SASP-DAAP Homework #1

March 2022

In this homework, you will be estimating the Room Impulse Response (RIR) of a small reverberant environment by means of a Wiener filter. Then, you will apply the filter thus obtained using the Overlap-and-Add (OLA) algorithm.

Task

You must develop a MATLAB script by completing the provided code stubs. Moreover, you should prepare a short report (1-page) explaining the main reasoning behind your implementation. For writing the report, you are invited to use LaTeX—this might also be a good chance to familiarize with existing tools such as `www.overleaf.com`.

Problem Formulation

Let's imagine to have a microphone located within a reverberant room at some distance form a known sound source. The microphone is assumed to be ideal and its frequency response is thus simply defined as $H(\omega) = 1$. Therefore, given a real-valued source signal $x(n)$, the signal $y(n)$ captured by the microphone only depends on the RIR $g(n)$, so that $y(n) = (x * g)(n)$.

Wiener Filters

You have been provided with a source guitar signal $x(n)$ and the microphone signal $y(n)$, both at a sampling frequency $f_s = 16000$ Hz. Your task is to estimate the RIR $g(n)$ by means of an adaptive digital filter $W(z)$. You will employ the Wiener filter framework to determine the optimal filter in the minimum mean-squared error (MSE) sense.

Since RIRs are usually very long, we would need a comparably long filter to be capable of capturing their behavior. In this homework, we will consider a Wiener filter with $M = 4000$ taps.

First, you will implement the closed-form Wiener Hopf solution:

$$\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p}$$

where $\mathbf{R} \in \mathbb{R}^{M\times M}$ is the auto-correlation matrix of the input signal and $\mathbf{p} \in \mathbb{R}^{M}$ is the cross-correlation vector between the input and the desired response.

Second, you will implement the Steepest Descent algorithm for iteratively determining the filter taps $\mathbf{w}$. In doing that, consider performing at least 2000 gradient updates. Additionally, you are tasked to determine:

- the step-size parameter μ to ensure the stability of the algorithm,
- the global time constant for the convergence of the algorithm,
- the value of the MSE function $J(\mathbf{w})$ after 2000 iterations,
- the ratio between the value of $J(\mathbf{w})$ computed above and the theoretical minimum $J_{\min}$.

For each point, please provide a couple of lines of code in the MATLAB script to supplement your choices.

Finally, produce a single figure containing three subplots for the true RIR and the filter taps obtained via Wiener-Hopf solution and Steepest Descent, respectively. Please make sure that each plot is correctly named and each axis is appropriately labeled. The abscissae should expressed in time units, e.g., in seconds. Notice that the RIR $g(n)$ is provided for visual reference only and it should not be used otherwise—consider that the true RIR is likely to be unknown in a real-life scenario!

Overlap-and-Add

Once the Wiener filter taps have been computed as discussed above, you will then apply $W(z)$ to the source signal $x(n)$.

First, you will perform time-domain filtering via the MATLAB function `filter`. Finally, use `soundsc` to reproduce the output.

Second, you will implement filtering in the frequency domain by computing the `fft` of both the signal and the filter. In doing so, please make sure to zero-pad the two by a suitable amount, possibly up to the smallest power of two that satisfies the temporal anti-aliasing condition. Finally, play the filtered waveform via `soundsc`.

Then, you will implement the OLA algorithm. Use a tapered window of length 256 samples and a hop-size satisfying the Constant Overlap-and-Add (COLA) condition—notice that the window is of even length! You are required to filter each frame in the frequency domain, possibly with zero-padding, and then return to the time domain via `ifft`. Finally, make sure to reconstruct the signal by correctly applying OLA to each filtered frame. Once again, use `soundsc` to play back the resulting signal.

Homework Submission

Groups of two people are allowed. Both the report and the MATLAB script should have a clear indication of your name(s) and student ID number(s). The materials should be submitted in the dedicated folder on WeBeep:

`Assignments/DAAP Homework #1`

Please submit a single compressed folder per group, e.g.,

`HW1_{name1}_{name2}.zip`