Adding Fiscal Instruments to the Model

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1 Berg et al.

In Berg et al., various tax instruments are used. The Value Added Tax (VAT), denoted as h, acts as a multiplier on consumption as indicated on page 13, equation 13. Additionally, user fees for infrastructure services are expressed as a fixed multiple or fraction f of recurrent costs. This relationship is represented by the equation:

$$\mu = f\delta P_{zo} \tag{1}$$

The fiscal instruments appear as follows in the household's budget constraint:

$$\begin{aligned} P_t b_t^{\varsigma} - b_t^{\varsigma *} = {} & r_{x,t} k_{x,t-1}^{\varsigma} + r_{n,t-1} k_{n,t-1}^{\varsigma} + w_t L_t^{\varsigma} + \frac{R_t}{1+a} + \frac{T_t}{1+a} \\ & - \frac{1+r_{t-1}^*}{1+g} b_{t-1}^{\varsigma *} + \frac{1+r_{t-1}}{1+g} P_t b_{t-1}^{\varsigma} \\ & - P_{k,t}\left(i_{x,t}^{\varsigma} + i_{n,t}^{\varsigma} + AC_{x,t}^{\varsigma} + AC_{n,t}^{\varsigma}\right) \\ & - P_t c_t^{\varsigma}(1+h_t) - \mu z_{t-1}^{e} - P_t^{\varsigma} - \Phi_t^{\varsigma}, \end{aligned} \tag{2}$$

Regarding expenditure instruments, the government budget constraint ensures that the present discounted value (PDV) of inflows equals the PDV of transfers plus debt service on the three forms of borrowing (domestic, external concessional, and external non-concessional) and public investment, as follows:

$$\begin{aligned} P_t \Delta b_t + \Delta d_{c,t} + \Delta d_t = {} & \frac{r_{t-1} - g}{1+g} P_t b_{t-1} + \frac{r_{d,t-1} - g}{1+g} d_{t-1} + \frac{r_{dc,t-1} - g}{1+g} d_{c,t-1} \\ & + P_{z,t}\mathbb{I}_{z,t} + \mathbb{T}_t - h_t P_t c_t - \mathbb{G}_t - \mathbb{N}_t - \mu z_{t-1}^{e} \end{aligned}$$

Infrastructure investment is exogenous. In the base case, they calibrate public investment to 0.06 of GDP and have it fall to 0.03 (i.e., it is front-loaded and falls, in a 'public investment scaling-up scenario'). This is in line with 2008 figures where SSA LICs spend average 6.09 percent investment to GDP and maintenance/operation to GDP was 3.4 percent. The implied stationary level of public infrastructure is the stock that can be perpetually maintained with gross public investment at 0.03 of GDP.

Transfers initial value exogenously set too. Adjust transfers (and dynamic tax rate) if gap between borrowing + revenue and spending too large.

Large cost overruns during the implementation phase of public investments so multiply NEW project investment $(i_{z,t} - \bar{i_z})$ by $\mathcal{H}_t = \left(1 + \frac{i_{z,t}}{z_{t-1}} - \delta - g\right)^{\phi}$, where $\phi \geq 0$ determines the severity of the absorptive capacity—or "bottleneck"—constraint in the public sector. In effect, net public investment is subject to adjustment costs (they do not appear to acommodate disinvestment in a sensible way; if $i_z < \bar{i}_z$, the capital stock falls more slowly than it would if there were no cost overruns. In their runs they set $\phi = 0$!). The constraint affects only implementation costs for new projects: in a steady state,

$$\left(1 + \frac{\bar{i_z}}{\bar{z}} - \delta - g\right)^{\phi} = 1, \quad \bar{i_z} = (\delta + g)\bar{z} \tag{3}$$

Public investment in infrastructure capital does not always translate one-to-one into effective productive capital due to inefficiencies. According to the following equation:

$$(1 + g)z_t = (1 - \delta)z_{t-1} + i_{z,t}, \tag{4}$$

newly built infrastructure may not be economically valuable productive capital. Instead, productive capital z_t^e is actually used in technologies, defined by:

$$z_t^e = \bar{s}\bar{z} + s(z_t - \bar{z}), \quad \text{with} \quad \bar{s} \in [0, 1] \quad \text{and} \quad s \in [0, 1], \tag{5}$$

where $\bar{s}$ and s are parameters of efficiency at and off steady state, and $\bar{z}$ is public capital. Combining equations (2) and (3), obtain:

$$(1 + g)z_t^e = (1 - \delta)z_{t-1}^e + s(i_{z,t} - \bar{i_z}) + \bar{s}\bar{z}, \tag{6}$$

Effective productive capital then enters the production function in the following form:

$$q_{x,t} = A_{x,t} \left(z_{t-1}^e\right)^{\psi_x} \left(k_{x,t-1}\right)^{\alpha_x} \left(L_{x,t}\right)^{1-\alpha_x}, \tag{7}$$

[In this setup, it should be reasonably straighforward to characterize the social marginal product of public infrastructure capital in the steady state, and ask whether it is larger than the private sector's opportunity cost of current consumption (the bond interest rate) or the public sector's opportunity cost of current spending (the nonconcessional interest rate).]

2 Li et al.

In Li et al., tax instruments such as the VAT (τ) are examined as multipliers on consumption, as detailed on page 14, equation 29. Households allocate their time (n) between producing goods and accumulating human capital ie. going to school (μ). [The household's budget constraint is:]

The government spends on transfers, economic infrastructures (such as roads, railways, and ports) (z^i), social infrastructures (such as schools and hospitals) (z^e), and infrastructure maintenance. The maintenance equation is provided on page 12, equation 20.

$$\Delta b_t^x + \Delta b_t^d = m_t^z + g_t^z + \mathbb{T}_t + \left(r_t^d - g\right)\frac{b_{t-1}^d}{1+g} + \left(r_t^x - g\right)\frac{b_{t-1}^x}{1+g} - \tau_t c_t - G_t, \tag{8}$$

$$\text{where} \quad m_t^z \equiv m_t^e + m_t^i, \tag{9}$$

$$g_t^z \equiv g_t^e + g_t^i, \tag{10}$$

In comparison to the Berg paper, equation (7) includes only one form of external debt, excludes natural resource rents, and makes an explicit distinction between public investment spending and maintenance spending (the latter, however, is simply defined as covering depreciation; the policy choice of interest is the split of total public investment in equation (9)). Much like Berg paper, they set initial investment to GDP to be 0.06 initially and then falling to 0.03. They assume 2/3 of this spending is on roads, other 1/3 on schools. A section of this paper is dedicated to numerically finding this "optimal ratio" which they find to be 76 percentage towards education in one model and 51 percent in another. [It will be interesting how they set this optimality exercise up: maybe they do a grid search of different mixes of the given front-loaded investment boom, taking the overall path of the boom as given; what welfare criterion do they use, etc?] they just numerically find optimal investment in roads vs schools. grid search not quite needed, it's linear

Schools contribute to long-term productivity, although it takes on average $1/\omega$ for their effects to become fully productive. The addition of human capital via schooling is modeled as:

$$A^e \left(z_{t-1}^e\right)^\phi \left(e_t^\chi u_t\right)^\nu, \tag{11}$$

where χ is positive and where the human capital accumulation function is expressed as:

$$(1+g)\xi_t = (1-\omega)\xi_{t-1} + A^e \left(z_{t-1}^e\right)^\phi \left(e_t^\chi u_t\right)^\nu. \tag{12}$$

The human capital that turns into finished schooling positively impacts labor effectiveness, as shown in the following equation:

$$e_t = (1-\delta_e)\frac{e_{t-1}}{1+g} + \omega\xi_{t-1}, \tag{13}$$

where δ_e is the depreciation rate of effectiveness of labor e. Notice that for given steady-state values of z^e and u, equations (11) and (12) can be jointly solved for the steady-state values of ξ and e. [It would be very interesting to calculate the private return to investing in education, at least in the steady state; this may involve looking at the first-order conditions for the choice of u.]

Government infrastructure (roads) (z_{t-1}^i), is taken to power of output elasticity of public capital. Roads increase production functions immediately as it's directly in production function. Effective labor $(e_t^\chi l_t)$ is part of the production function, representing how human capital transforms raw labor into effective units of labor. This relationship is modeled as:

$$y_t = A^y \left(z_{t-1}^i\right)^{\psi} (k_{t-1})^{\alpha} \left(e_t^{\chi} l_t\right)^{1-\alpha}, \tag{14}$$

So, economic infrastructure affects output directly and immediately, while social infrastructure feeds through slowly into the productivity of labor in producing both human capital and output.