Incorporating a simple yet more complicated human capital structure and fiscal block

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1 Model

This dynamic general equilibrium overlapping generations model incorporates human capital in a way that allows allows for the analysis of trade-offs in government human capital investment. This model is meant to capture that human capital accumulation in the young is a result of both parental input and government input, the dynamic complimentarity of human capital investment especially for the young, and the reality that those with higher human capital levels invest more human capital into their offspring.

1.1 Firms

There is a representative firm whose output is

$$Y_t = A_t \left(Z_{t-1}\right)^{\psi} \left(K_{t-1}\right)^{\alpha} \left(H_{t-1} L_{t-1}\right)^{1-\alpha} \tag{1}$$

where Z is the public infrastructure stock and H is the aggregate human capital stock.

Firms are profit maximizing and wages are paid at the marginal product of labor and physical capital is rented out at the marginal product of capital:

$$w_t = H_{t-1} \left(1-\alpha\right) A_t \left(Z_{t-1}\right)^{\psi} \left(K_{t-1}\right)^{\alpha} \left(H_{t-1} L_{t-1}\right)^{-\alpha}, \tag{2}$$

$$r_t = (1-\alpha) A_t \left(Z_{t-1}\right)^{\psi} \left(K_{t-1}\right)^{\alpha-1} \left(H_{t_1} L_{t-1}\right)^{1-\alpha}, \tag{3}$$

$$\Pi_t = Y_t - w_t H_t L_t - r_t K_t - \delta K_t \tag{4}$$

1.2 Government

The government firstly spends by making public infrastructure investments (I^z). The investment gets tranformed into the public infrastructure stock with a inefficiency parameter ($\bar{s}$) as follows:

$$Z_t = (1-\delta) Z_{t-1} + (1-\bar{s}) I_t^z \tag{5}$$

The government also spends on education investment (I^e), transfers (T), and interest payments on public debt (rD). There is a uniform tax rate (τ) levied on individual income, individual capital gains, and goods/services (consumption tax) as almost two-thirds of tax revenue in Sub-Saharan African HIPC countries came from those sources in the 2010s. Government revenue also comes from grants (G), and the government can make-up for any shortfalls by borrowing:

$$\Delta D_t = r_{t-1} D_{t-1} + I_t^e + I_t^z + T_t - G_t - \tau w_t H_t L_t - \tau r_{t-1} K_t - \tau C_t \tag{6}$$

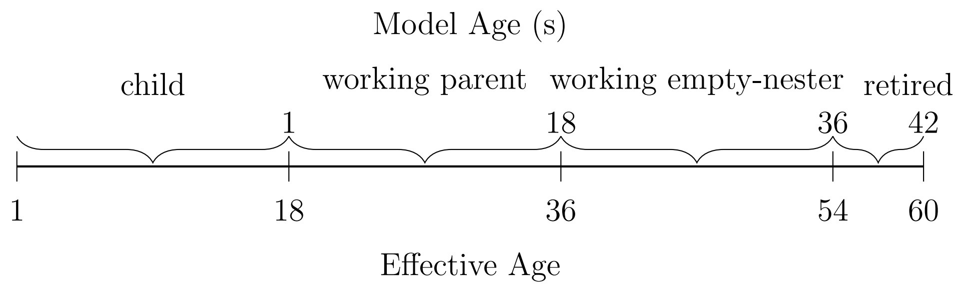
where the debt stock (D_t) cannot exceed a proportion of GDP:

$$D_t \leq \nu Y_t \tag{7}$$

1.3 Households

There are 60 different overlapping generations. Time (t) is discrete where each time unit represents one year. Each generation is represented by a representative agent. Each agent lives for 60 time periods before dying, effectively 60 years of life. However, there are only 42 distinct households, with the first model age $s = 1$ starting at adulthood, effectively aged 19. From $s = 1$, effectively aged 19, until $s = 18$, effectively aged 37, an agent is a working parent needing to optimize their consumption while also providing education to their child. Working parents have a child in their household starting at $s = 1$ and this child leaves to form their own household at the working parent's $s = 18$ time. Children make no decisions and only live in their parents' household and accumulate human capital based on the investments made by their parents and the government. From $s = 19$, effectively aged 37, until $s = 37$, effectively aged 54, an agent is a working empty-nester. Their child has left the household and they no longer need to provide for their child's education. From $s = 38$, effectively aged 55, until $s = 42$, effectively aged 60, an agent is retired and provides no labor. An overview of this provided in the figure below.

Figure 1: 60 Period Age Structure



All households have the same utility function where there is non-separable preferences in consumption and non-working hours:

$$u(c, 1-n) = \frac{(c(1-n)^{\gamma})^{1-\eta} - 1}{1-\eta}. \tag{8}$$

Non-working hours can be spent towards leisure (l) or educating their child (e) where e is 0 unless the household is a working parent:

$$n = l + e, \quad e = 0 \;\; \forall s \in [19, 42] \tag{9}$$

The overwhelming literature shows that more educated parents invest more time into their children's education and lead to better learning outcomes. (Guryan et al., 2008) (Martinez et al., 2022) This is reflected in the model with an agent's ratio of time spent educating vs. leisure being a function of their human capital level:

$$\frac{e}{l} = \omega * log(h) \tag{10}$$

*NOTE: that function is not final, can change. i can also calibrate eta and omega such that n is about 0.33 (8 hours of day) for the average worker and e is about 0.08 (2 hours)

Physical capital is analogous to wealth and agents are allowed to borrow up to the borrowing limit $\underline{k} \leq 0$. Agents start life with capital $k_t^{s=1} = 0$ and leave life with $k_t^{s=42} = 0$.

*NOTE: have questions about underline k

Households' budget constraints are

$$\begin{gathered} k_{t+1}^{s+1} + (1-\tau_t)c_t^s + e_t^s = (1-\tau_t)w_t h_t^s l_t^s + (1+r_t)k_t^s - \tau r_t \max\{k_t^s, 0\} + T, \\ e_t^s = 0 \;\; \forall s \in [19, 42], \quad l_t^s = 0 \;\; \forall s \in [37, 42] \end{gathered} \tag{11}$$

where agents don't spend any time educating their children after model age $s = 18$. Furthurmore, agents are retired and don't spend any time working after model age $s = 36$. The Bellman equation is

$$V^s(h_t^s, k_t^s) = \max_{c_t \geq 0, a_{t+1} \geq a, n_t \in [0,1]} \left\{ u(c, 1-(e+l)) + \beta V^{s+1}(h_{t+1}^{s+1}, k_{t+1}^{s+1}) \right\} \tag{12}$$

subject to the budget constraint (11) and

$$V^{43}(\cdot) = 0 \tag{13}$$

At the start of adulthood at model age $s = 1$, a child is born in the household starting out with a human capital level of 1 ($h^{c,s=1} = 1$). Throughout the child's childhood, they can accumulate human capital through educational investments. These investments consist of parental contributions (e) and government investments (I^e), which follow a constant elasticity of substitution (CES) function:

$$I_t^{c,s} = \left\{ \theta^p \left(\varsigma_e e_t^s\right)^{\psi} + (1-\theta^p) \left(\varsigma_{e,g} \frac{I_t^e}{18}\right)^{\psi} \right\}^{\frac{1}{\psi}} \tag{14}$$

where ς_e denotes the productivity of parental time input, $\varsigma_{e,g}$ denotes the productivity of governmental education investment, and θ is the share of parental input relative to schools affecting human capital accumulation. Government education investment is averaged over the 18 households with children. This is a simplified and modified version of Jang and Yum's (2024) human capital investment structure. Aggregated education investment $I_t^{c,s}$ is then transformed into human capital through

$$h_{t+1}^{c,s+1} = (I_t^{c,s})^{\theta_I^s} (h_t^{c,s})^{1-\theta_I^s}. \tag{15}$$

This human capital accumulation model, similar to Jang and Yum (2024) and Cunha and Heckman (2007), exhibits dynamic complimentary: higher human capital increases the effectiveness of human capital investments thereby increasing the human capital stock. Therefore, earlier investments in human capital enhance the effectiveness of later investments, leading to greater returns over time.

*NOTE: could make theta decrease as age increases to emphasize importance of younger investment so younger investments more crucial?

1.4 Equilibrium

1. Individual and aggregate behavior are consistent:

$$L_t = \sum_{s=1}^{36} \frac{l_t^s}{36}, \tag{16}$$

$$K_t = \sum_{s=1}^{42} \frac{k_t^s}{42}. \tag{17}$$

$$H_t = \sum_{s=1}^{42} \frac{h_t^s}{42}. \tag{18}$$

   such that the aggregate stocks are the weighted sum of the stocks of each generation.

2. Factor prices (w_t, r_t) are completely determined by (2) and (3).

3. Given the government policies and factor prices, households solve the value functions (12) over their life cycle with perfect foresight.

4. The goods market clears:

$$Y_t = \sum_{s=1}^{T=42} \frac{c_t^s}{T} + K_{t+1} - (1-\delta)K_t. \tag{19}$$

2 Calibration