$$\mathcal{L}(\Theta,\lambda) = \Theta^H G \hat{\Xi} G \Theta - \lambda(\Theta^H G \Theta - 1)$$

- Instead of calculating the derivative wrt. Θ, we can treat $\mathbb{C}^k \equiv \mathbb{R}^{2k}$ and calculate the derivatives wrt. $Re(\Theta)$ and $Im(\Theta)$.

$$\begin{aligned}\mathcal{L}(Re(\Theta), Im(\Theta), \lambda) = {} & Re(\Theta)^T GSG\, Re(\Theta) + i\, Re(\Theta)^T GSG\, Im(\Theta) \\ & - i\, Im(\Theta)^T GSG\, Re(\Theta) + Im(\Theta)^T GSG\, Im(\Theta) \\ & - \lambda\left(Re(\Theta)^T G\, Re(\Theta) + Im(\Theta)^T G\, Im(\Theta) - 1\right)\end{aligned}$$

Then:

$$(1)\quad \frac{\partial \mathcal{L}}{\partial Re(\Theta)} = G(S+S^T)G\, Re(\Theta) + i\, G(S-S^T)G\, Im(\Theta) - 2\lambda G\, Re(\Theta) \overset{!}{=} 0$$

$$(2)\quad \frac{\partial \mathcal{L}}{\partial Im(\Theta)} = G(S+S^T)G\, Im(\Theta) + i\, G(S-S^T)G\, Re(\Theta) - 2\lambda G\, Im(\Theta) \overset{!}{=} 0$$

using $S+S^T = 2R$, $S-S^T = 2i\underline{I}$:

$$(1)'\quad \cancel{2}\, GRG\, Re(\Theta) - GIG\, Im(\Theta) = \lambda\, G\, Re(\Theta)$$

$$(2)'\quad GRG\, Im(\Theta) - GIG\, Re(\Theta) = \lambda\, G\, Im(\Theta)$$

or:

$$\begin{pmatrix} GRG & -GIG \\ -GIG & GRG \end{pmatrix} \begin{pmatrix} Re(\Theta) \\ Im(\Theta) \end{pmatrix} = \lambda \cdot \begin{pmatrix} G & 0 \\ 0 & G \end{pmatrix} \begin{pmatrix} Re(\Theta) \\ Im(\Theta) \end{pmatrix}$$

with the solution being the leading eigenvector of

$$\begin{pmatrix} G^{-1} & 0 \\ 0 & G^{-1} \end{pmatrix} \begin{pmatrix} GRG & -GIG \\ -GIG & GRG \end{pmatrix} = \begin{pmatrix} RG & -\underline{I}G \\ -\underline{I}G & RG \end{pmatrix}$$