Qiskit

# Project 33: Comprehensive Tutorial on Two Qubit Decomposition (KAK + Weyl Chamber)

## Final Showcase

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# Paper #1: Constructive Quantum Shannon Decomposition from Cartan Involutions

- Two qubit decomposition using Cartan KAK decomposition
- Base case for more general compilation routine

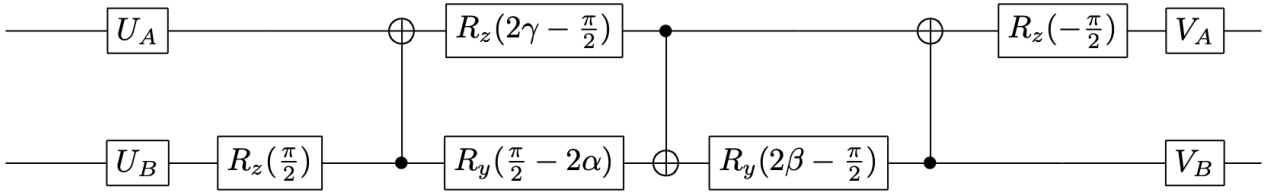



**Figure 1.** The CNOT optimized universal two qubit circuit; $U_A$, $U_B$, $V_A$, and $V_B$ may be decomposed into 3 single qubit rotations each by the Euler angle decomposition given above, and $V_A$ and $U_B$ may absorb the z-rotations preceding and following them respectively yielding a circuit consisting of 3 CNOT gates and 15 single qubit rotations.

# What is KAK Decomposition?

- KAK is a method to decompose two-qubit gates into entangling gates (such as CNOT) and single qubit gates. Using Cartan decomposition from Lie Algebra, KAK has the form:

$$U(4) = e^{i\phi}(A_0 \otimes A_1)e^{i(k_1\sigma_X^2+k_2\sigma_Y^2+k_3\sigma_Z^2)}(B_0 \otimes B_1)$$

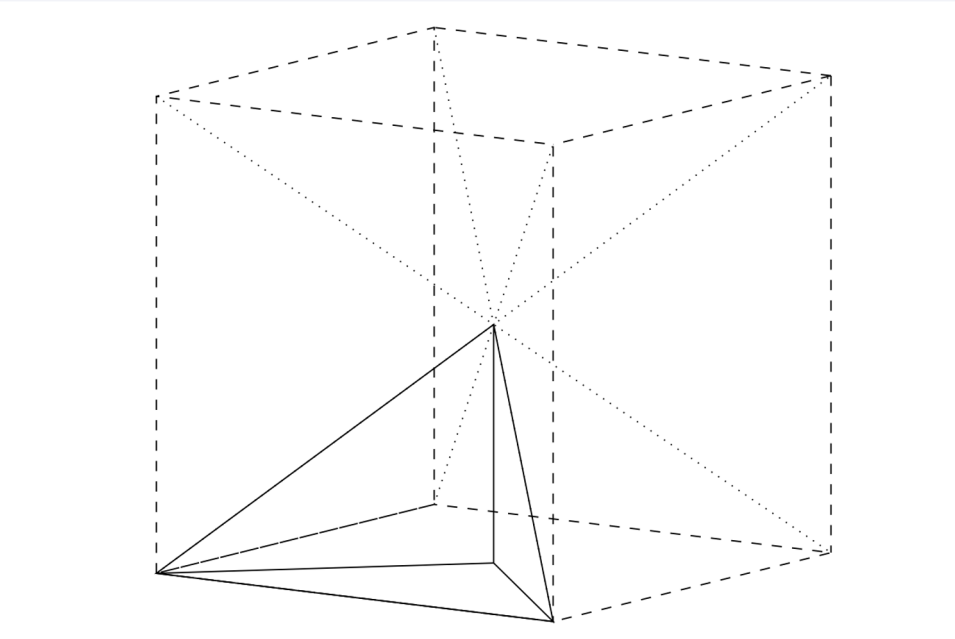
where

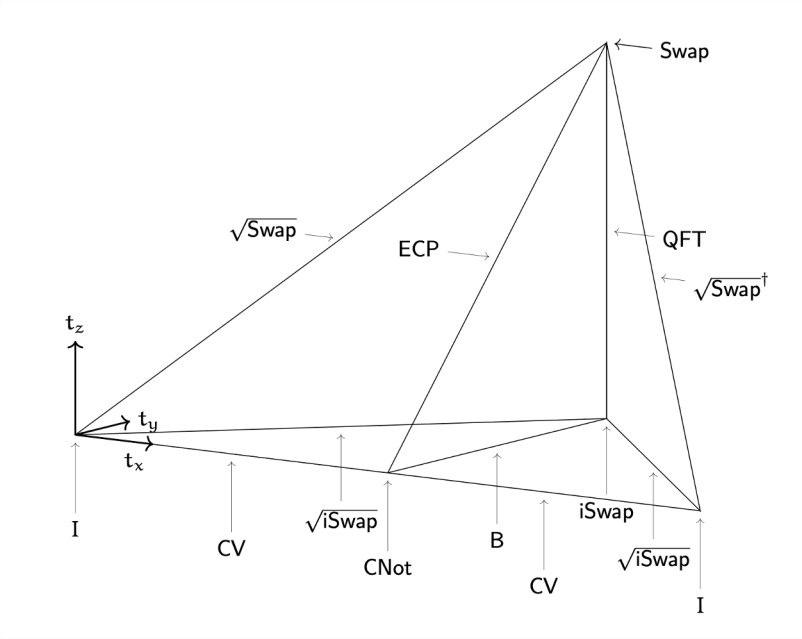
- U(4) is the two-qubit unitary matrix to be decomposed
- Φ is the global phase
- $A_0$, $A_1$, $B_0$, $B_1$ are single qubit gates
- $k_1$, $k_2$, $k_3$ are “canonical” parameters

Sources:
[1] Drury, B., & Love, P. (2008). Constructive quantum Shannon decomposition from Cartan involutions. *Journal of Physics A: Mathematical and Theoretical*, *41*(39), 395305. https://doi.org/10.1088/1751-8113/41/39/395305

# Paper #2: A geometric theory of non-local two-qubit operations

- The Weyl chamber is a geometric tools to “clean up” the KAK decomposition. Formally, it reduces the symmetry symmetry of the “canonical” parameters $k_1$, $k_2$, $k_3$ and allows for “almost unique KAK decomposition.

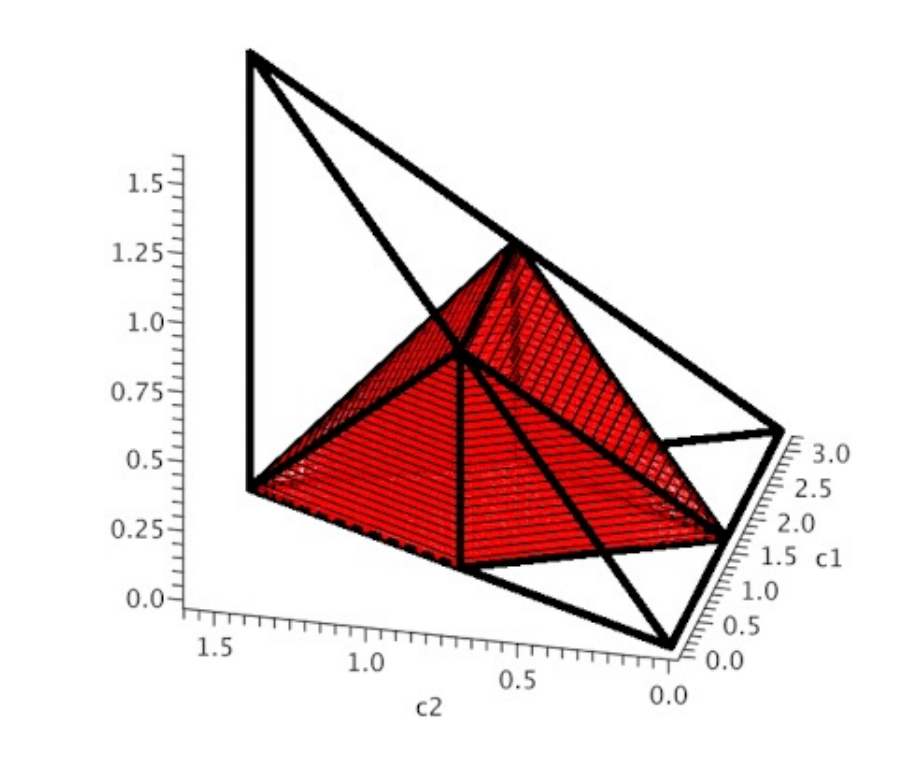





[2] Zhang, J., Vala, J., Sastry, S., & Whaley, K. B. (2003). Geometric theory of nonlocal two-qubit operations. *Physical Review A*, *67*(4). https://doi.org/10.1103/physreva.67.042313

# Weyl Chamber

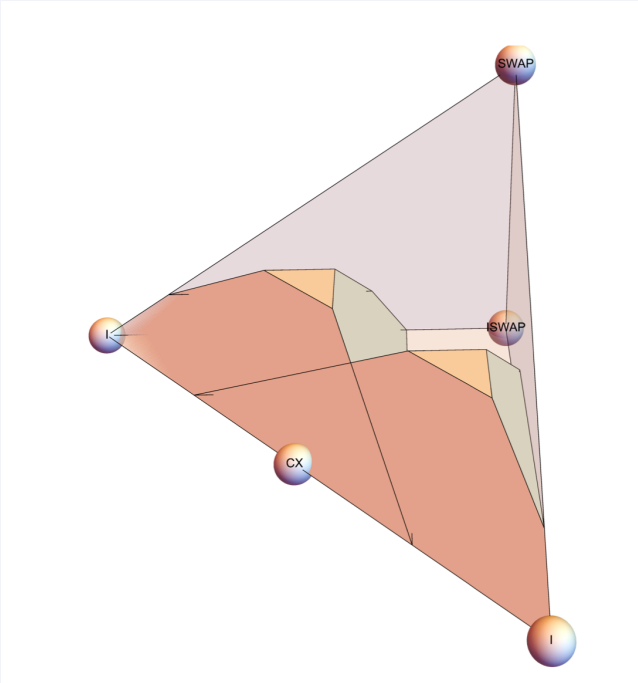
- The Weyl chamber allows Zhang et al. to show that “exactly half the non-local [two-qubit] gates are perfect entanglers”.

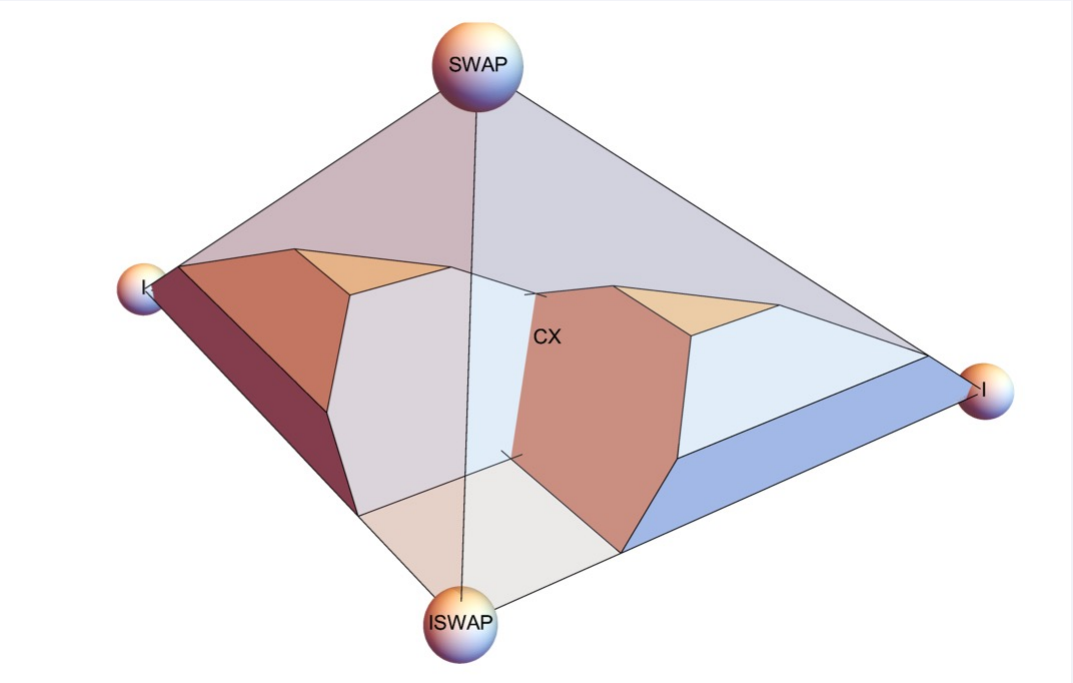



[2] Zhang, J., Vala, J., Sastry, S., & Whaley, K. B. (2003). Geometric theory of nonlocal two-qubit operations. *Physical Review A*, *67*(4). https://doi.org/10.1103/physreva.67.042313

# Future Directions

- Medium article with high level overview and visualizations
- Explore papers that builds up from KAK and Weyl chamber




[3] Peterson, E., Bishop, L., & Javadi-Abhari, A. (2022). Optimal synthesis into fixed XX interactions. https://doi.org/10.22331/q-2022-04-27-696