A Minimal Book Example

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Contents

Chapter 1

Prerequisites

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports, e.g., a math equation $a^2 + b^2 = c^2$.

The **bookdown** package can be installed from CRAN or Github:

```
install.packages("bookdown")
# or the development version
# devtools::install_github("rstudio/bookdown")
```

Remember each Rmd file contains one and only one chapter, and a chapter is defined by the first-level heading `#`.

To compile this example to PDF, you need XeLaTeX. You are recommended to install TinyTeX (which includes XeLaTeX): https://yihui.org/tinytex/.

Chapter 2

Basics

The sample space is the set of all possible outcomes. An event is a subset of the sample space, either an outcome (simple event) or a collection of outcomes (compound event).

Events are **independent** if an occurrence of one has no effect on the probability of the other. Events are **exclusive** if $P(AB) = 0$. Use the multiplication rule to calculate combined probabilities for non-exclusive events ($P(A|B) = P(A)P(B)$ for independent events, and $P*()(), and the addition rule to calculate combined probabilities for exclusive events.

)

Chapter 3

Random Variables and Distributions

3.1 Binomial

Binomial sampling is used to model counts of one level of a categorical variable over a *fixed sample size*.

> If X is the count of successful events in n identical and independent Bernoulli trials of success probability π, then X is a random variable with a binomial distribution $X \sim \text{Bin}(n, \pi)$
>
> $$f(X = k; n, \pi) = \frac{n!}{k!(n-k)!}\pi^k(1-\pi)^{n-k} \qquad k \in (0, 1, ..., n), \quad \pi \in [0, 1]$$
>
> with $E(X) = n\pi$ and $Var(X) = n\pi(1-\pi)$.

Example

Data set `dat` contains frequencies of high-risk drinkers vs non-high-risk drinkers in a college survey.

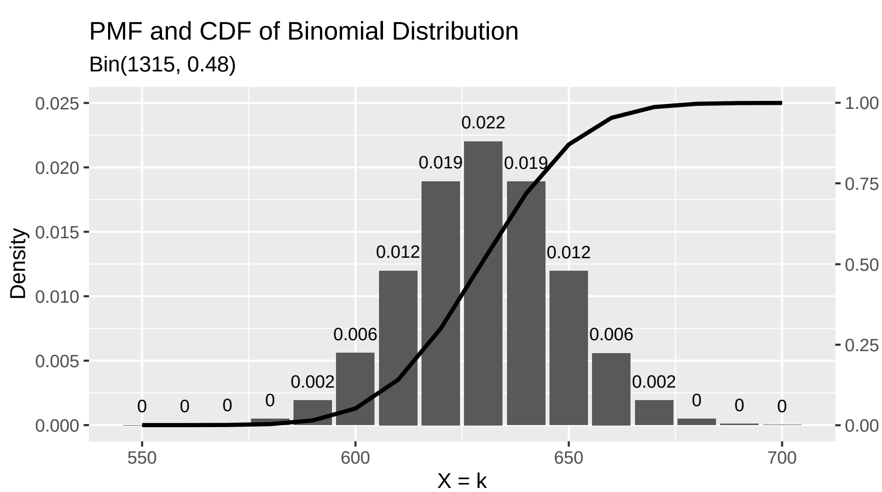
```
## dat$high_risk   n   percent
##            No 685 0.5209125
##           Yes 630 0.4790875
```

The MLE of π from the Binomial distribution is the sample mean.

```r
x <- sum(dat$high_risk == "Yes")
n <- nrow(dat)
p <- x / n
print(p)
```

```
## [1] 0.4790875
```

```r
data.frame(events = seq(550, 700, 10),
           pmf = dbinom(seq(550, 700, 10), n, p),
           cdf = pbinom(seq(550, 700, 10), n, p, lower.tail = TRUE)) %>%
ggplot() +
  geom_col(aes(x = events, y = pmf)) +
  geom_text(aes(x = events, label = round(pmf, 3), y = pmf + 0.001),
            position = position_dodge(0.9), size = 3, vjust = 0) +
  geom_line(aes(x = events, y = cdf/40), size = 1) +
  scale_y_continuous(limits = c(0, .025),
                     sec.axis = sec_axis(~ . * 40, name = "Cum Prob")) +
  labs(title = "PMF and CDF of Binomial Distribution",
       subtitle = paste0("Bin(", n, ", ", scales::comma(p, accuracy = .01), ")"),
       x = "X = k", y = "Density")
```



There are several ways to calculate a confidence interval for π. One method is the **normal approximation** (Wald) interval.

$$\pi = p \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

```r
alpha <- .05
z <- qnorm(1 - alpha / 2)
se <- sqrt(p * (1 - p) / n)
p + c(-z*se, z*se)
```

```
## [1] 0.4520868 0.5060882
```

This method is easy to understand and calculate by hand, but its accuracy suffers when $np < 5$ or $n(1-p) < 5$ and it does not work at all when $p = 0$ or $p = 1$. Option two is the **Wilson** method.

$$\frac{p + \frac{z^2}{2n}}{1 + \frac{z^2}{n}} \pm \frac{z}{1 + \frac{z^2}{n}} \sqrt{\frac{p(1-p)}{n} + \frac{z^2}{4n^2}}$$

```r
est <- (p + (z^2)/(2*n)) / (1 + (z^2) / n)
pm <- z / (1 + (z^2)/n) * sqrt(p*(1-p)/n + (z^2) / (4*(n^2)))
est + c(-pm, pm)
```

```
## [1] 0.4521869 0.5061098
```

This is what `prop.test()` does when you set `correct = FALSE`.

```r
prop.test(x = x, n = n, correct = FALSE)
```

```
##
##  1-sample proportions test without continuity correction
##
## data:  x out of n, null probability 0.5
## X-squared = 2.3004, df = 1, p-value = 0.1293
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.4521869 0.5061098
## sample estimates:
##         p
## 0.4790875
```

There is a second version of the Wilson interval that applies a "continuity correction" that aligns the "minimum coverage probability", rather than the "average probability", with the nominal value.

```r
prop.test(x = x, n = n)
```

```
##
##  1-sample proportions test with continuity correction
##
## data:  x out of n, null probability 0.5
## X-squared = 2.2175, df = 1, p-value = 0.1365
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.4518087 0.5064898
## sample estimates:
##         p
## 0.4790875
```

Finally, there is the Clopper-Pearson **exact confidence interval**. Clopper-Pearson inverts two single-tailed binomial tests at the desired alpha. This is a non-trivial calculation, so there is no easy formula to crank through. Just use the `binom.test()` function and pray no one asks for an explanation.

```r
binom.test(x = x, n = n)
```

```
##
##  Exact binomial test
##
## data:  x and n
## number of successes = 630, number of trials = 1315, p-value = 0.1364
## alternative hypothesis: true probability of success is not equal to 0.5
## 95 percent confidence interval:
##  0.4517790 0.5064896
## sample estimates:
## probability of success
##              0.4790875
```

The expected probability of no one being a high-risk drinker is $f(0; 0.479) = \frac{1315!}{0!(1315-0)!} 0.479^0 (1-0.479)^{1315-0} = 0$.

```r
dbinom(x = 0, size = n, p = p)
```

```
## [1] 0
```

The expected probability of at least half the population being a high-risk drinker, $f(658, 0.479)$, is impossible to write out, and slow to calculate.

```
pbinom(q = .5*n, size = n, prob = p, lower.tail = FALSE)
```

```
## [1] 0.06455096
```

As n increases for fixed π, the binomial distribution approaches normal distribution $N(n\pi, n\pi(1-\pi))$. The normal distribution is a good approximation when n is large.

```
pnorm(q = 0.5, mean = p, sd = se, lower.tail = FALSE)
```

```
## [1] 0.06450357
```

Suppose a second survey gets a slightly different result.

```
dat2 <- data.frame(
  subject = 1:1315,
  high_risk = factor(c(rep("Yes", 600), rep("No", 715)))
)
(dat2_tabyl <- janitor::tabyl(dat2, high_risk))
```

```
##  high_risk   n   percent
##         No 715 0.5437262
##        Yes 600 0.4562738
```

The two survey counts of *high risk* = "Yes" are independent random variables distributed $X \sim \text{Bin}(1315, .48)$ and $Y \sim \text{Bin}(1315, .46)$. What is the probability that either variable is $X \geq 650$?

Let $P(A) = P(X \leq 650)$ and $P(B) = P(Y \leq 650)$. Then $P(A|B) = P(A) + P(B) - P(AB)$, and because the events are independent, $P(AB) = P(A)P(B)$.

```
p_a <- pbinom(q = 650, size = 1315, prob = 0.48, lower.tail = FALSE)
p_b <- pbinom(q = 650, size = 1315, prob = 0.46, lower.tail = FALSE)
p_a + p_b - (p_a * p_b)
```

```
## [1] 0.1484061
```

Here's an empirical test.

```
df <- data.frame(
  x = rbinom(10000, 1315, 0.48),
  y = rbinom(10000, 1315, 0.46)
  )
mean(if_else(df$x >= 650 | df$y >= 650, 1, 0))
```