Politecnico di Milano

Homework Lab #2 - Helmholtz Resonator Tree

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Course: Musical Acoustics – Professor: Fabio Antonacci
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Exercise 1. Model the response of a single Helmholtz resonator.

In order to model the response of the Helmholtz resonator, we computed the electric analog of each component, exploiting the given parameters, as follows:

- The input pressure p_1 takes the role of the voltage in our equivalent circuit
- The neck of the resonator can be seen as a short pipe of length l_0 and cross-section S. Its electric analog is an inductance and its value can be computed as

$$L = \frac{\rho l_{tot}}{S} = 0.1\,\mathrm{H}$$

  where ρ is the density of the air flowing through the pipe while l_{tot} takes into account also the end corrections and can be written as

$$l_{tot} = l_0 + \delta_{ex} + \delta_{in} \Rightarrow l_{tot} = l_0 + 0.61 \cdot r + \frac{8}{3\pi} \cdot r$$

  with $r = \sqrt{\frac{S}{\pi}}$.
  Due to the unusual resonator's dimensions, these corrections are not negligible and deeply influence the final result.
- The losses caused by the viscous movement of the fluid in the neck are represented through a resistance. Its value is

$$R = \frac{\rho c}{S} = 4.12\,\Omega$$

- The air cavity of the resonator can be seen as a volume that expands and compresses, showing an elastic behavior. The corresponding electric analog is a capacitance whose value can be computed as

$$C = \frac{V_0 \rho}{c^2} = 7.08 \times 10^{-7}\,\mathrm{F}$$

a) Set a simulation in Simscape and plot the frequency response $H(\omega) = \frac{U_1(\omega)}{p_1(\omega)}$

In order to simplify the computation of exercise 2), we created a Simulink library `"polimi.slx"` which contains the Helmholtz resonator's subsystem and the discrete impulse, shown in figures 1 and 2 respectively.
As regards the Helmholtz resonator's subsystem, it's composed of the previously mentioned components, plus a current and a voltage sensors, used to export the values of the desired quantities to the ports of the subsystem; on the other hand, to compute the discrete impulse, we added a controlled voltage source, whose voltage has been obtained considering the difference between two step functions, with step time equal to 1 and $1 + 1/F_s$ respectively, where $F_s = 44\,100\,\mathrm{Hz}$ is the chosen sampling frequency.
In addition, we considered ideal inductance and capacitance, thus neglecting their series resistances.

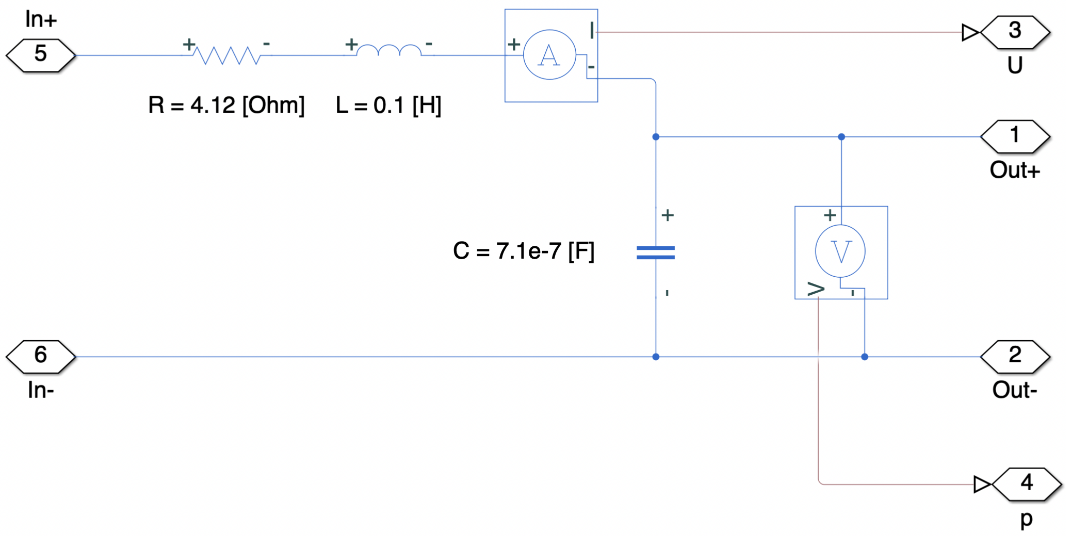


Figure 1: Helmholtz resonator's scheme

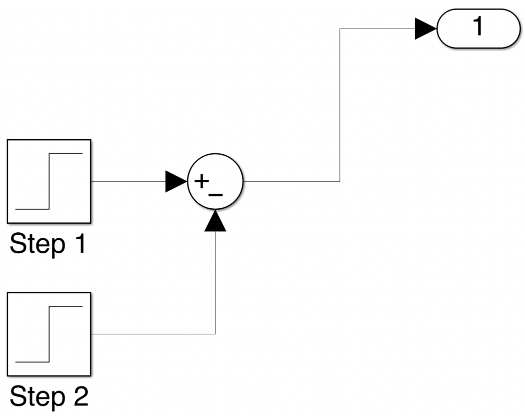


Figure 2: Discrete input scheme

This particular approach gave us the possibility to consider the single Helmholtz resonator (fig.3) as the simplest case of Helmholtz resonator tree (point 2.a), and thus to condensate the computation of the two exercises.
After the implementation of the Simulink library, we wrote the following Matlab function:
`function [model_name] = generate_helmholtz_tree_model(N, K, tree_type, Fs)`
which takes as input the values of N,K, the type of tree (balanced or unbalanced) and the sampling frequency F_s, and automatically generates the desired tree `.slx` model file.
Once the models are created, they are used as input for the Matlab `sim` function, to automatically run the simulation.
This method avoids the direct usage of Simulink interface, since it provides a programmatic way to compute the results directly in Matlab.
As mentioned before, by imposing $N = 0$ and $K = 1$, the single Helmholtz resonator is obtained; the corresponding frequency response function is represented in fig.4.

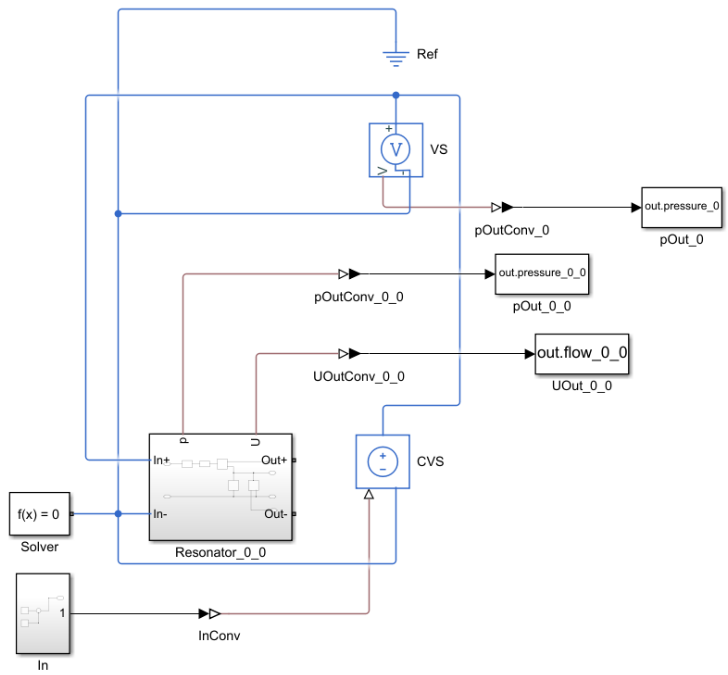


Figure 3: Electric analog scheme of the single Helmholtz resonator

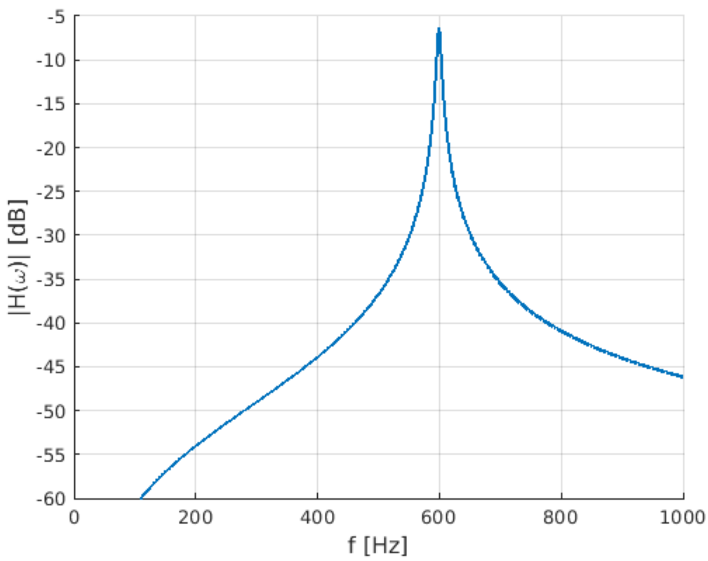


Figure 4: Frequency response function of the single Helmholtz resonator

b) Compute the natural frequency of the resonator analytically and verify that it matches the results of the simulation

The frequency response function we're interested in is the ratio between the acoustic flow $U_1(\omega)$ (equivalent to the electric current flowing through the circuit) and the pressure $p_1(\omega)$ (i.e. the input voltage). In other words, $H(\omega)$ corresponds to the admittance of the system, and can be therefore computed as the inverse of the overall impedance $Z(\omega)$. Since the electric components are in series, $Z(\omega)$ can be in turn computed as the sum of the impedance of each element, as follows:

$$Z(\omega) = j\omega L + R + \frac{1}{j\omega C}$$

The system is analogous to a damped oscillator, where the resistance, inductance and capacitance act as the damper, mass and spring respectively. Its natural frequency w_0 and the correspondent damped one w_d can be therefore computed as:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 3758\,\mathrm{rad/s}$$

$$\omega_d = \omega_0\sqrt{1-\alpha^2} = 3757.9\,\mathrm{rad/s} \Rightarrow f_d = 598.096\,\mathrm{Hz}$$

where the damping factor $\alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$.
By finding the maximum of the frequency response function depicted in fig. 4 by mean of the `findpeaks` Matlab function, we can observe that it matches the analytical result, meaning that the use of lumped components provides a good approximation for this system.

Exercise 2. Combine more resonators in a tree and analyze the response obtained.

a) Use the RLC circuit defined in Ex.1 and connect its replicas to build a NxK tree, where N is height of the tree and K is branch division (= how many leaves for each branch). Use the same parameters for each component and analyze the frequency response using as output the current in one of the leaves ($U_{n,k}$) and pressure p_0 as input

From an analytical point of view, the general input impedance of the $K-th$ layer is given by:

$$Z_K(\omega) = j\omega L + R + \frac{1}{j\omega C + \sum_{n=1}^{N}\frac{1}{Z_{K-1}(\omega)}}$$

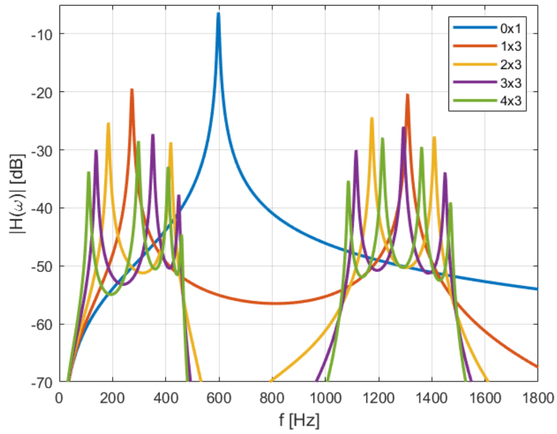
To generate the trees we used the Matlab functions described in exercise 1.
The simulations were run for both balanced and unbalanced trees.

b) Try with different N and K and highlight what these parameters control in the final response

We analyzed the frequency response functions related to the balanced and unbalanced structures obtained varying N from 0 to 4 and K from 1 to 4 separately. By looking at the results obtained and plotted below, the following observations can be made:

- When varying the height of the tree N (fig. 5a, 5b):
  - The configuration is the same for the balanced and unbalanced cases
  - The number of peaks is equal to $2 \cdot N$ (except for the single resonator)
  - The positions of the peaks related to the Nth level are almost symmetric with respect to the $(N-1)$th ones
  - The magnitude of the peaks progressively decreases at increasing of N

(a) Balanced tree



(b) Unbalanced tree

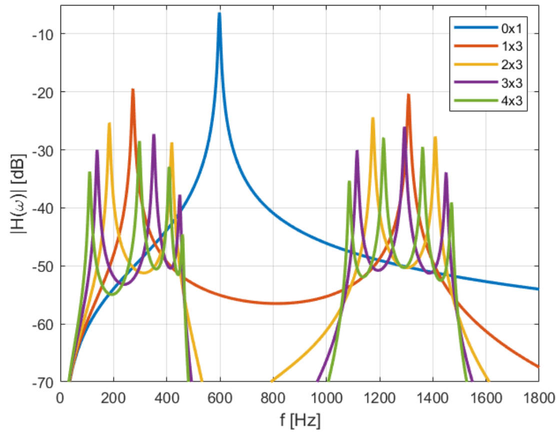
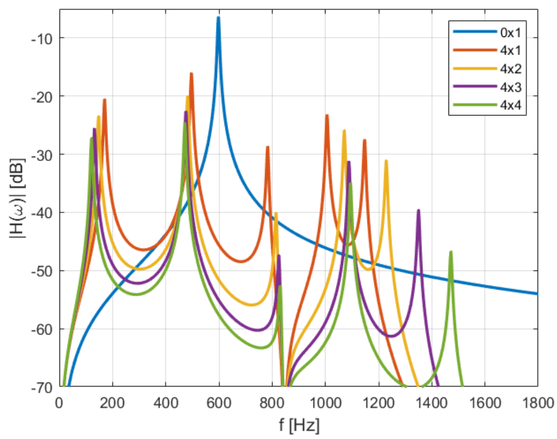


Figure 5: Fixed K, varying N

- When varying the number of branches K (fig. 6a, 6b):
  - In the balanced case, the number of resonances stays the same independently on the number of branch divisions, while in the unbalanced one, a further increasing in the number of peaks is detected
  - Both in the balanced and unbalanced case the resonance's positions are slightly shifted with respect to the parents' one; in particular for $f > f_d$ the shift happens towards the right direction, vice versa for $f < f_d$ the resonance frequencies are diminished
  - The peaks in the response are more equally spread across the frequency axis compared to the previous cases
  - The magnitude of the peaks progressively decreases at increasing of K

(a) Balanced tree



(b) Unbalanced tree

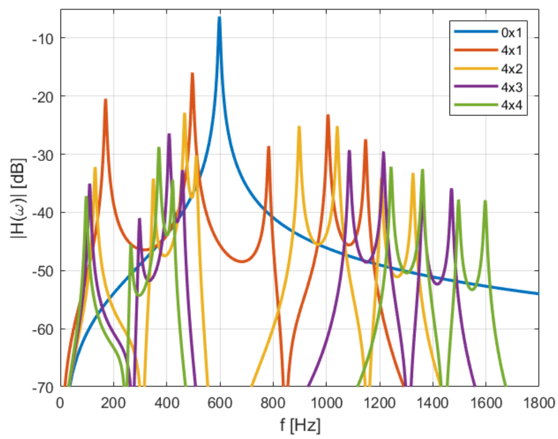


Figure 6: Fixed N, varying K

c) What happens if you change the location in which you evaluate the frequency response inside the tree hierarchy? Elaborate on that, showing some examples

In order to evaluate how the frequency response changes if measured at different points of the tree hierarchy, we chose to analyze different balanced and unbalanced trees, varying N and K, to derive an empirical pattern related to the number and the position of the antiresonances in the several cases.
For the unbalanced structures, we placed the current sensor of the equivalent circuit in correspondence of the truncated branches (i.e. the ones which no longer branch off); all the obtained frequency responses are plotted below.
Since each Helmholtz resonator's subsystem already includes two output ports for the pressure and acoustic flow, the computation was done automatically, without manually changing the position of the measurement point for each iteration. The following observations have been made:

- All the considered configurations show an almost linear decrease in the number of antiresonances, when considering increasingly external levels
- In the balanced cases the peaks are more equally spread along the x axis and the outer level doesn't show any antiresonances (both for N or K fixed)
- In the unbalanced cases, for all the levels (except for the first one) there's an antiresonance in correspondence of the resonant frequency of the single Helmholtz resonator f_d

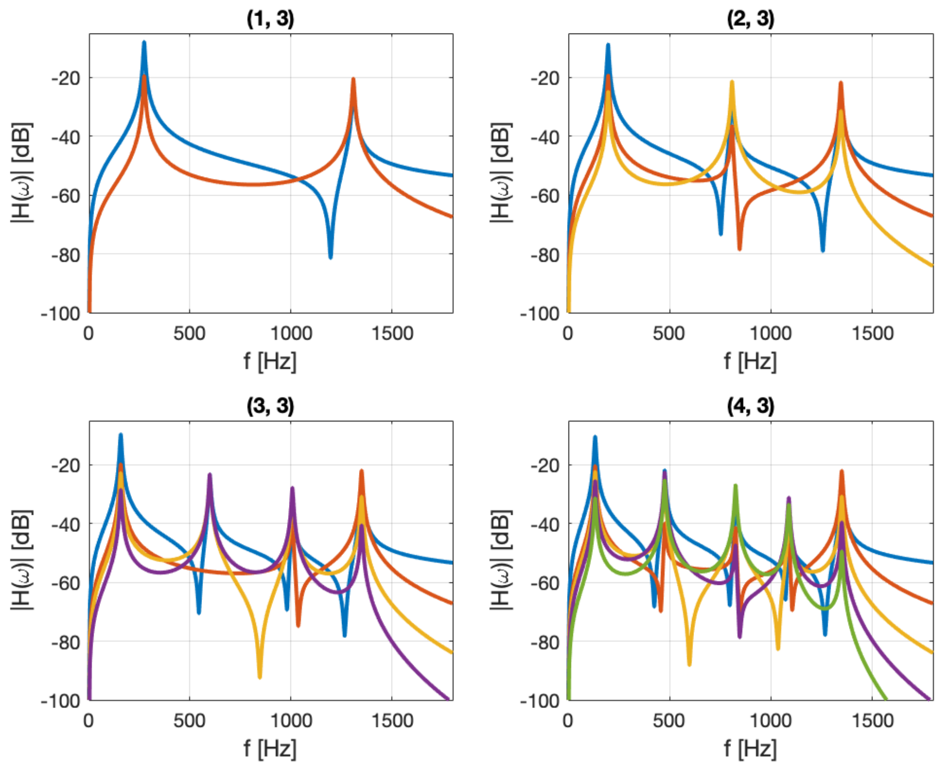


Figure 7: Different measurement levels for fixed K, balanced configuration

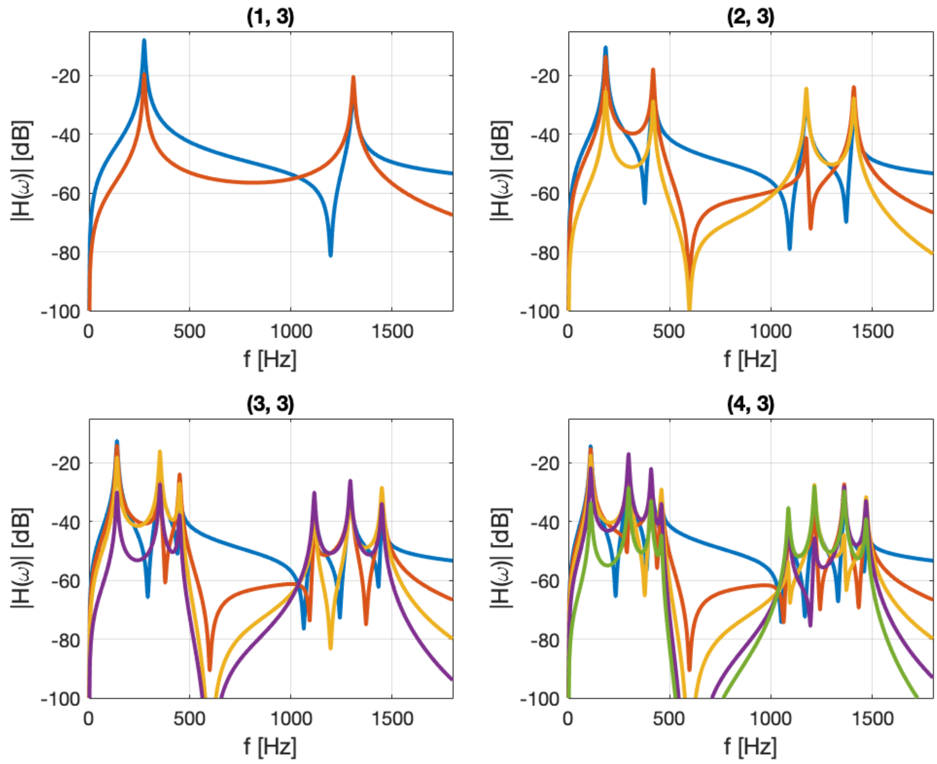


Figure 8: Different measurement levels for fixed K, unbalanced configuration

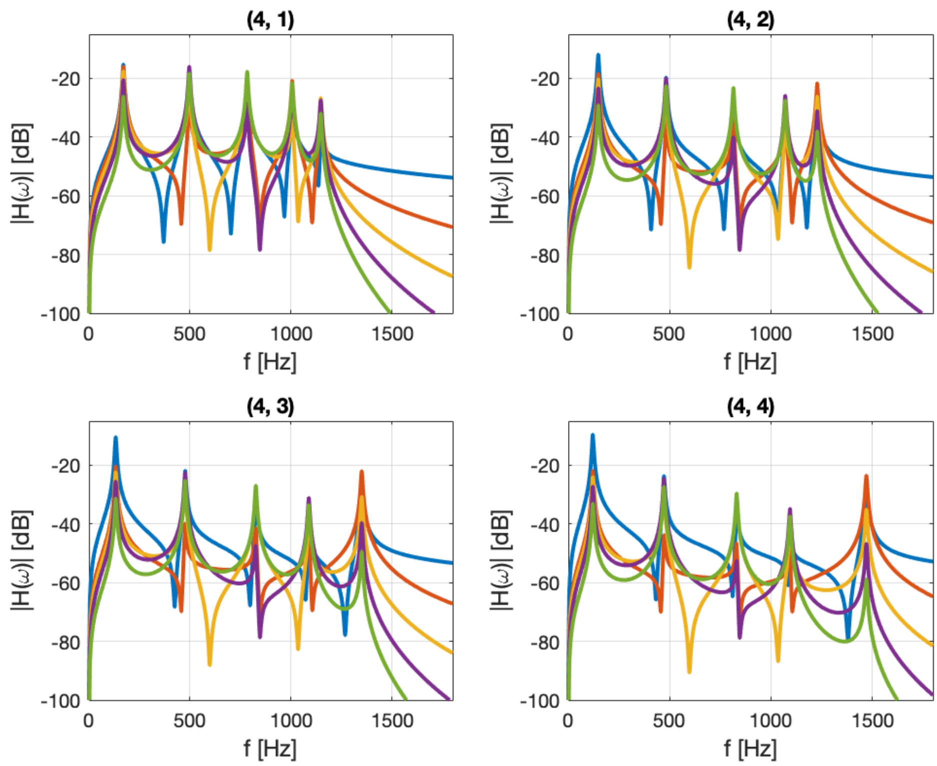


Figure 9: Different measurement levels for fixed N, balanced configuration

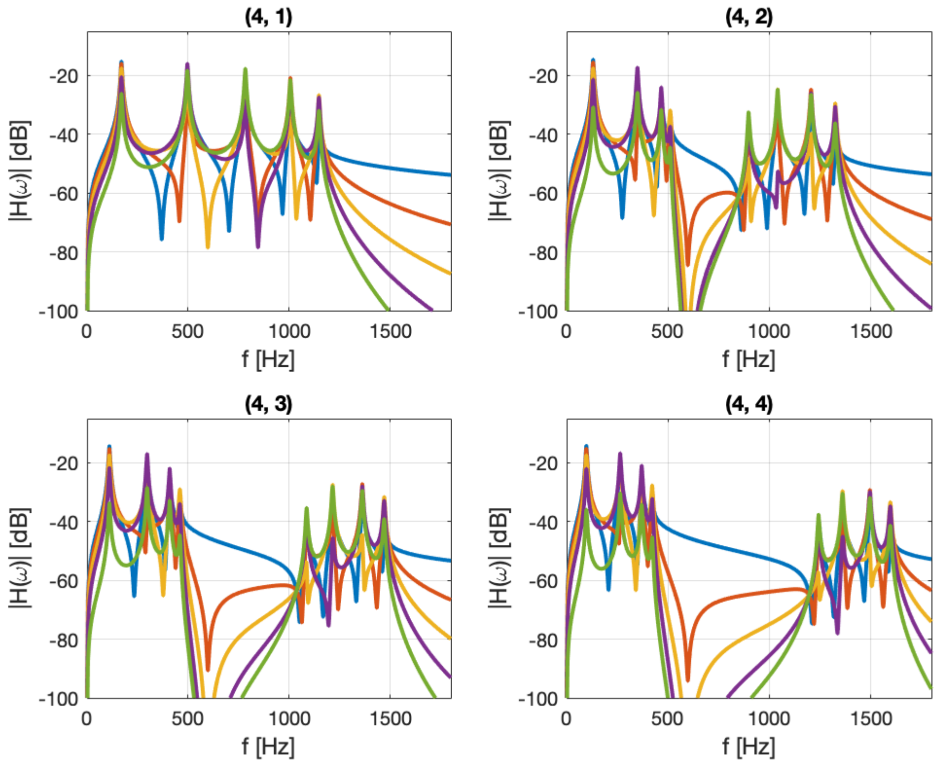


Figure 10: Different measurement levels for fixed N, unbalanced configuration