Politecnico di Milano

Homework Lab #1 - Glass Harp

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Course: Musical Acoustics – Professor: Fabio Antonacci
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1. Exercise 1. Generate a 3D model of the Wineglass

a) Build the 3D model of the Wineglass

For the wineglass COMSOL 3D model, we realized the geometry using *Sketch Tools* to draw the glass harp's cross section in the yz work plane (Fig.2), then we revolved it around the z axis and converted the result into solid (Fig.1).

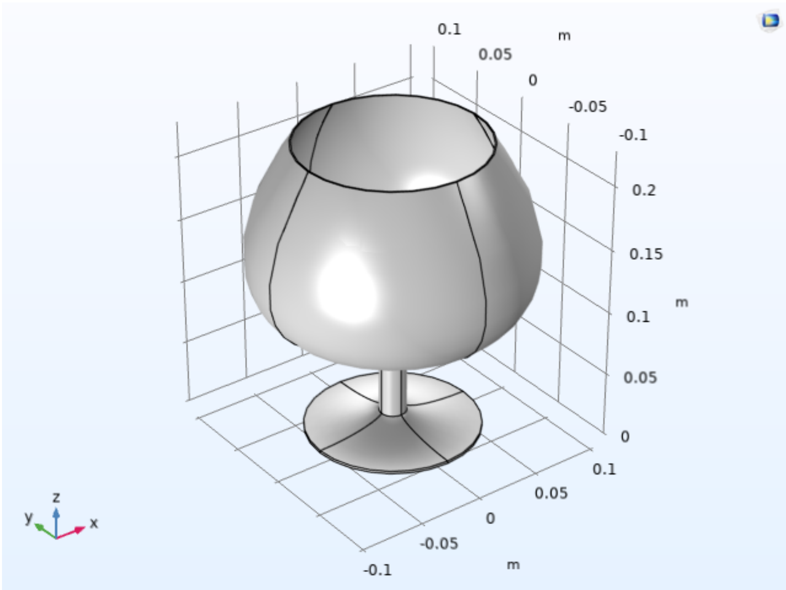

Figure 1: 3D model of the wineglass

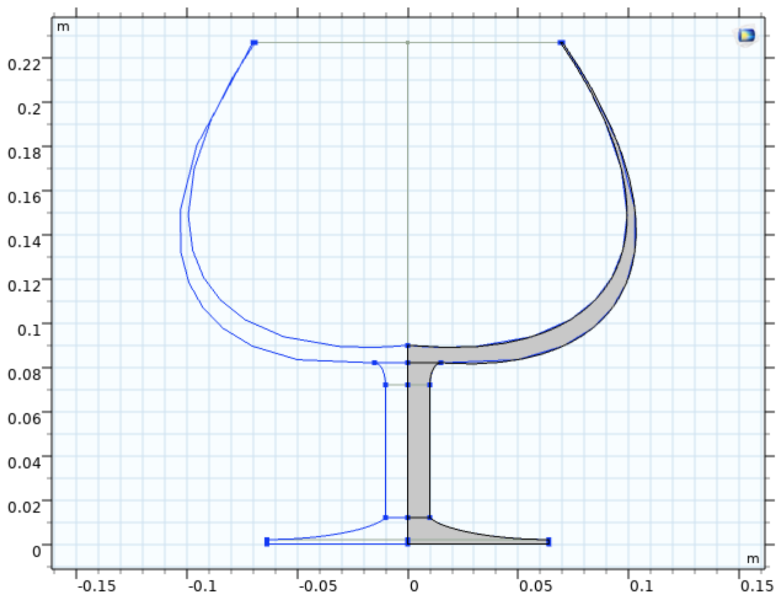

Figure 2: Cross section of the wineglass in the yz plane

The guide lines for the dimensions of the various geometry entities (base, stem, bowl) were obtained by means of a series of overlapped rectangles, which acted as reference to compute the curved final geometry. The material we chose has the following parameters: Young's Modulus 73.1 GPa, density 2203 kg/m^3, Poisson Ratio 0.17.

b) Perform eigenfrequency simulation for the Wineglass using the pinned boundary condition for the bottom face. Searching for at least 20 eigenfrequencies and choose yourself around which frequency (when you specify the study parameters). What do the first eigenfrequency represent? Export some animations related to the frequency motion

For the boundary condition of the wineglass' bottom face we applied the *Prescribed Displacement* node, setting a zero prescribed displacement along the z direction: this allows a rigid motion of the glass only along the xy plane.
We built a mesh (Fig.3) which enabled a trade off between the computational efficiency and the accuracy of the results. This has been done by selecting a composite tetrahedral mesh, consisting of three different element sizes: finer for the bowl, normal for the stem and coarser for the base of the wineglass. The reason that lays behind this logic is that, being the glass' bottom surface fixed with the boundary conditions, it can't vibrate and therefore doesn't need an accurate discretization. The same also applies for the stem which, although not pinned, shows a significant displacement only for high frequencies. Due to the small thickness of the bowl's border we also used the mesh's boundary generator to build a triangular mesh that better fits the model. Lastly we computed the study searching for 20 eigenfrequencies around 1 kHz.

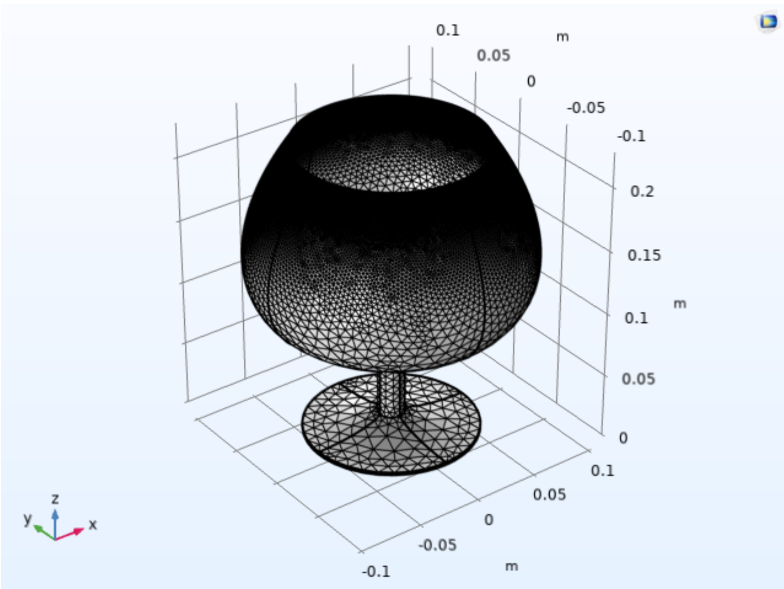

Figure 3: Mesh of the 3D model of the wineglass

By looking at the results listed in the Table **??**, the following observations can be highlighted:

- Due to the imposed constraints, only three out of the six possible rigid modes expected for a 3D model are visible and they are represented by the first three values of Table **??**: as we can see from the table, the values of the first three eigenfrequencies are purely imaginary and almost zero. In general, the computed rigid body modes will not be recognizable as having pure translation or rotation; in fact, from the exported animations, it's possible to see that they contain linear combinations of all the fundamental rigid body motions.

- Some of the results are equal in pairs; actually, they refer to two degenerate modes, that shares the same eigenfrequency. As we can see from the animations, they describe the same kind of motion, displayed from a different angle.

Eigenfrequencies (Hz)
$0.00239i$
$0.00248i$
$0.00314i$
173.07
173.07
550.30
550.36
811.09
811.09
1055.52

Eigenfrequencies (Hz)
1078.57
1111.77
1111.85
1549.43
1549.43
1549.68
1549.74
2106.16
2106.20
2759.67

Table 1: 3D model eigenfrequencies study results

2. Exercise 2. Generate an axysimmetric model of the Wineglass

a) Model the Wineglass geometry using an axysimmetric model

In order to obtain the exact same geometry for the wineglass, we exported it from the yz work plane of the 3D model (*wineglass_geom.mphbin* file) and imported it into the 2D axysimmetric project.

b) Repeat the eigenfrequency study with the same configurations as the ones chosen for Ex.1, fix the bottom of the wineglass using the pinned boundary condition for the bottom face

Since now we're dealing with a 2D axysimmetric model, the previous tetrahedral mesh is no longer suitable, while a triangular one is adopted (Fig.4). Despite the difference in the shape of the elements of the discretization, the same logic as before has been applied (finer elements for the bowl, normal for the stem and coarser for the base).

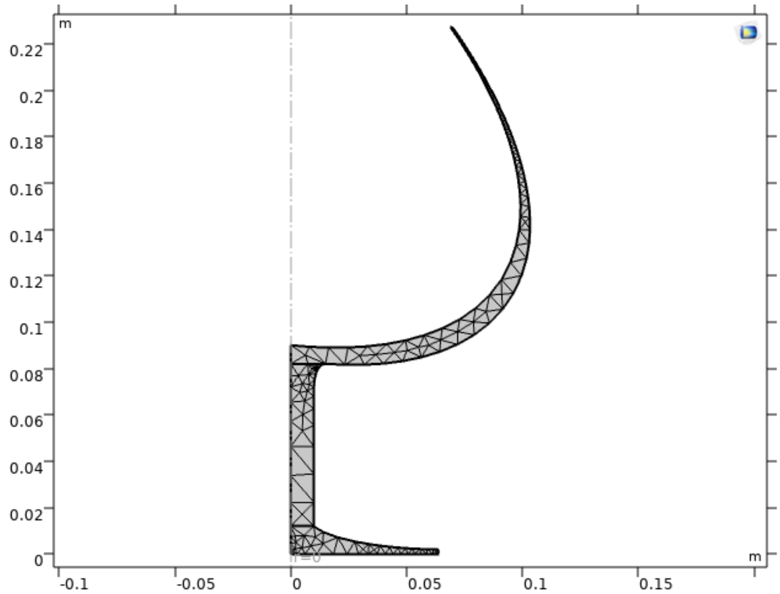

Figure 4: Mesh of the axysimmetric model of the wineglass

The results of this study are shown in Table **??**.

c) Repeat the eigenfrequency study, this time using the circumferential mode extension in the axial symmetry approximation in solid Mechanics

Including the circumferential mode extension means adding to the study the computation of torsional eigenmodes (which are not axially symmetric). Table 3 reports the results.

d) Are you able to obtain the same results of Ex.1 in the case of (b) and (c)? Elaborate on that

The results obtained with the axysimmetric approximation didn't totally match the previous 3D ones. In particular, for case (b), we found one common value (1076 Hz): this happened since this 2D computation only takes into account the modes that are symmetric with respect to the z axis, as we can notice from the exported animation

($2D\_full\_no\_extension$). This incoherence can be fixed exploiting the circumferential mode extension of case (c). By varying the azimuthal mode number m, it becomes possible to study cases that would normally require a full 3D analysis. For low values of m, only torsional and axial modes are included, while for bigger ones also bending and higher-order torsional modes are displayed. In our case, by repeating the same study with m progressively increasing from 0 to 7, we were able to discover all the modes of our previous 3D analysis, as shown in Table 4.

Eigenfrequencies (Hz)
1076.16
6961.30
8885.81
9675.79
10313.99
11018.77
11326.37
12731.99
14576.77
16759.93

Eigenfrequencies (Hz)
16939.74
19419.11
22464.64
25887.52
25997.98
29657.61
33677.11
36174.92
37977.10
39116.82

Table 2: 2D model eigenfrequencies study results without circ. mode extension

Eigenfrequencies (Hz)
0.0021
172.87
1546.79
6616.98
8108.12
9199.98
9852.01
10484.42
11363.62
12406.80

Eigenfrequencies (Hz)
13578.43
14195.76
15932.20
18522.39
21029.06
21519.67
23527.19
24092.13
25265.27
27805.91

Table 3: 2D model eigenfrequencies study results with circ. mode extension (m=1)

Circumferential mode number	**3D analysis common modes (Hz)**
$m = 0$	rigid mode 1055.66 1076.16
$m = 1$	172.87 1546.79
$m = 2$	549.67
$m = 3$	810.21
$m = 4$	1111.10
$m = 5$	1548.80
$m = 6$	2105.50
$m = 7$	2758.70

Table 4: 3D vs 2D axisimmetric analysis eigenfrequencies matching