Politecnico di Milano

Homework Lab #4 - Brass instrument simulation

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Course: Musical Acoustics – Professor: Fabio Antonacci

Due date: *January 9th, 2023*

Task

Implement the model of a brass instrument (trumpet) in COMSOL Multiphysics in order to simulate its acoustic response. Through FEM simulation compute the input impedance of the trumpet in the frequency domain. Describe in details the implementation and your choices for setting the simulation, you can use screenshots of the software.

COMSOL Multiphysics

Geometry

All the components of our model (tube, bell and mouthpiece) are symmetric with respect to the center of the tube: we therefore set a 2D axisymmetric simulation to reduce the complexity and computation time of our study.
Since all the simulations are computed progressively adding complexity to the same initial component, we defined a set of parameters for every element of the model and created all the elements as geometry parts, which were later added to the *Geometry* node. A geometry part is a geometry sequence which takes as input a set of input parameters and gives as output a set of geometry objects. This approach allowed us to reproduce and parameterize each component, by simply loading the corresponding instances and specifying the desired parameters' values inside the geometry of the model.
We modelled each element's section as follows:

- Tube: rectangular polygon with base $r_T = 0.6\,\mathrm{cm}$ and height $L_T = 1.37\,\mathrm{m}$.
- Bell: parametric curve with an exponential profile, described by the following equation:
$$r = \sqrt{\frac{S_T}{\pi} e^{mz}}$$
where $m = 28$ and $S_T = r_T^2\pi$. The bell's length is $L_h = 0.2\,\mathrm{m}$.
- Mouthpiece: polygon (narrow throat) with short side equal to $1/8r_T$ and length $L_M = 10\,\mathrm{cm}$ overlapped to a half-circle with radius $r_M = r_T + 0.3\,\mathrm{cm}$ (cup where the lips sit).

Together with the tube, mouthpiece and bell, we also modelled an additional geometry part, in order to mimic an open and nonreflecting infinite domain and simulate the free field conditions. The latter consists of a sphere of radius $r_S = 2\,\mathrm{m}$, with an empty space of length $L_{space} = 20\,\mathrm{mm}$ and cross-section equal to the tube's one in correspondence of the input region. We also defined a layer of thickness $0.25\,\mathrm{m}$ inside the *Sphere* sub node, which allowed us to apply the PML condition described later.

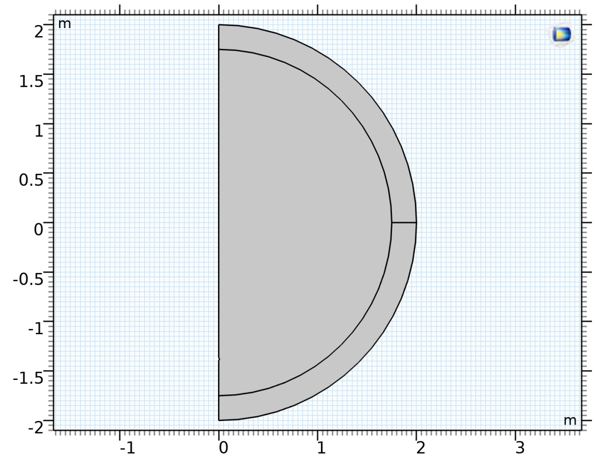

Figure 1: Free field geometry

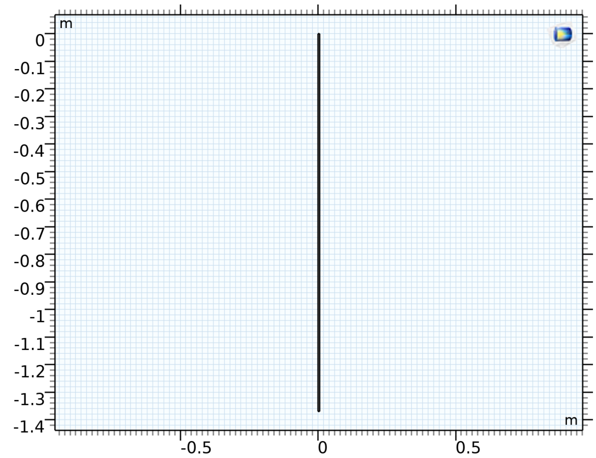

Figure 2: Tube geometry

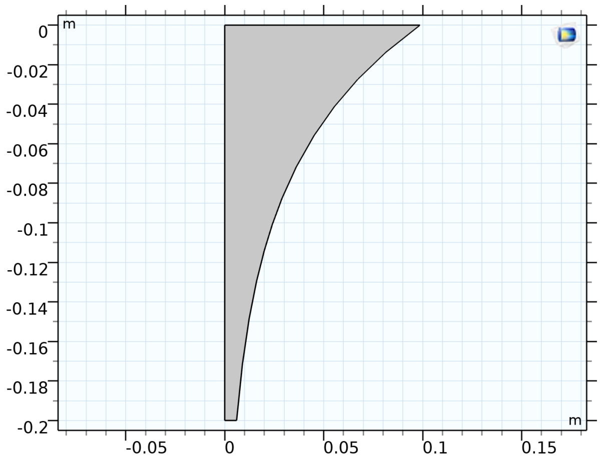

Figure 3: Bell geometry

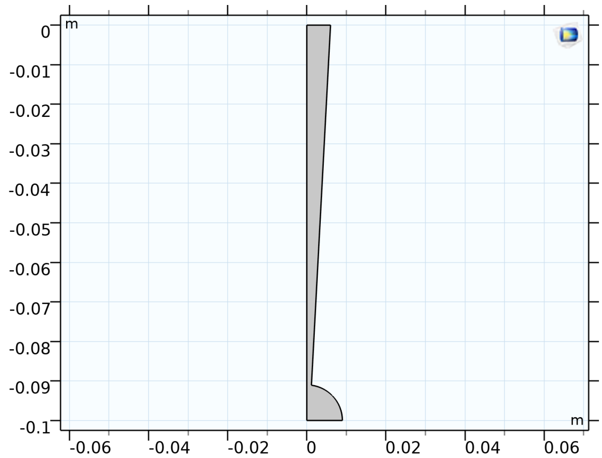

Figure 4: Mouthpiece geometry

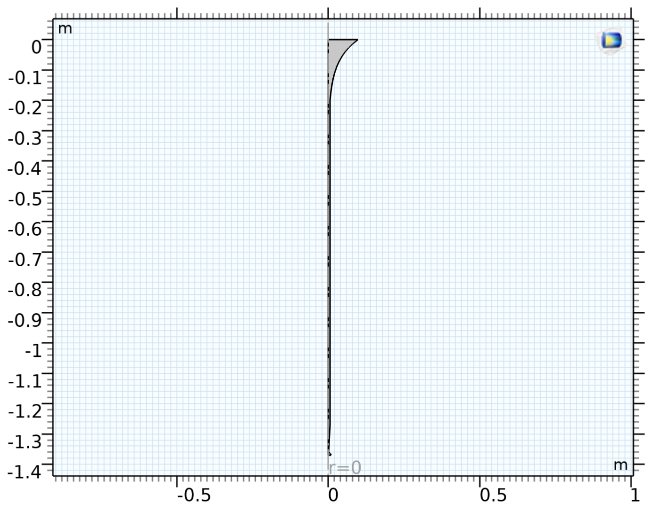

Figure 5: Complete trumpet geometry

Materials and Physics

Once we defined all the geometry parts and related components, we added the *Material* and *Physics* to our model.
With regards to the first one, we globally selected *Air* from COMSOL library and applied it to all domains. To simulate free field, we also set the PML in correspondence of the above defined layer of the sphere, using COMSOL *Perfectly Matched Layer* sub node. The latter represents a non-physical, computational layer which mimics the absorption of acoustic energy propagating through the air.
For the physics we chose *Pressure Acoustic, Frequency Domain.* We specified the *Interior Sound Hard Boundary (Wall)* to all the components, to ensure null total normal velocity on both the up and down side of the selected boundary, while applying a slit on the pressure (which is discontinuous across the boundary). We also applied *Port boundary condition* to excite the acoustic waves that enter our acoustic model, specifying the port type (circular), consistently with the circular cross sections of the elements of our model, and the input pressure of 1.1 Pa. Fig. 6-7 show the *Sound hard and Port boundary condition* applied on the cup of the mouthpiece.

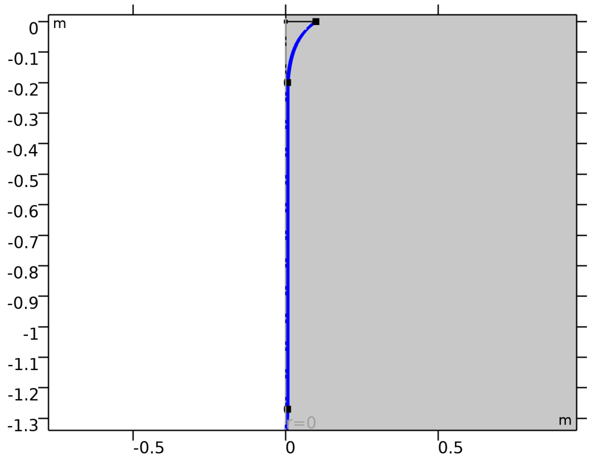

Figure 6: *Sound hard boundary condition*

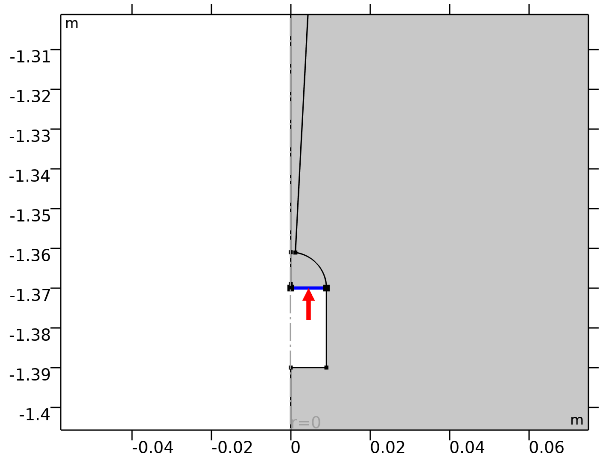

Figure 7: *Port boundary condition*

Mesh

We adopted a free triangular mesh for all the components and then used the *Mapped* sub node to extend it to the adjacent PML.
In order to satisfy the 5 points per wavelength condition, we set the *Maximum Element Size* equal to:

$$max = \lambda_{max}/5$$

where λ_{max} is the wavelength for the maximum frequency f_{max}, given by:

$$\lambda_{max} = \frac{c_0}{f_{max}} = 0.289\,\mathrm{m}$$

and $c_0 = 343\,\mathrm{m/s}$ is the wave speed.

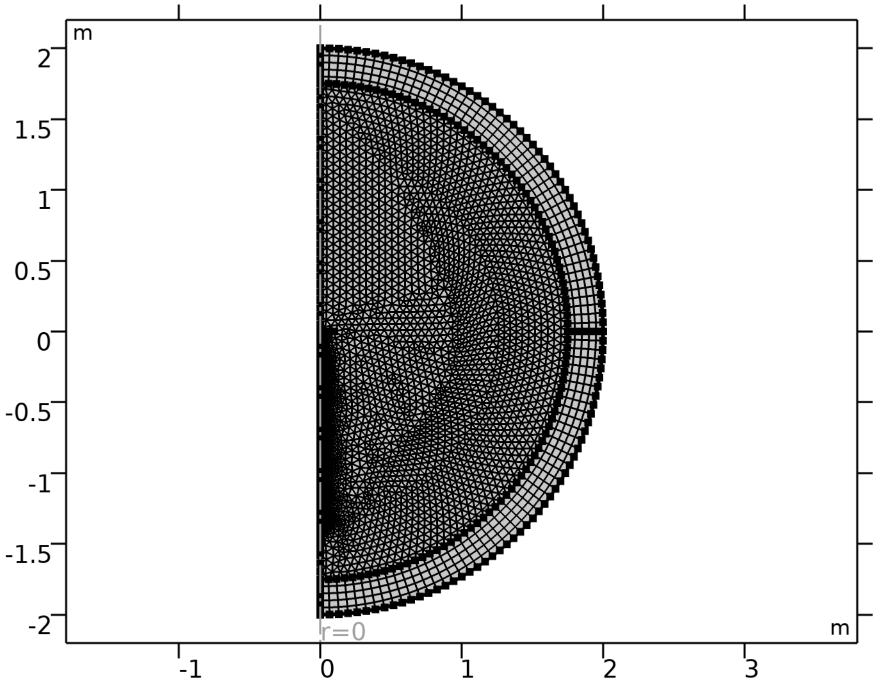

Figure 8: Mesh of the complete model

Study

We computed four analogous studies which analyze the tube, tube with bell, mouthpiece and complete trumpet respectively. As mentioned before, each of them has a greater complexity, due to the presence of additional geometrical entities, with respect to the previous one.
We implemented a *Frequency Domain* study for each component, going from a minimum frequency $f_{min} = 50\,\text{Hz}$ up to $f_{max} = 1200\,\text{Hz}$ in $N = 255$ steps.
The aim of our studies was the computation of the input acoustic impedance, defined as the ratio between pressure $P(\omega)$ and air flow velocity $U(\omega)$ at the input. To compute the impedance in dB across the input interface, starting from its general definition, we expressed it as follows:

$$Z(\omega) = \frac{P(\omega)}{U(\omega)} \Rightarrow Z = 10\log_{10}\left(\left|\frac{\int_S < p_T > dS}{\int_S < v_Z > dS}\right|\right)$$

where $< p_T >$ is the total acoustic pressure, $< v_Z >$ is the component of the velocity along the z-axis and S is the input surface (meaning the tube's cross section for the first two cases, the mouthpiece's one for the second ones). To obtain the conversion in dB we multiplied by 10 since we're dealing with RMS values.
In order to compute the integral over the interface, we defined it in the *Integration* sub node for each component, specifying the desired surface from time to time.
For the mouthpiece study, we adopted a smaller domain to simulate the free field condition, halving the radius of the air sphere.
For each individual study, the physics of the other components were disabled and in order to easily run all the studies in one shot, we also setup a study node that contains the references to all the other ones.

Results and Comments

The sound generated in brass instruments is strongly related to the input acoustic impedance. The latter is a measure of resistance to putting a pressure wave through the tube and describes the effect of the retro-action of the sound energy to the input: in correspondence of the output of the tube, the waves encounter an acoustic interface and are reflected back. These reflections inform the player on how to modify the input (consisting of buzzing lips) to obtain different notes. For these reasons, the alignment of input impedance peaks for a brass instrument is very important for its acoustic response and harmonic generation. High input impedance instruments (such as the trumpet) are tuned so that the notes in correspondence of the frequencies of the peaks (maxima of acoustic impedance) are much easier to play with respect to the others.
Through our analysis we studied the acoustic input impedance associated to each component, so to understand their individual contribution to the resulting sound.

Tube

The tube represents the main part of the brass instrument, where pressure waves are created. Since in a tube only standing waves can propagate, it can only support a set of fixed tones, corresponding to the eigenfrequencies (impedance maxima). In other words, the tube selects which notes can be easily played (meaning the ones corresponding to the eigenfrequencies).
The tube acoustic input impedance (fig.9) presents sharp clear peaks located in correspondence of the fundamental frequency f_1 and the odd harmonics.

$$f_1 \approx \frac{c_0}{4L_T} = 62.59\,\mathrm{Hz}$$

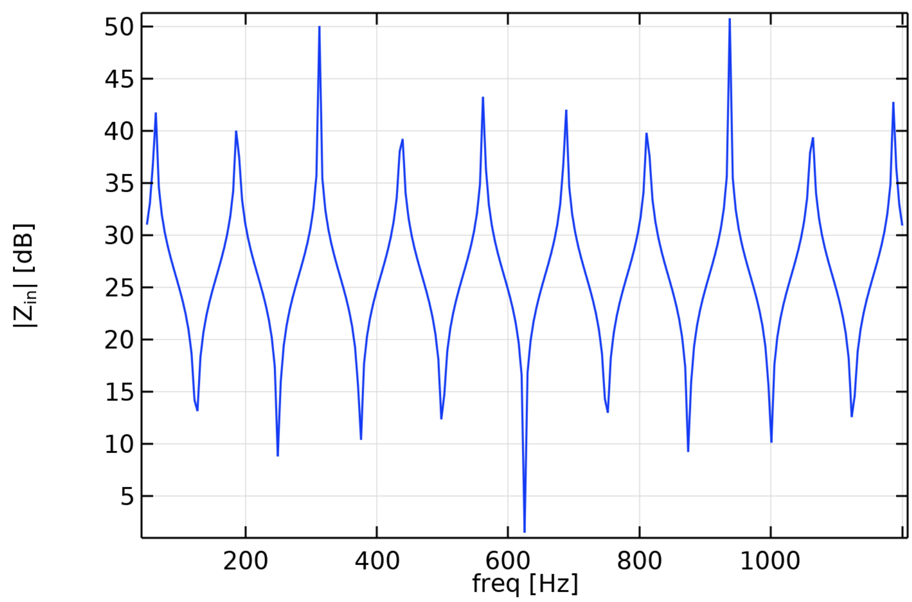

Figure 9: Tube input impedance

Tube with bell

The bell is a conical section at the end of the tube and acts as an impedance adaptor between the tube and the open air. The presence of the bell (fig.10) increases the overall acoustic impedance (the sound will be louder) and improves the radiation smoothing high frequencies (the sound will be brighter).
The geometry profile of the bell slightly shifts the resonance frequencies towards the left, as shown in fig.13.

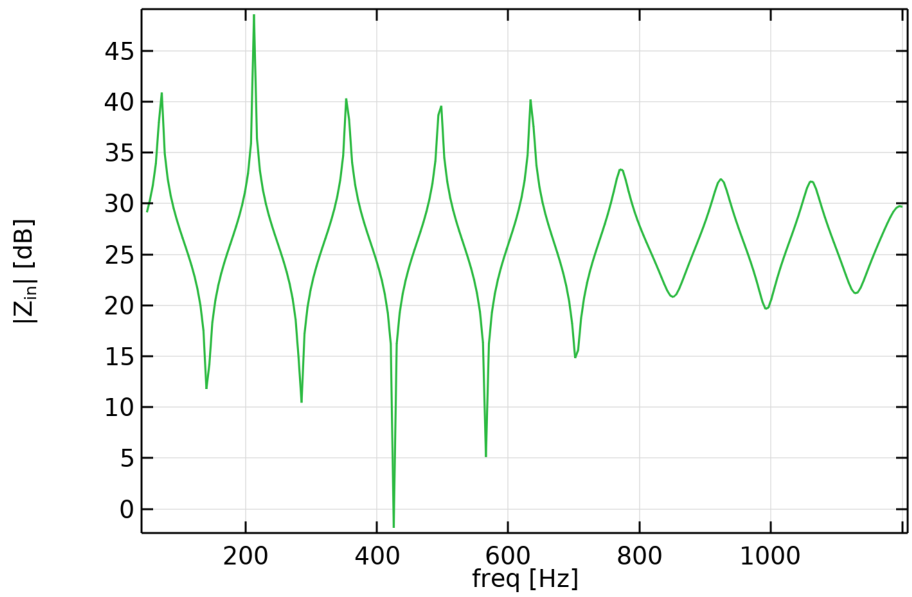

Figure 10: Tube with bell input impedance

Mouthpiece

The mouthpiece acts as an impedance adaptor between the lips and the tube, modifying the resonances of the instrument (and so its final sound). In particular, the mouthpiece acts on the middle frequency range (fig.11), increasing the impedance and changing the envelope of its maxima.
Fig. 11 only shows one resonance peak: this is due to the fact that the mouthpiece in isolation behaves like an internally excited Helmholtz resonator with a peak driving-point impedance at the normal resonance frequency

$$\omega_0 = (LC)^{-1/2}$$

where L and C corresponds to its inertance and capacitance respectively. Since both L and C depend on the mouthpiece geometry, so do the position and height of the mouthpiece's resonance peak.

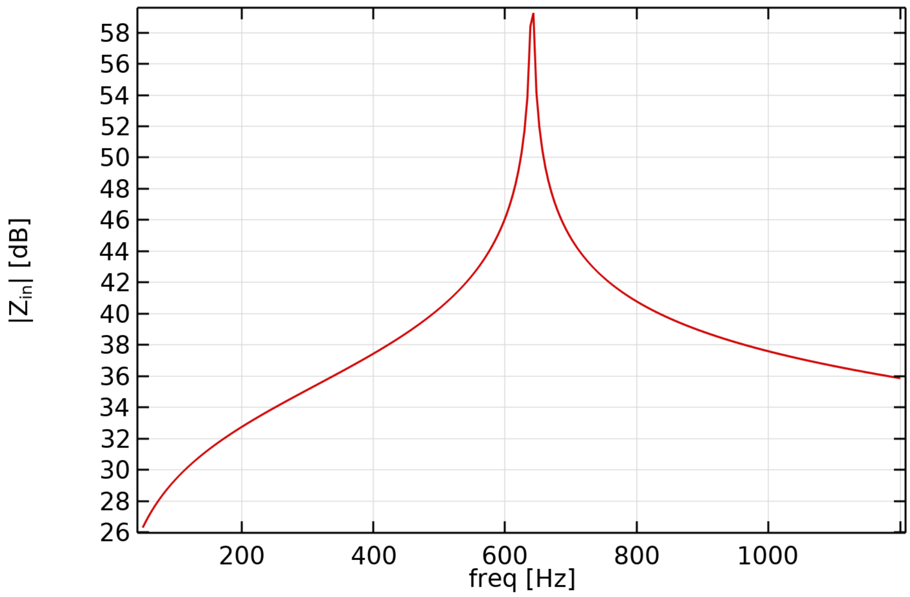

Figure 11: Mouthpiece input impedance

Complete model

This last result is given by the sum of the three above-mentioned elements in a complete model.
We can notice how each element progressively increased the quality of the instrument: we started from a simple tube, which only allows to play a set of fixed notes and had no impedance matching with the outside air, then included the bell so to add overtones and finally completed the model with the mouthpiece, which helps the musician playing the notes in the middle frequency range.
Fig. 12 shows that at high frequencies the acoustic impedance is almost flat while at lower frequencies we have a stair case behaviour. This means that, in the upper frequency range, all the notes will be equally easy to play for the musician; in the lower one, instead, only a discrete number of frequencies will be accessible. The graph also presents a magnitude increase in correspondence of the mid range, due to the presence of the mouthpiece.

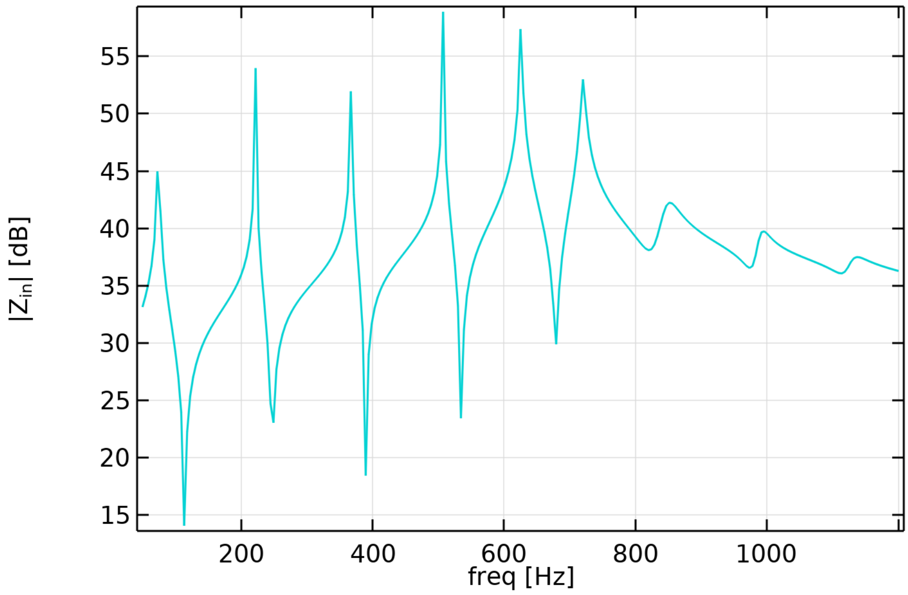

Figure 12: Input impedance of the complete model of the trumpet

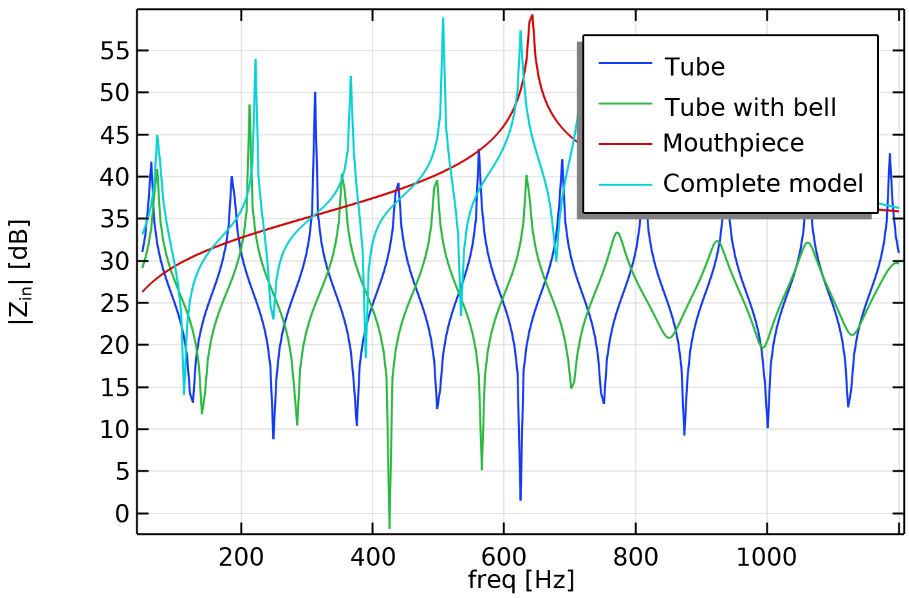

Figure 13: Input impedance comparison