. Set $\varepsilon_{SW} = \varepsilon_{Th}$, and determine the maximally allowed distortion ε_{ThRD} using Equation (8).
5. Choose a rate-distortion code of minimum rate R_Y with $\varepsilon_{RD} \leq \varepsilon_{ThRD}$.

In *Method 2* we first choose the rate-distortion code and then adapt the LDPC code:

1. Fix the side-information rate R_Y.
2. Select a rate-distortion code of rate R_Y with minimum distortion ε_{RD}.
3. Determine the resulting probability ε_{SW} using (5).
4. Choose an LDPC code for minimum binning rate R_X that achieves $P(X \neq \hat{X}) \leq \eta$ for $\varepsilon_{Th} \geq \varepsilon_{SW}$.

These two methods for code design are applied in the following section to obtain numerical results and illustrate the performance.

V. Numerical Results

For the following investigation, we restricted ourselves to the rate-distortion codes listed in Table II. Furthermore, we considered a set of finite-length LDPC codes determined in the following way. We first optimised infinite-length LDPC codes of different rates for the BSC. Then, for each code, we determined the crossover probability of the BSC, ε_{Th}, such that the finite-length code achieves a bit error rate less than or equal to $\eta = 10^{-3}$. The corresponding LDPC codes and performance parameters are listed in Table III. These parameters lead in the overall coding scheme to the reconstruction criterion $P(\hat{X} \neq X) \leq \eta = 10^{-3}$.

Fig. 6 shows the boundary of the region and rate points achievable for three cases:

(a) practical rate-distortion codes and perfect binning codes;
(b) practical binning codes and perfect rate-distortion codes; and
(c) practical binning codes and practical rate-distortion codes.

These cases are now discussed.

Consider first Method 1, where we fix the binning rate R_X, and thus the rate of the LDPC code. We focus on the zoomed part of Fig. 6 to explain the relationships. The line from point A to point E is the boundary of the achievable rate region, requiring a perfect binning code and a perfect

TABLE III
LDPC codes and their finite-length performance

Rate R_X	ε_{Th} for $P(X \neq \hat{X}) \leq 10^{-3}$
0.50	0.0892
0.55	0.1051
0.60	0.1166
0.65	0.1340
0.70	0.1518
0.75	0.1726
0.80	0.1981
0.85	0.2226
0.90	0.2629
0.95	0.2908

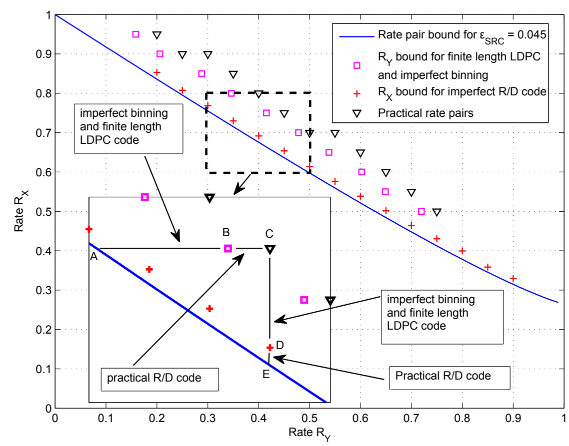

Fig. 6. Boundary of rate region, achievable rate pairs, and rate gaps due to imperfection of binning code and rate-distortion code ($P(X \neq \hat{X}) \leq 10^{-3}$).

rate-distortion code. If we go from a perfect binning code to a practical one, namely to a finite-length LDPC code, the rate R_Y needs to be increased, in order to still achieve the reconstruction criterion $P(\hat{X} \neq X) \leq \eta$. Under the assumption that perfect rate-distortion codes are available, the rate pair point then moves from point A to point B. A practical rate-distortion code requires to further increase R_Y, and we obtain the practical rate pair at point C in the figure.

Consider now Method 2, where we fix the encoding rate R_Y of the side-information, i.e., rate of the rate-distortion code. Similar to above, we focus on the zoomed part of Fig. 6. Going from a perfect rate-distortion code to a practical one increases the distortion and thus requires a higher binning rate. Thus the rate pair point moves from E to D in order to achieve the reconstruction criterion, where we assume that a perfect binning code is available. Considering a finite-length LDPC code for binning, the rate R_X needs to be further increased and we obtain again the practical rate pair, indicated by point C.

In Method 1 we fix the binning rate R_X and then adapt the rate of the convolutional code. The convolutional code rate may easily be adapted by puncturing; finding the best puncturing scheme, however, requires a search over possible puncturing patterns. On the other hand, Method 2 starts with a fixed rate of the convolutional code. The infinite-length LDPC code may be optimised by standard methods, like density evolution or EXIT charts; finding the best finite-length code, however, again requires a search over candidate codes.

Above explanations clarify the impact of imperfections of the binning code and the rate-distortion code on the achievable rate pairs, and they allow to separate and study the individual contributions.

VI. Summary

In this paper we have proposed a practical code construction for the coded side-information problem, where we consider uniformly distributed sources with symmetric correlation. The construction is motivated by the approach taken in the achievability part of the coding theorem, and it is implemented by LDPC codes for the binning part and convolutional codes for the rate-distortion (side-information) part. We have provided numerical examples for our approach and as well demonstrated the impact of the imperfectness of the two codes on the achievable rate pairs. In future research we will look at similar constructions for arbitrary binary sources. The problems to be solved are rate-distortion codes and binning codes for non-uniform sources.

Acknowledgements

This work was supported in parts by the Australian Research Council under the ARC Discovery Project DP120102123.

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