manu_proc2d_means

December 9, 2020

1 Elastic Full Proc2D Means

Zwei Probleme: 1. Mean ist an den Enden etwas zu “lang”. 2. Rotation und Scaling der Procrustes SRV Kurven lässt sich nicht einfach auf die Data Curves übertragen.

Veranschaulichung:

[1]:
```r
library(elasdics)
```

ProcrustesLoad some datasets with random rotation and scaling.

[2]:
```r
source("/home/mnl/Statistik/masterthesis/code/datasets.R")
set.seed(18)

data_curves <- curves.spiral(n_curves=4, rotate=TRUE, scale=TRUE, center=TRUE)
data_curves2 <- curves.digit3(rotate=TRUE, scale=TRUE, center=TRUE)
```

Take a look at the data curves.

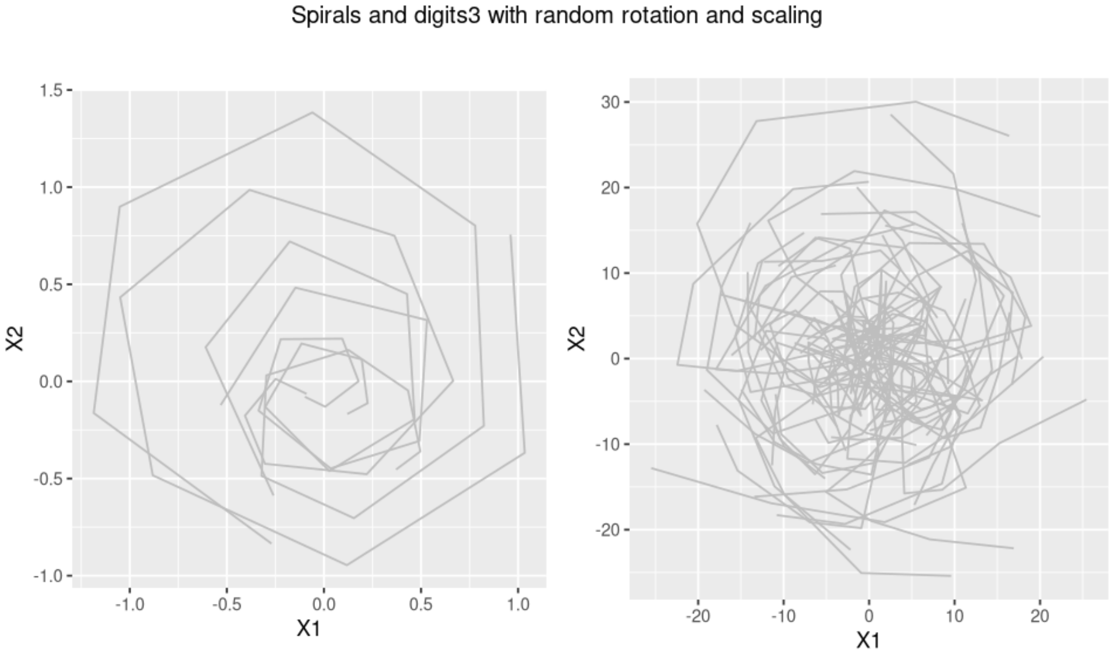
[9]:
```r
library(ggplot2)
library(gridExtra)
library(dplyr)

p1 <- ggplot(bind_rows(data_curves, .id="id"), aes(x=X1, y=X2)) +
    geom_path(size=0.5, aes(group=id), color="grey") +
    coord_fixed()

p2 <- ggplot(bind_rows(data_curves2, .id="id"), aes(x=X1, y=X2)) +
    geom_path(size=0.5, aes(group=id), color="grey") +
    coord_fixed()

# Display plots in smaller size
options(repr.plot.width=8, repr.plot.height=5)

grid.arrange(p1, p2, nrow=1, top="Spirals and digits3 with random rotation and␣
 ↪scaling")
```



Calculate Smooth Elastic Mean Switch out `compute_elastic_mean` function in Lisa's `elasdics` package namespace with my own version that implements procrustes 2d means.

[50]:

```r
source("/home/mnl/Statistik/masterthesis/code/modify_compute_elastic_mean.R") # Switcheroo~

# Turn of warnings
defaultW <- getOption("warn")
options(warn = -1)

# Compute Means
knots <- seq(0,1, length = 11)
smooth_elastic_mean <- compute_elastic_mean(data_curves, knots = knots, proc2d = TRUE)
knots2 <- seq(0,1, length = 31)
smooth_elastic_mean2 <- compute_elastic_mean(data_curves2, knots = knots2, proc2d = TRUE)

# Turn on warnings
options(warn = defaultW)
```

Zwei Varianten die Procrustes fits der data curves zu berechnen

1. Naiver Ansatz: Procrustes fits of curve $\beta_i(t)$ calculated directly as $\beta_i^p(t) = \beta_i^* \hat{\mu} \beta_i / (\beta_i^* \beta_i)$

Anmerkung: Das klappt schon recht gut. Der berechnete procrustes mean $\hat{\mu}$ sieht aber an den "Enden" etwas zu lang aus. Da bin ich gerade noch am überlegen, woran das liegt.

[51]:

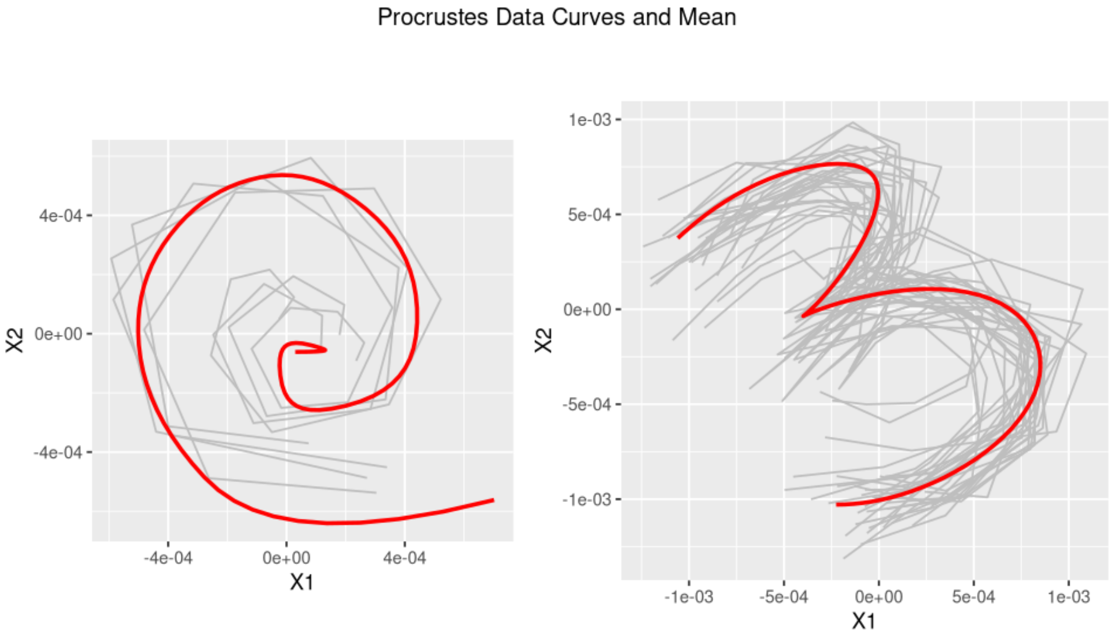
```
align_curve_proc2d <- function(data_curve, mean, grid.len = 101){
    # Evaluate mean function on grid.
    arg.grid = seq(0,1,len = grid.len)
    mean_eval = get_evals(mean, t=arg.grid)
    mean_eval = complex(real = mean_eval[,1], imaginary = mean_eval[,2])
    # Evaluate curve on arg.grid.
    b_coefs <- as.matrix(data_curve[,c(-1,-2)])
    # ToDo: How to smooth here? Use type="smooth" for now.
    b_eval <- elasdics:::make_design(arg.grid, knots = data_curve$t_optim) %*%␣
 ↪b_coefs
    b_eval <- complex(real = b_eval[,1], imaginary = b_eval[,2])
    # Calculate proc2d fit on evaluated curve.
    # Note: Functional scalar products here, that's why I evaluate
    #       mean and data_curve on dense arg.grid to approximate the
    #       integral with a sum. Is that correct?
    bm <- Conj(b_eval) %*% mean_eval
    bb <- Conj(b_eval) %*% b_eval
    # Apply rotation+scaling to original curve, return.
    b_compl <- complex(real = b_coefs[,1], imaginary = b_coefs[,2])
    pfit <- as.vector(bm) * b_compl / as.vector(bb)
    data.frame(t = data_curve$t, t_optims = data_curve$t, X1 = Re(pfit), X2 =␣
 ↪Im(pfit))
}

pfits <- lapply(smooth_elastic_mean$data_curves, function(x) {
        align_curve_proc2d(x, smooth_elastic_mean)
})
pfits2 <- lapply(smooth_elastic_mean2$data_curves, function(x) {
    align_curve_proc2d(x, smooth_elastic_mean2)
})

p1 <- ggplot(bind_rows(pfits, .id="id"), aes(x=X1, y=X2)) +
    geom_path(aes(group=id), size = 0.5, color="grey") +
    geom_path(data=get_evals(smooth_elastic_mean), aes(x=X1, y=X2), color =␣
 ↪"red", size = 1) +
    coord_fixed()

p2 <- ggplot(bind_rows(pfits2, .id="id"), aes(x=X1, y=X2)) +
    geom_path(aes(group=id), size = 0.5, color="grey") +
    geom_path(data=get_evals(smooth_elastic_mean2), aes(x=X1, y=X2), color =␣
 ↪"red", size = 1) +
    coord_fixed()

grid.arrange(p1, p2, nrow=1, widths=8:9, top="Procrustes Data Curves and Mean")
```



2. Procrustes fit of curve $\beta_i^p(t)$ calculated as $\beta_i^p(t) = \mathrm{b}^2 e^{i\mathrm{G}}\, \beta_i(t)$. With b and G calculated from the procrustes fit of the SRV curves $q_i^p(t) = \mathrm{b}e^{i\mathrm{G}} q_i(t)$.

Anmerkung: Das passt leider überhaupt nicht. Ich verstehe aber nicht genau warum. Meine (zu simple?) Überlegung war:

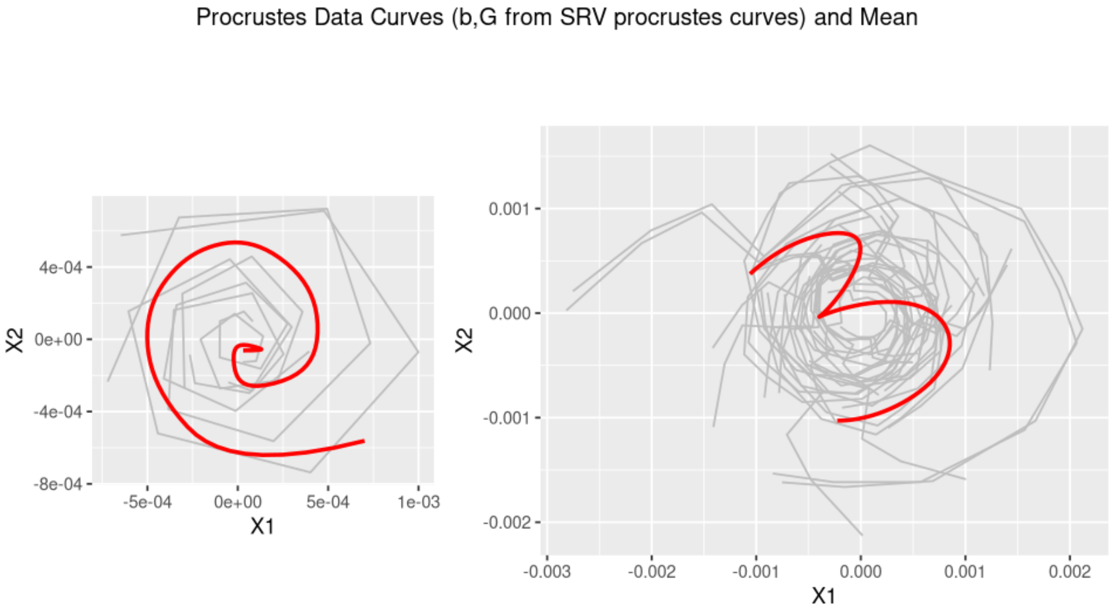
$$\beta_i^p(t)+T = \int_0^t q_i^p(s)\,||q_i^p(s)||\,\mathrm{d}s = \int_0^t \mathrm{b}e^{i\mathrm{G}} q_i(s)\,||\mathrm{b}e^{i\mathrm{G}} q_i(s)||\,\mathrm{d}s = \mathrm{b}^2 e^{i\mathrm{G}} \int_0^t q_i(s)\,||q_i(s)||\,\mathrm{d}s = \mathrm{b}^2 e^{i\mathrm{G}}\, \beta_i(t)$$

da $\mathrm{b} > 0$ und $||e^{i\mathrm{G}}|| = 1$, mit Translation T (und $T = 0$ assuming centered curves).

[53]:
```
p1 <- ggplot(bind_rows(smooth_elastic_mean$procrustes_fits, .id="id"), aes(x=X1, y=X2)) +
    geom_path(aes(group=id), size = 0.5, color="grey") +
    geom_path(data=get_evals(smooth_elastic_mean), aes(x=X1, y=X2), color = "red", size = 1) +
    coord_fixed()

p2 <- ggplot(bind_rows(smooth_elastic_mean2$procrustes_fits, .id="id"), aes(x=X1, y=X2)) +
    geom_path(aes(group=id), size = 0.5, color="grey") +
    geom_path(data=get_evals(smooth_elastic_mean2), aes(x=X1, y=X2), color = "red", size = 1) +
    coord_fixed()
```

```
grid.arrange(p1, p2, nrow=1, widths = 2:3, top="Procrustes Data Curves (b,G␣
↪from SRV procrustes curves) and Mean")
```



```
[ ]:
```