manu_mean_directly_from_coefs

February 4, 2021

1 Estimating the Proc2D mean in the Cov.-Smoothing Basis

Import libraries.

[78]:
```
# Required
library(mgcv)
library(sparseFLMM)
library(dplyr)

# To deal with SRV framework
library(elasdics)

# Plotting
library(ggplot2)
library(gridExtra)
library(viridis)
library(rgl)
library(fields)

# Datasets
source("/home/mnl/Statistik/masterthesis/code/datasets.R")

# Seed
set.seed(18)
```

1.1 Prepare Dataset

Load dataset and transform curves to SRV framework.

[2]:
```
# Simulate 2D spirals.
data_curves <- curves.spiral(n_curves=4, rotate=TRUE, scale=TRUE, center=TRUE)

# Create arc length parametrization.
data_curves <- lapply(data_curves, function(data_curve) {
    data.frame(t = get_arc_length_param(data_curve), data_curve)
})

# Get SRV data curves.
```

```
srv_data_curves <- lapply(data_curves, get_srv_from_points)

# Reparametrize SRV data curves and put curves into long (stacked) format.
t_optims <- lapply(srv_data_curves, function(srv_data_curve) {␣
↪c(srv_data_curve$t, 1) })
model_data <- elasdics:::get_model_data(t_optims, srv_data_curves, knots=c(),␣
↪type="smooth")
```

Transform curves from real plane to complex. Add a curve id column (for the covariance estimation).

[3]:
```
# Create curve id column for model_data. (hacky!)
ids <- do.call(c, lapply(t_optims, function(x) length(x)-1))
ids <- rbind(1:length(ids), ids)
ids <- apply(ids, 2, function(x) rep(x[1], times = x[2]))
ids <- do.call(c, as.list(ids))

# x,y to complex. Add id column.
model_data_complex <- complex(re=model_data[,2], im=model_data[,3]) %>%␣
↪matrix(nrow=dim(model_data)[1])
model_data_complex <- data.frame(id = ids, m_long = model_data$m_long, q_m_long␣
↪= model_data_complex)
```

1.2 Estimate Covariance Surface

Build covariance response on (s,t) grid.

[4]:
```
cov_dat <- lapply(split(model_data_complex, model_data_complex$id), function(x)␣
↪{
    combs <- combn(1:nrow(x),2)
    data.frame(
        t = x$m_long[combs[1,]],
        s = x$m_long[combs[2,]],
        qq = x$q_m_long[combs[1,]] * Conj(x$q_m_long[combs[2,]])
    )
})
cov_dat <- do.call(rbind,cov_dat)
```

Fit covariance surface using `mgcv`.

[51]:
```
# Parameters for covariance smoothing
knots = seq(0,1,length=9)
cov.m = 2 # basis order (spline degree-1)
cov.d = 0 # penalty
# Using knots
cov.knots = c(rep(0,cov.m+1), knots, rep(1,cov.m+1))
cov.k = length(cov.knots) - cov.m - 2 # basis dimension.
```

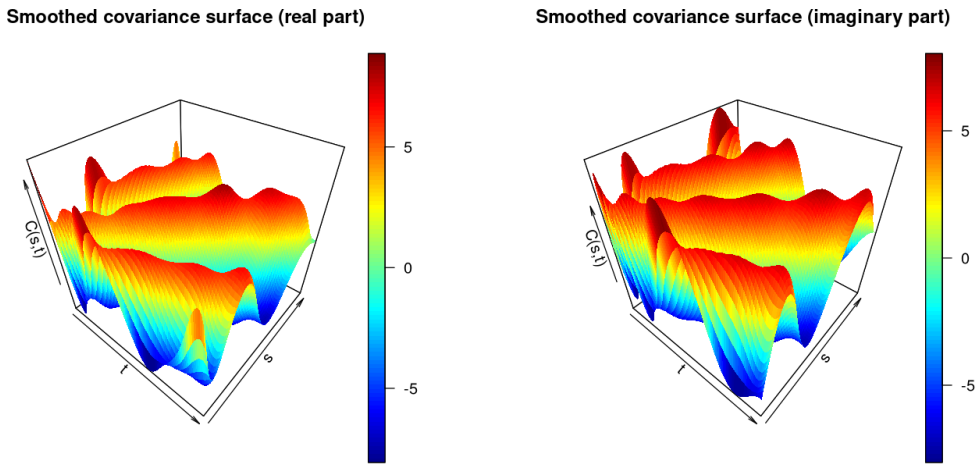
```
# Smooth covariance surface
cov_fit_re <- bam(Re(qq) ~ s(t, s, bs="symm", k = cov.k, m = c(cov.m, cov.d),
                             fx = FALSE, xt = list(skew = FALSE)),
                  data = cov_dat, method = "REML", knots=list(t = cov.knots, s
 ↪= cov.knots), outer.ok = TRUE)
cov_fit_im <- bam(Im(qq) ~ -1 + s(t, s, bs="symm", k = cov.k, m = c(cov.m, cov.
 ↪d),
                                  fx = FALSE, xt = list(skew = TRUE)),
                  data = cov_dat, method = "REML", knots=list(t = cov.knots, s
 ↪= cov.knots), outer.ok = TRUE)
```

```
Warning message in
smooth.construct.ps.smooth.spec(eval(as.call(list(as.symbol("s"), :
"there is *no* information about some basis coefficients"
Warning message in
smooth.construct.ps.smooth.spec(eval(as.call(list(as.symbol("s"), :
"there is *no* information about some basis coefficients"
```

Plot covariance surface on a grid.

[52]:
```
# Define covariance surface grid (s,t).
arg.grid = seq(0, 1, len=101)
cov.grid = expand.grid(t = arg.grid, s = arg.grid)
# Evaluate fit on grid.
cov.re = predict(cov_fit_re, newdata = cov.grid)
cov.im = predict(cov_fit_im, newdata = cov.grid)
```

[53]:
```
options(repr.plot.width=12, repr.plot.height=6)
par(mfrow=c(1,2), mar=c(4,4,4,1), oma=c(0.5,0.5,0.5,0))
# From 'fdapace/src/R/CreateCovPlot.R'
args1 <- list(
    xlab='t', ylab='s', zlab = 'C(s,t)',
    main = 'Smoothed covariance surface (real part)'
)
args2 = list (x = arg.grid, y = arg.grid, z = matrix(cov.re,nrow=101))
do.call(plot3D::persp3D, c(args2, args1))

# From 'fdapace/src/R/CreateCovPlot.R'
args1 <- list(
    xlab='t', ylab='s', zlab = 'C(s,t)',
    main = 'Smoothed covariance surface (imaginary part)'
)
args2 = list (x = arg.grid, y = arg.grid, z = matrix(cov.im,nrow=101))
do.call(plot3D::persp3D, c(args2, args1))
```

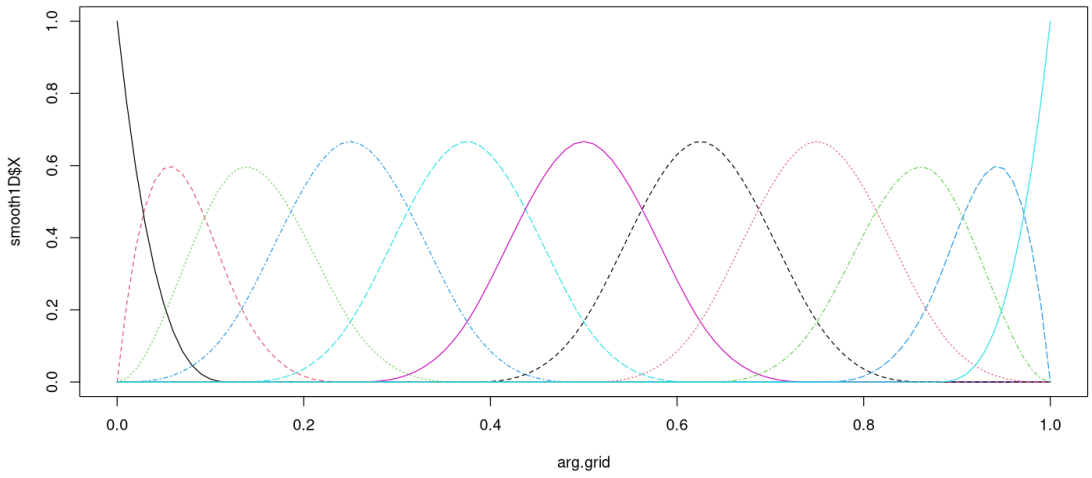


1.3 Estimate Procrustes Mean Shape in fixed Basis

Extract basis functions from covariance smoothing.

Note: Using `bs="ps"` here.

[54]:
```
smooth1D <- smooth.construct(s(t, bs="ps", k = cov.k, m = c(cov.m, cov.d),
                               fx = FALSE, xt = list(skew=TRUE)),
                             data = list(t=arg.grid), knots=list(t=cov.knots))
matplot(arg.grid, smooth1D$X, t = "l")
```



[55]:
```
get_coef_matrix <- function (model){
    # Get tensor basis coefficients.
    F <- model$smooth[[1]]$bs.dim
    skew <- model$smooth[[1]]$xt$skew
    beta <- model$coefficients

    # Build index vector for coeffs.
    ind_mat <- matrix(seq_len(F^2), ncol = F, nrow = F)
    pairs <- cbind(c(ind_mat), c(t(ind_mat)))
    cons <- pairs[pairs[, 1] < pairs[, 2], , drop = FALSE]
    indv <- pairs[pairs[, 1] <= pairs[, 2], 1, drop = FALSE]
    diag <- pairs[pairs[, 1] == pairs[, 2], 1, drop = FALSE]

    # Build coefficient matrix from coeffs and index vector.
    beta.mat <- seq_len(F^2)
    if(skew) {
        beta.mat[cons[,1]] <- beta
        beta.mat[diag] <- 0  # Diagonal is zer0 I guess?
        beta.mat <- matrix(beta.mat, nrow=F)
        beta.mat[upper.tri(beta.mat)] <- -t(beta.mat)[upper.tri(beta.mat)]
    } else {
        beta.mat[indv] <- beta
        beta.mat <- matrix(beta.mat, nrow=F)
        beta.mat[upper.tri(beta.mat)] <- t(beta.mat)[upper.tri(beta.mat)]
    }
    beta.mat
}
```

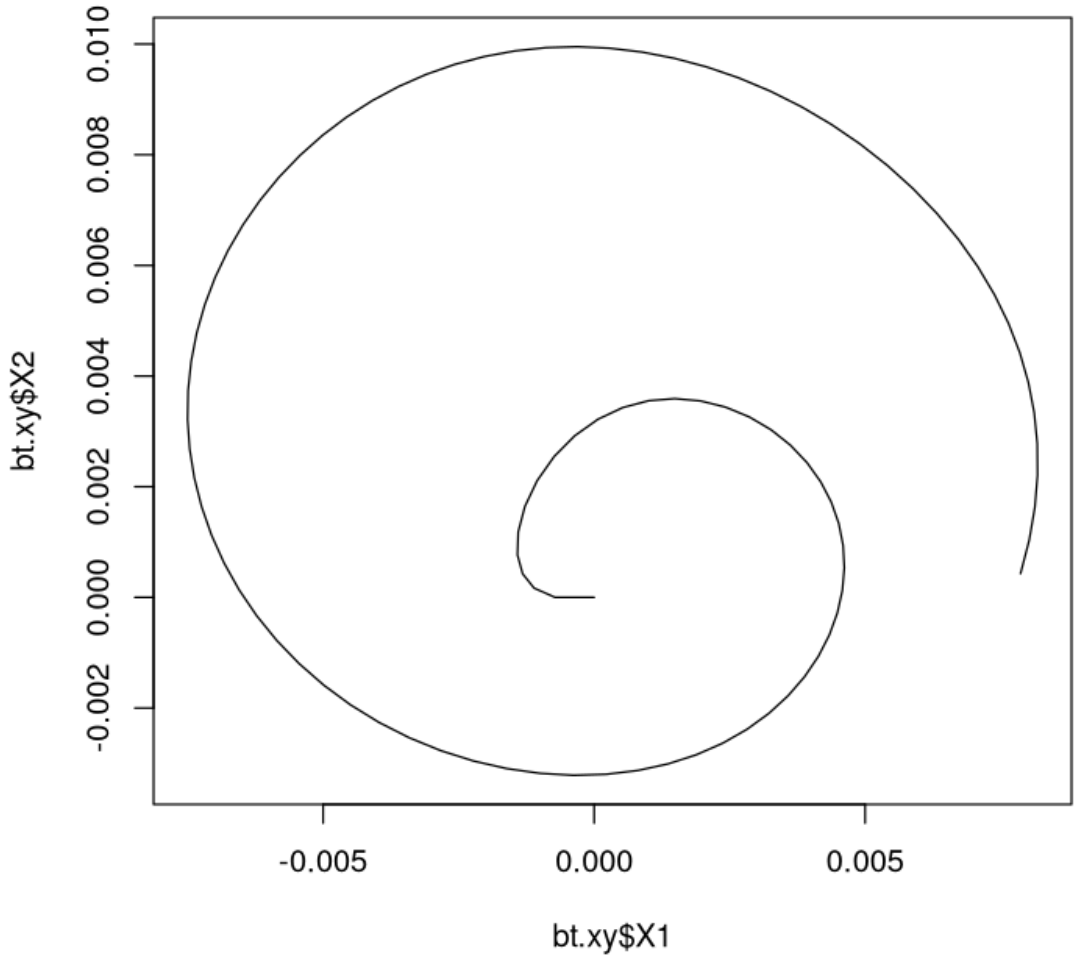
[56]:
```
beta.mat.re <- get_coef_matrix(cov_fit_re)
beta.mat.im <- get_coef_matrix(cov_fit_im)

beta.mat <- matrix(
    complex(real = as.vector(beta.mat.re), imaginary = as.vector(beta.mat.im)),
    ncol = cov.k)
```

[57]:
```
# Calculate largest eigenvector
coefs.mean <- eigen(beta.mat)$vectors[,1]
```

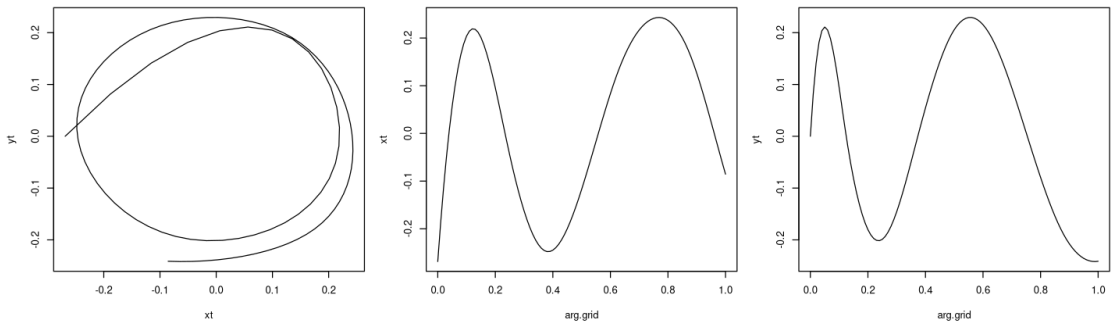
[61]:
```
qt <- smooth1D$X %*% coefs.mean
xt <- Re(qt)
yt <- Im(qt)

qt.xy <- data.frame(t=arg.grid, X1=xt, X2=yt)
bt.xy <- get_points_from_srv(qt.xy)
```

[63]:
```
options(repr.plot.width=6, repr.plot.height=6)
par(mfrow=c(1,1), mar=c(4,4,4,1), oma=c(0.5,0.5,0.5,0))
```

```
matplot(bt.xy$X1, bt.xy$X2, t="l")
```



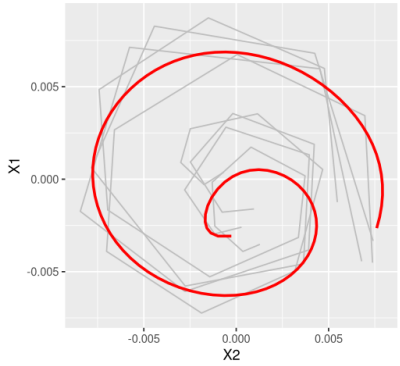
[64]:
```
options(repr.plot.width=12, repr.plot.height=4)
par(mfrow=c(1,3), mar=c(4,4,4,1), oma=c(0.5,0.5,0.5,0))
matplot(xt,yt,t="l")
matplot(arg.grid, xt, t="l")
matplot(arg.grid, yt, t="l")
```



1.3.1 Plot Procrustes Fits + Mean from fixed basis (Note: No warping!)

Note: this is super hacky and just for illustration purposes. ^_^

[123]:
```
bt.xy <- center_curve(bt.xy)

align_curve_proc2d <- function(data_curve, mean){
    mean_coefs = as.matrix(mean)
    mean_eval = elasdics:::make_design(arg.grid, knots = seq(0,1,length = 102))␣
 ↪%*% mean_coefs
    mean_eval = complex(real = mean_eval[,2], imaginary = mean_eval[,1])
    b_coefs <- as.matrix(data_curve[,-1])
    b_eval <- elasdics:::make_design(arg.grid, knots = data_curve$t) %*% b_coefs
    b_eval <- complex(real = b_eval[,1], imaginary = b_eval[,2])
    bm <- Conj(b_eval) %*% mean_eval
    bb <- Conj(b_eval) %*% b_eval
    # Apply rotation+scaling to original curve, return.
    b_compl <- complex(real = b_coefs[,1], imaginary = b_coefs[,2])
    pfit <- as.vector(bm) * b_compl / as.vector(bb)
    data.frame(t = data_curve$t, X1 = Re(pfit), X2 = Im(pfit))
}

pfits <- lapply(data_curves, function(x) {
    pfit <- align_curve_proc2d(x, bt.xy)
    center_curve(pfit)
})

ggplot(bind_rows(pfits, .id="id"), aes(x=X2, y=X1)) +
    geom_path(aes(group=id), size = 0.5, color="grey") +
    geom_path(data=bt.xy, aes(x=X1, y=X2), color = "red", size = 1) +
    coord_fixed()
```


0.005
0.000
-0.005
X1
-0.005
0.000
0.005
X2

[ ]:

[ ]: