Incorporating a Human Capital Externality, Simplest Case

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1 Introduction

At the end of "Debt Relief With and Without Policy Reform" we incorporate human capital into a neoclassical growth model and study various simulations of the impact of debt relief. In the current note, we extend this to a situation in which a neoclassical aggregate production function shows constant returns to scale in labor, physical capital, and human capital, but where a portion of the contribution of human capital may be external to the firm. Households accumulate physical and human capital in this model, and rent the services of these capital stocks to firms. But because firms cannot appropriate the value of the human-capital spillovers they generate, households are compensated in competitive factor markets only for the direct contribution of their human capital to output. Human capital is therefore under-accumulated in equilibrium, an outcome that reduces the return to physical capital accumulation as well. Policies that incentivize human capital accumulation, even on a temporary basis, have more favorable welfare effects in the presence of the human capital externality than when it is absent.

2 Model Equations

2.1 Firms

The aggregate production function is

$$Y_t = A_t K_{t-1}^{\alpha} H_{t-1}^{\omega-\phi} L^{1-\alpha-\omega}, \tag{1e}$$

When there is no human-capital externality ($\phi = 0$), equation (1e) is identical to equation (1). In the presence of a human capital externality, however, total factor productivity is given by

$$A_t = BH_{t-1}^{\phi}, \quad \text{for} \quad 0 \le \phi < \omega. \tag{*}$$

The profits of firms are given by the counterpart of equation (2),

$$\Pi_t = (1-\tau_t)Y_t - r_t^k K_{t-1} - r_t^h H_{t-1} - w_t L, \tag{2e}$$

where r_t^h is the payment for a unit of the services of human capital. Firms treat A_t as given, which means that the owners of human capital are not compensated for the productivity externality they provide. The firm's optimal choices of capital and labor satisfy a modified version of equation [7]:

$$r_t^k K_{t-1} = \alpha(1-\tau_t)Y_t \tag{7e.1}$$

$$r_t^h H_{t-1} = (\omega-\phi)(1-\tau_t)Y_t \tag{7e.2}$$

$$w_t L = (1-\alpha-\omega)(1-\tau_t)Y_t \tag{7e.3}$$

*We are grateful to Michael Pham for expert research assistance.

2.2 Households

The households' problem is to choose $C_t, I_s^k, I_s^h, K_t, H_t, B_t$, and D_t to maximize

$$\sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma}$$

subject to a flow budget constraint, the dynamic equations for human and physical capital, and the debt constraint. The Lagrangian for this problem, in the case of a fixed external debt constraint $D_t \leq D$, can be written

$$\begin{aligned} L = \sum_{s=t}^{\infty} \beta^{s-t} \Bigg\{ & \frac{C_s^{1-\gamma} - 1}{1-\gamma} \\ & + \lambda_s \Big(\Pi_s + w_s L + r_s^k K_{s-1} + r_s^h H_{s-1} + Z_s - I_s^k - I_s^h \\ & + R_{s-1}^* B_{s-1} - R_{s-1}^* D_{s-1} + F_s - B_s + D_s - C_s \Big) \\ & + \theta_s^k \left((1-\delta) K_{s-1} + I_s^k - \frac{\mu}{2} \left(\frac{I_s^k}{K_{s-1}} - \delta \right)^2 K_{s-1} - K_s \right) \\ & + \theta_s^h \left((1-\delta) H_{s-1} + I_s^h - \frac{\mu}{2} \left(\frac{I_s^h}{H_{s-1}} - \delta \right)^2 H_{s-1} - H_s \right) \\ & + \nu_s (\eta K_s - D_s) \Bigg\} \end{aligned}$$

where the transfer from government (Z_s), profits (Π_s), and external debt cancellation (F_s) are all treated as exogenous.

The first-order conditions governing the household's choice variables are

$$\lambda_t = C_t^{-\gamma} \quad \text{(Consumption FOC)} \tag{M1}$$

$$\lambda_t = \theta_t^k \left[1 - \mu \left(\frac{I_t^k}{K_{t-1}} - \delta \right) \right] \quad \text{(Physical capital investment FOC)} \tag{M2}$$

$$\lambda_t = \theta_t^h \left[1 - \mu \left(\frac{I_t^h}{H_{t-1}} - \delta \right) \right] \quad \text{(Human capital investment FOC)} \tag{M3}$$

$$\theta_t^k = \beta \left[\lambda_{t+1} r_{t+1}^k + \theta_{t+1}^k \left\{ \frac{\mu}{2} \left[\left(\frac{I_{t+1}^k}{K_t} \right)^2 - \delta^2 \right] + (1-\delta) \right\} \right] + \eta \nu_t \quad \text{(Physical capital FOC)} \tag{M4}$$

$$\theta_t^h = \beta \left[\lambda_{t+1} r_{t+1}^h + \theta_{t+1}^h \left\{ \frac{\mu}{2} \left[\left(\frac{I_{t+1}^h}{H_t} \right)^2 - \delta^2 \right] + (1-\delta) \right\} \right] \quad \text{(Human capital FOC)} \tag{M5}$$

$$\lambda_t = \beta R_t \lambda_{t+1} \quad \text{(Domestic debt FOC)} \tag{M6}$$

$$\lambda_t = \beta R_t^* \lambda_{t+1} + \nu_t \quad \text{(Foreign debt FOC)} \tag{M7}$$

With the shadow price λ_t strictly positive, complementary slackness requires that the household's flow budget constraint hold with equality. This gives us

$$C_t = \Pi_t + w_t L + r_t^k K_{t-1} + r_t^h H_{t-1} + Z_t - I_t^k - I_t^h - R_{t-1} B_{t-1} - R_{t-1}^* D_{t-1} + F_t - B_t + D_t$$

Using [2e] to substitute for Π_t, the household's budget constraint can be written as

$$C_t = (1-\tau_t)Y_t + Z_s - I_t^k - I_t^h + R_{t-1}B_t - 1 - R_{t-1}^* D_{t-1} + F_t - B_t + D_t$$

This can be combined with the government's budget constraint

$$B_t = (G_t + Z_t) - \tau_t Y_t + R_{t-1}B_{t-1}$$

to yield the open-economy GDP identity

$$C_t = Y_t - G_t - I_t^k - I_t^h - R_{t-1}B_{t-1} + F_t + D_t \tag{M8}$$

GDP and total factor productivity come from [1a]:

$$Y_t = A_t K_{t-1}^{\alpha} H_{t-1}^{\omega-\phi} L^{1-\alpha-\omega} \tag{M9}$$

$$A_t \equiv B H_{t-1}^{\phi} \tag{M10}$$

The investment dynamics give us the two equations

$$K_t = (1-\delta)K_{t-1} + I_t^k - \frac{\mu}{2}\left(\frac{I_t^k}{K_{t-1}} - \delta\right)^2 K_{t-1} \tag{M11}$$

$$H_t = (1-\delta)H_{t-1} + I_t^h - \frac{\mu}{2}\left(\frac{I_t^h}{H_{t-1}} - \delta\right)^2 H_{t-1} - H_t \tag{M12}$$

and the factor demands of firms give us

$$r_t^k K_{t-1} = \alpha(1-\tau_t)Y_t \tag{M13}$$

$$r_t^h H_{t-1} = (\omega - \phi)(1-\tau_t)Y_t. \tag{M14}$$

The external debt constraint is not always binding, but the complementary slackness condition

$$\nu_t(\eta K_t - D_t) = 0 \tag{M15}$$

must hold in every period.

Equations [M1]-[M15] constitute 15 dynamic equations in the 15 variables

$$A_t, Y_t, C_t, I_t^k, I_t^h, K_t, H_t, D_t, r_t^k, r_t^h, R_t, \lambda_t, \theta_t^k, \theta_t^h, \nu_t.$$

3 Steady-State Equations

To characterize the steady state, we start with (M1) and (M6), which imply $\bar{R} = \beta^{-1}$ or

$$\bar{r} = \rho, \tag{SS1}$$

where ρ is the rate of time preference.

From the stock-flow equations for physical and human capital (M11) and (M12), steady-state gross investment in each type of capital is equal to depreciation:

$$\bar{I}^k = \delta\bar{K}, \qquad \bar{I}^h = \delta\bar{H}. \tag{SS2}$$

From (M2) and (M3), the shadow prices on physical and human capital are equal in the steady state to the shadow value of an additional unit of disposable income:

$$\bar{\theta}^k = \bar{\theta}^h = \bar{\lambda} = \bar{C}^{-\gamma} \tag{SS3}$$

After imposing $\bar{\theta_k} = \bar{\lambda}$ from (SS3), equation (M4) gives us $r^k = \rho + \delta - \eta(\frac{\bar{\upsilon}}{\beta\bar{\lambda}})$. The steady-state shadow price on the debt constraint, in turn, can be obtained from (M6), (M7), (SS1) and (SS3) as

$$\bar{\upsilon} = \beta(\rho - r^*)\bar{\lambda} > 0, \tag{SS10}$$

where the inequality follows from $\rho > r^*$. Net of depreciation, therefore, the returns on physical and human capital come into line with the domestic real interest rate in the steady state:

$$\bar{r}^k - \delta = \bar{r} - (\bar{r} - r^*)\eta, \qquad \bar{r}^h - \delta = \bar{r}. \tag{SS4}$$

A binding debt constraint ($\eta > 0$) is associated with a lower marginal product of capital in the steady state.

From equations (1e) and (7), the after-tax returns to human and physical capital in the steady state are given by

$$\bar{r}^k = (1-\bar{\tau})\alpha\bar{B}\bar{K}^{\alpha-1}\bar{H}^{\omega}, \qquad \bar{r}^h = (1-\bar{\tau})(\omega-\phi)\bar{B}\bar{K}^{\alpha}\bar{H}^{\omega-1}. \tag{SS5}$$

From (M13) and (M14), the steady-state ratio of human to physical capital is given by

$$\frac{\bar{H}}{\bar{K}} = \frac{\omega - \phi}{\alpha}\frac{\bar{r}^k}{\bar{r}^h},$$

or, substituting for r_k and r_h from (SS5), by

$$\frac{\bar{H}}{\bar{K}} = \frac{\omega - \phi}{\alpha}\left[1 - \left(\frac{\bar{r} - r^*}{\bar{r} + \rho}\right)\eta\right] \tag{SS12}$$

The steady-state values of physical and human capital can then be derived from (SS5) and (SS12) as

$$\bar{K} = \left[\frac{(1-\bar{\tau})\alpha\bar{B}\bar{H}^{\omega}}{\bar{r}_k}\right]^{\frac{1}{1-\alpha}} = \left[\frac{(1-\bar{\tau})\alpha^{1-\omega}(\omega-\phi)^{\omega}\bar{B}}{\bar{r}_k^{1-\omega}\bar{r}_h^{\omega}}\right]^{\frac{1}{1-\alpha-\omega}} \tag{SS7K}$$

and

$$\bar{H} = \left[\frac{(1-\bar{\tau})(\omega-\phi)\bar{B}\bar{K}^{\alpha}}{\bar{r}_h}\right]^{\frac{1}{1-\omega}} = \left[\frac{(1-\bar{\tau})\alpha^{\alpha}(\omega-\phi)^{1-\alpha}\bar{B}}{\bar{r}_k^{\alpha}\bar{r}_h^{1-\alpha}}\right]^{\frac{1}{1-\alpha-\omega}} \tag{SS7H}$$

With the shadow price on the borrowing constraint strictly positive in the steady state, that constraint is binding and the level of debt is therefore

$$\bar{D} = \eta\bar{K}. \tag{SS11}$$

From (M8), steady-state consumption is then given by

$$\bar{C} = \bar{B}\bar{K}^{\alpha}\bar{H}^{\omega} - \delta\bar{K} - \delta\bar{H} - \bar{G} - r^*\eta\bar{K} + F. \tag{SS8}$$

4 Calibration

Table 1: Parameter values

Parameter	Value	Notes
α	0.33	Physical Capital's share
γ	2	Coefficient of relative risk aversion
δ	0.2	Depreciation rate
ρ	0.07	Discount rate
μ	0.50	Adjustment cost parameter
$\overline{\tau}$	0.20	Tax rate on output, reduced to 0.15 under policy reform
$\overline{G}$	$0.15 \cdot \overline{Y}$	Government spending
r^*	0.05	Global interest rate
ω	0.33	Human Capital's Share
ϕ	0.1	Human Capital Externality
B	1	Total Factor Productivity in Absence of Human Capital Externality
$\overline{D}$	$0.4 \cdot \overline{Y}$	Debt constraint $D \leq \overline{D}$
η	$0.4 \cdot \frac{\overline{Y}}{\overline{K}}$	Debt constraint $D \leq \eta K$
φ	0.4	Debt constraint $D \leq \varphi Y$

Note: Debt-limit parameters are set to generate a debt level of 40 percent of steady-state GDP.

5 Welfare

The baseline value of household welfare, starting in a stationary state with consumption equal to $\overline{c}_B$, is

$$\sum_{s=1}^{\infty} \beta^{s-1} \frac{\overline{c}^{1-\gamma}}{1-\gamma} = \frac{1+\rho}{\rho} \cdot \frac{\overline{c}_B^{1-\gamma}}{1-\gamma}.$$

To compare post-intervention consumption with the baseline, we define the annuity value of consumption as the constant consumption level $\overline{c}(\{c_s : s = 1, ..., \infty\}$ that generates the same welfare as the sequence $\{c_s : s = 1, ..., \infty\}$. This is given by the solution to

$$\sum_{s=1}^{\infty} \beta^{s-1} \frac{c_s^{1-\gamma}}{1-\gamma} = \sum_{s=1}^{\infty} \beta^{s-1} \frac{\overline{c}^{1-\gamma}}{1-\gamma},$$

or, equivalently, by

$$\overline{c} = \left(\frac{\rho}{1+\rho} (1-\gamma) \sum_{s=1}^{\infty} \beta^{s-1} \frac{c_s^{1-\gamma}}{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

The ratio of this annuity value to baseline consumption is therefore

$$\frac{\overline{c}}{\overline{c}_B} = \left\{ \sum_{s=1}^{\infty} \nu_s \left(\frac{c_s}{\overline{c}_B} \right)^{1-\gamma} \right\}^{\frac{1}{1-\gamma}},$$

where $\nu_s = \rho \cdot \beta^s$ and $\sum \nu_s = 1$. The weighted average inside the curly backets can be readily computed from the period-by period deviations of the post-intervention path from the baseline.

In the section that follows, we report the welfare impacts of interventions as the associated percentage changes in the annuuity value of consumption,

$$100 \cdot \left(\frac{\overline{c}}{\overline{c}_B} - 1 \right). \tag{1}$$

These calculations employ the private discount rate and can therefore be interpreted as changes in private welfare. In Table 2 below, we calculate welfare changes both at the private discount rate and at a lower discount rate that corresponds to the global real interest rate.

6 Results

In a set of Appendices, we show the dynamic paths of consumption and other variables in response to interventions that comprise various combinations of debt relief and policy reform, and alternative specifications of the debt limit. We examine the impact of interventions in models that both exclude and include human capital. By comparison with the baseline parameter values in Table 1, the policy reforms we consider are a reduction in the tax rate from 0.2 to 0.15 or a reduction in the discount rate from 0.07 to 0.06. Debt relief consists of a one-time cancellation that reduces outstanding debt from 40 percent to 15 percent of baseline GDP.

The table below summarizes the welfare impacts of the various interventions.

Table 2: Welfare Changes

Model	**Welfare Change %** $\boldsymbol{\rho = 0.07}$	**Welfare Change %** $\boldsymbol{\rho = 0.05}$	**Corresponding Figure**
$D \leq \bar{D}$: Debt Relief Only	2.74	2.15	Figure 1
$D \leq \bar{D}$: Policy Reform (τ) Only	0.56	0.83	Figure 1
$D \leq \bar{D}$: Debt Relief & Policy Reform (τ)	3.36	3.03	Figure 1
$D \leq \varphi Y$: Debt Relief Only	2.70	2.11	Figure 2
$D \leq \varphi Y$: Policy Reform (τ) Only	0.57	0.82	Figure 2
$D \leq \varphi Y$: Debt Relief & Policy Reform (τ)	3.34	2.98	Figure 2
$D \leq \eta K$: Debt Relief Only	2.57	1.98	Figure 3
$D \leq \eta K$: Policy Reform (τ) Only	0.63	0.83	Figure 3
$D \leq \eta K$: Debt Relief & Policy Reform (τ)	3.26	2.84	Figure 3
H ($\phi = 0$), $D \leq \eta K$: Debt Relief Only	3.85	3.18	Figure 7
H ($\phi = 0$), $D \leq \eta K$: Policy Reform (τ) Only	0.30	1.19	Figure 7
H ($\phi = 0$), $D \leq \eta K$: Debt Relief & Policy Reform (τ)	4.24	4.47	Figure 7
H ($\phi = 0.1$), $D \leq \eta K$: Debt Relief Only	5.78	4.77	Figure 8
H ($\phi = 0.1$), $D \leq \eta K$: Policy Reform (τ) Only	1.71	2.63	Figure 8
H ($\phi = 0.1$), $D \leq \eta K$: Debt Relief & Policy Reform (τ)	7.77	7.65	Figure 8
H ($\phi = 0$), $D \leq \eta K$: Debt Relief Only	3.85	3.18	Figure 9
H ($\phi = 0$), $D \leq \eta K$: Policy Reform (ρ) Only	0.25	0.73	Figure 9
H ($\phi = 0$), $D \leq \eta K$: Debt Relief & Policy Reform (ρ)	4.14	3.95	Figure 9
H ($\phi = 0.1$), $D \leq \eta K$: Debt Relief Only	5.78	4.77	Figure 10
H ($\phi = 0.1$), $D \leq \eta K$: Policy Reform (ρ) Only	1.04	1.53	Figure 10
H ($\phi = 0.1$), $D \leq \eta K$: Debt Relief & Policy Reform (ρ)	6.86	6.36	Figure 10

Note: Models are identified by the form of their debt limit (e.g., $D \leq \bar{D}$ refers to a model with a fixed debt limit), and by whether they include human capital (modesl that include human capital are labeled H). Policy reforms consist of a reduction in the tax rate (τ) or a reduction in the discount rate (ρ).

The smallest welfare increments come from policy-reform-only interventions, where the intervention is either a reduction in the tax rate on total revenue or a reduction in the private sector's discount rate. Tax receipts are rebated lump-sum to the household, so for a given path of investment neither reform alters the disposable resources available to the household. But these reforms permanently increase the incentive to invest in both human and physical capital. Consumption therefore initially falls below its baseline value, before gradually rising to a new and permanently higher level.

Debt relief on its own also generates an investment boom, but this is accompanied by a consumption path that is uniformly above the baseline. Given the magnitude of debt cancellation in our interventions, this produces a larger increment to welfare than the tax reform intervention. Debt relief also more than offsets the initial decline in consumption due to a tax cut, so that consumption exceeds the baseline level along the entire adjustment path when both interventions are implemented simultaneously.

Incorporating human capital expands the elasticity of output with respect to accumulable factors of production from 0.33 to 0.66. The long-run impact of tax reform on output and consumption is therefore larger when human capital is incorporated. But the incentive to invest is also stronger, leading to a larger fall in short-run consumption when the tax rate is reduced. The net impact of tax reform is therefore to generate a slightly smaller increment to the annuity value of consumption when human capital is included than when it is not. Debt relief, in contrast - or the two interventions implemented together - produces a larger welfare gain when human capital is included than when it is not.

The final block of human capital entries in Table 2 show the impact of incorporating a human capital externality. The externality we incorporate is substantial: the privately-appropriated elasticity of output with respect to human capital falls by nearly a third, from 0.33 to 0.23, with the result that human capital is severely under-accumulated in the steady state (the baseline ratio of human to physical capital falls from 0.975 to 0.680). A cut in the tax rate on revenue now produces a much larger gain in the annuity value of consumption, associated in turn with an increment to human capital that is nearly 20 percent larger than in the absence of the externality - a development that crowds in greater physical capital accumulation as well. The welfare impacts of debt relief, with or without the tax cut, are correspondingly much larger than in the absence of the externality.

At the bottom of Table 2 we consider policy reforms that reduce the private discount rate, leaving the tax rate unchanged. The change in the discount rate is meant to capture any policy reform that extends the planning horizons of consumers. The 'debt relief only' simulations in these rows are identical by construction to the corresponding earlier rows in models with human capital. The simulations that incorporate the reduction in the discount rate show effects that are similar to those of a tax cut: they stimulate investment and growth, increasing capital stocks in both the short run and the long run.

7 Appendix

7.1 Appendix 1: Fixed Debt

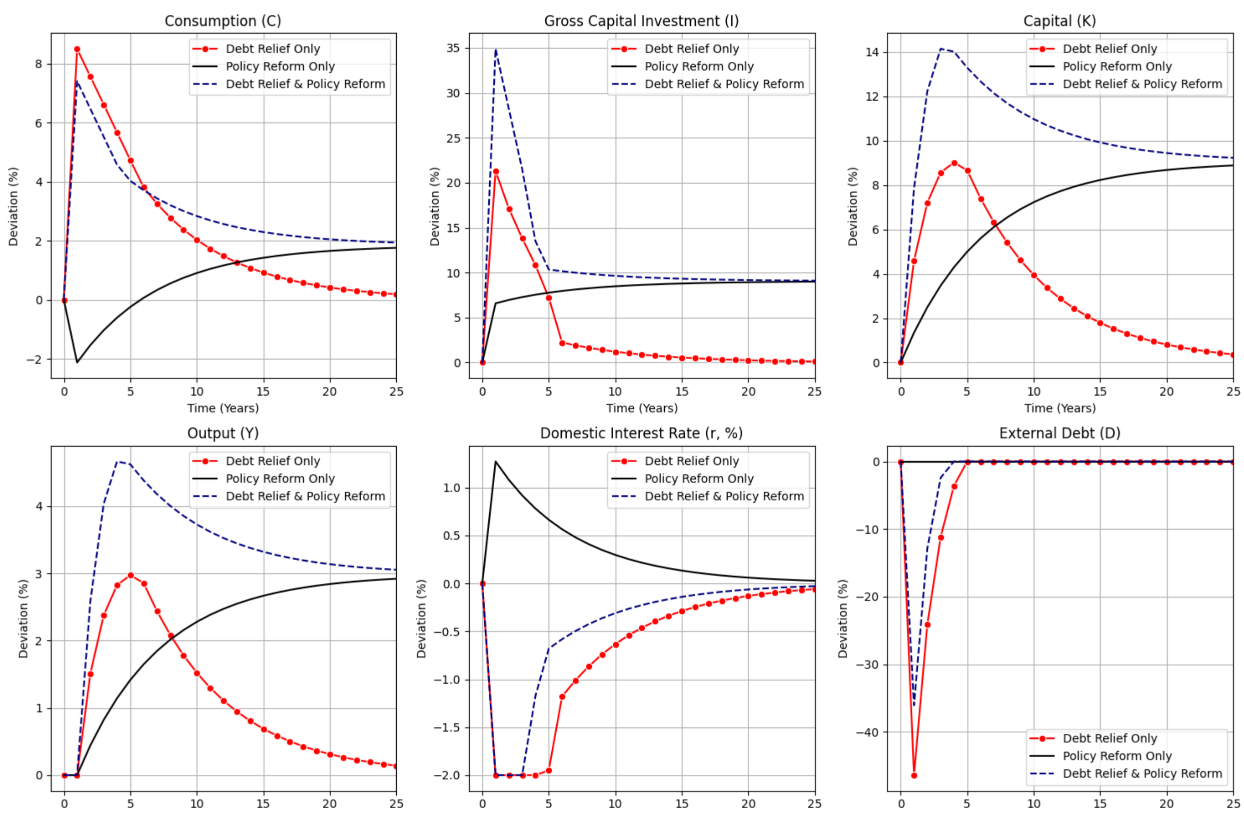

Figure 1: Policy reform with and without debt relief, with partly fixed debt limit

Non-stochastic simulations using Dynare (dr_fixed_jul5_2020.mod). The policy reform is a cut in the tax rate on GDP from 20% to 15%. Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. The figure above is produced by plots2.ipynb.

7.2 Appendix 3: Alternative Debt Constraints

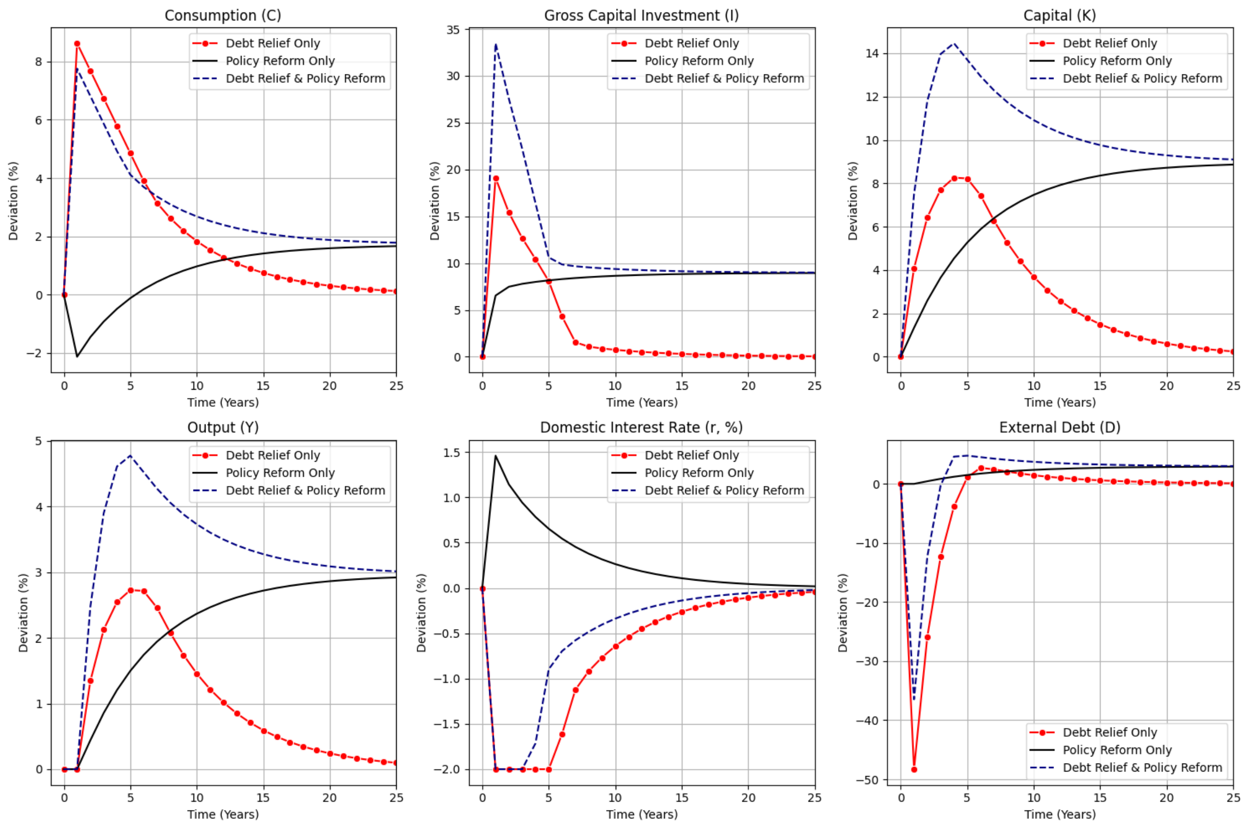

Figure 2: Policy reform with and without debt relief, with debt limit as proportion of GDP

Non-stochastic simulations using Dynare (dr_ylim_jul8_2020.mod). The policy reform is a cut in the tax rate on GDP from 20% to 15%. Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. The figure above is produced by plots2.ipynb.

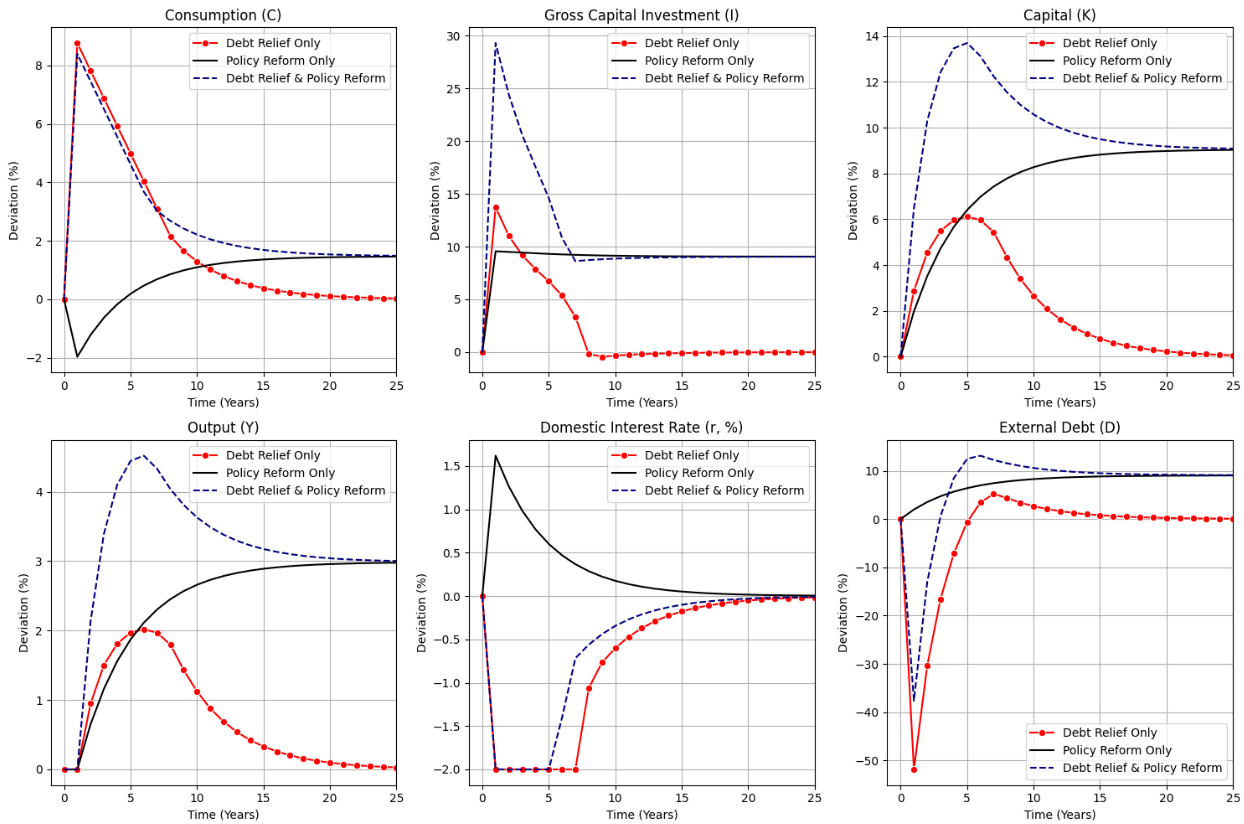

Figure 3: Policy reform with and without debt relief, with debt limit as proportion of physical capital

Non-stochastic simulations using Dynare (dr_klim_jul11_2020.mod). The policy reform is a cut in the tax rate on GDP from 20% to 15%. Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. The figure above is produced by plots2.ipynb.

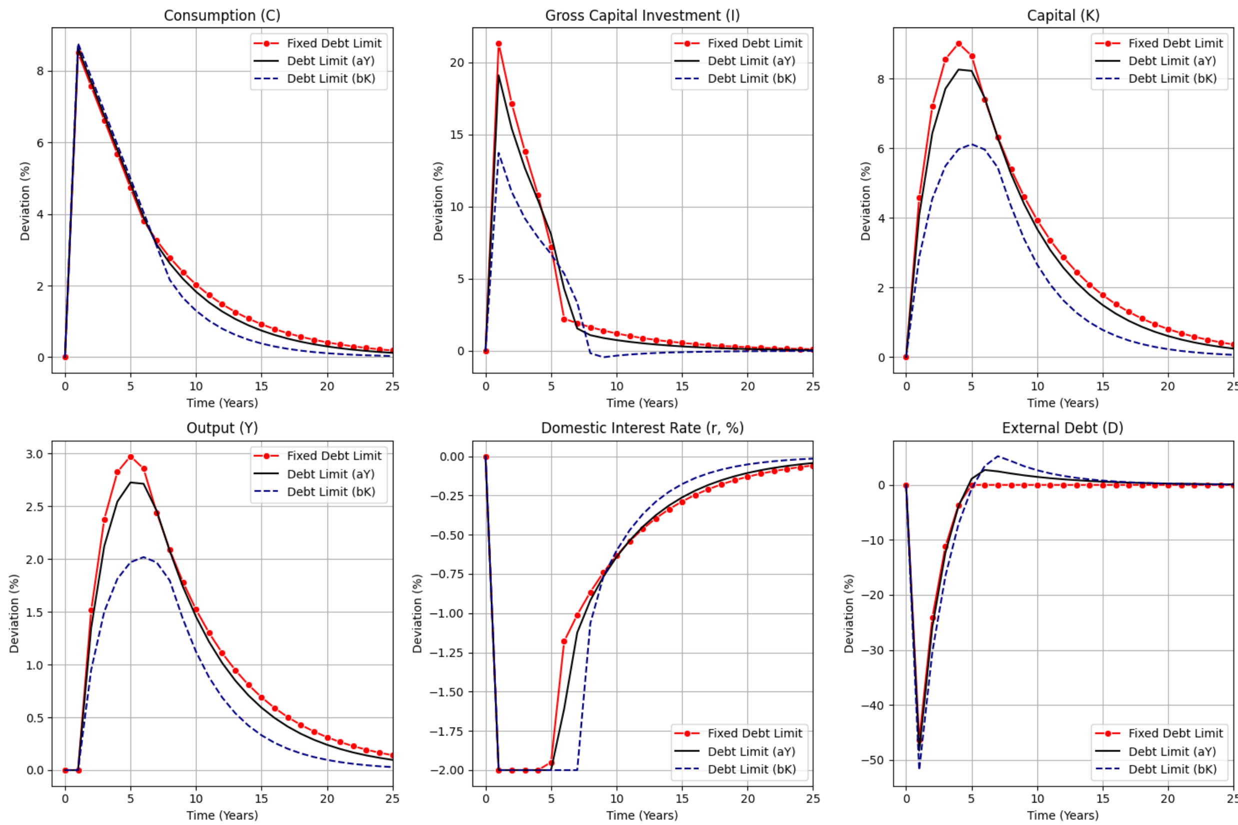

Figure 4: Debt relief under various debt constraints

Non-stochastic simulations using Dynare (dr_fixed_jul5_2020.mod, dr_ylim_jul8_2020.mod, dr_klim_jul11_2020.mod). Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. The figure above is produced by plots3.ipynb.

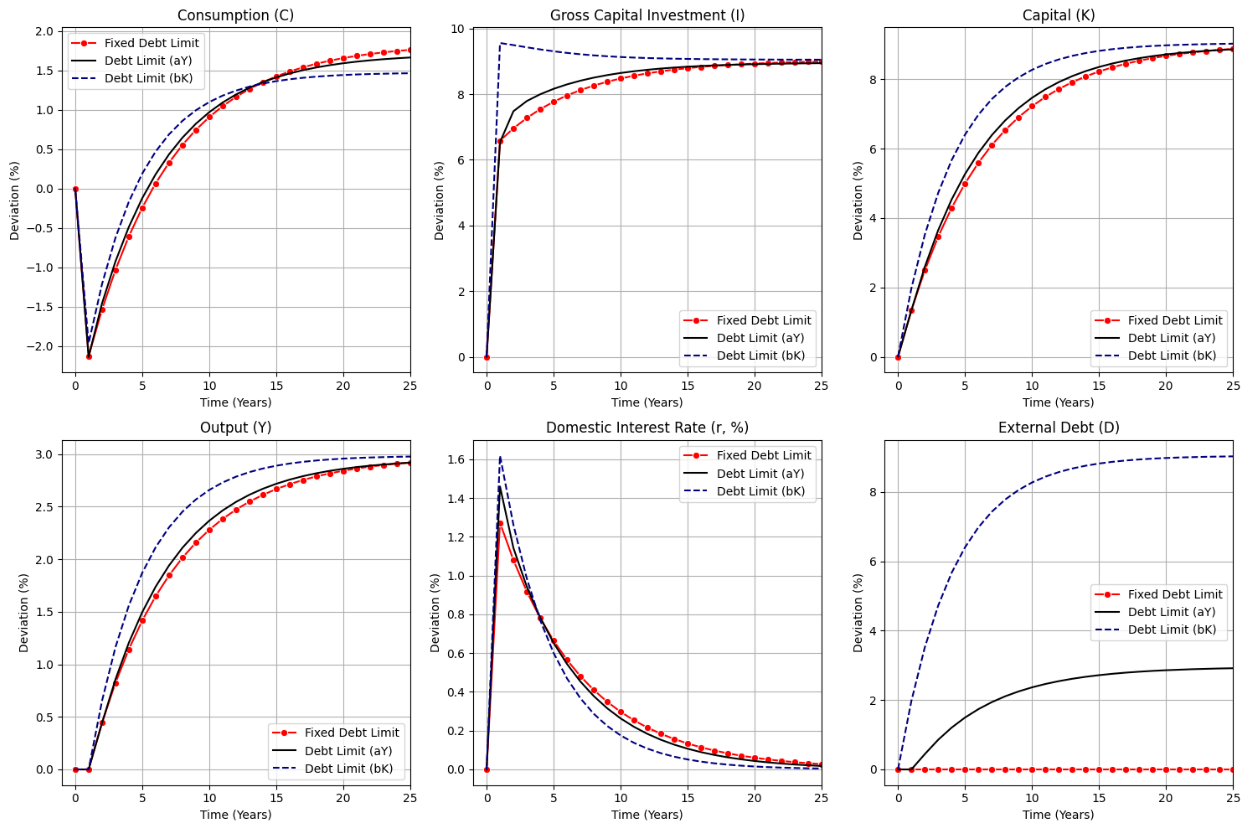

Figure 5: Policy Reform under various debt constraints

Non-stochastic simulations using Dynare (dr_fixed_jul5_2020.mod, dr_ylim_jul8_2020.mod, dr_klim_jul11_2020.mod). The policy reform is a cut in the tax rate on GDP from 20% to 15%. The figure above is produced by plots3.ipynb.

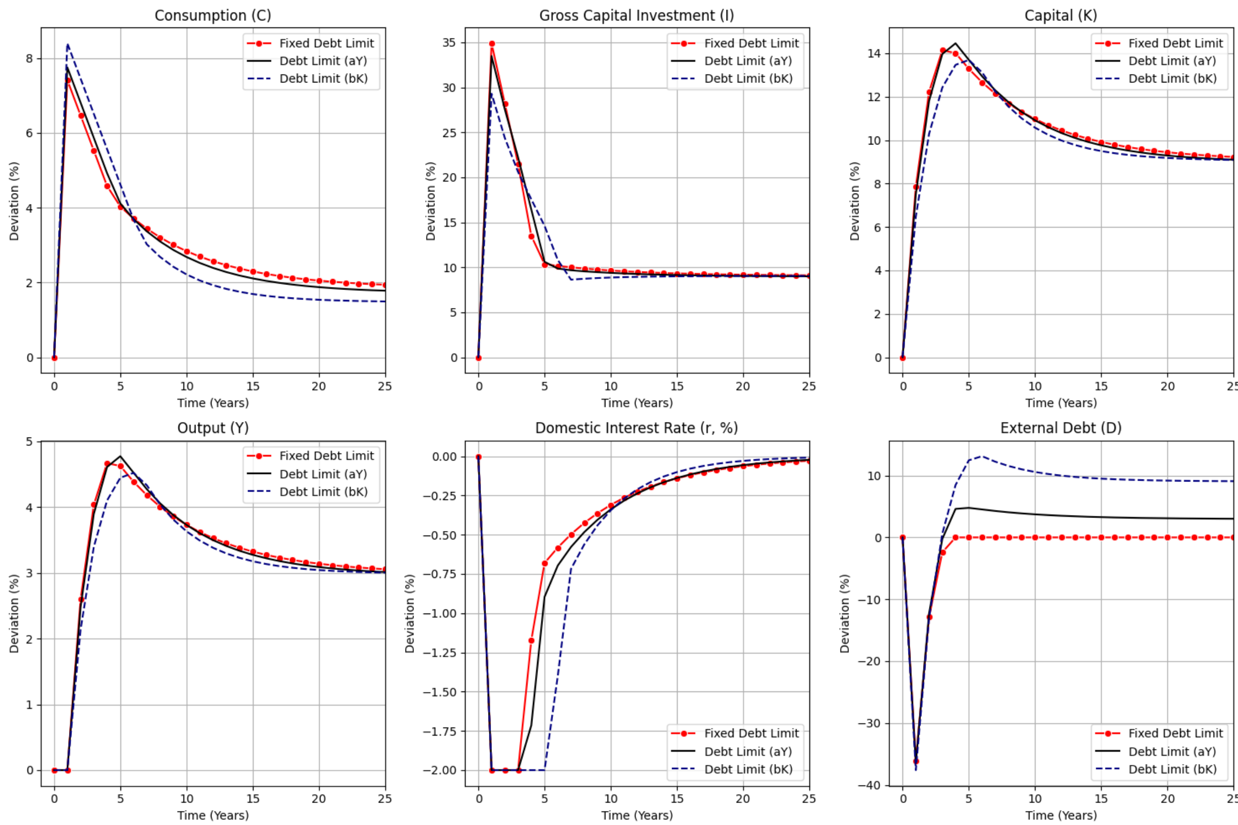

Figure 6: Policy Reform and Debt Relief under various debt constraints

Non-stochastic simulations using Dynare (dr_fixed_jul5_2020.mod, dr_ylim_jul8_2020.mod, dr_klim_jul11_2020.mod). The policy reform is a cut in the tax rate on GDP from 20% to 15%. Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. The figure above is produced by plots3.ipynb.

7.3 Appendix 4: Incorporating non-collateralizable human capital

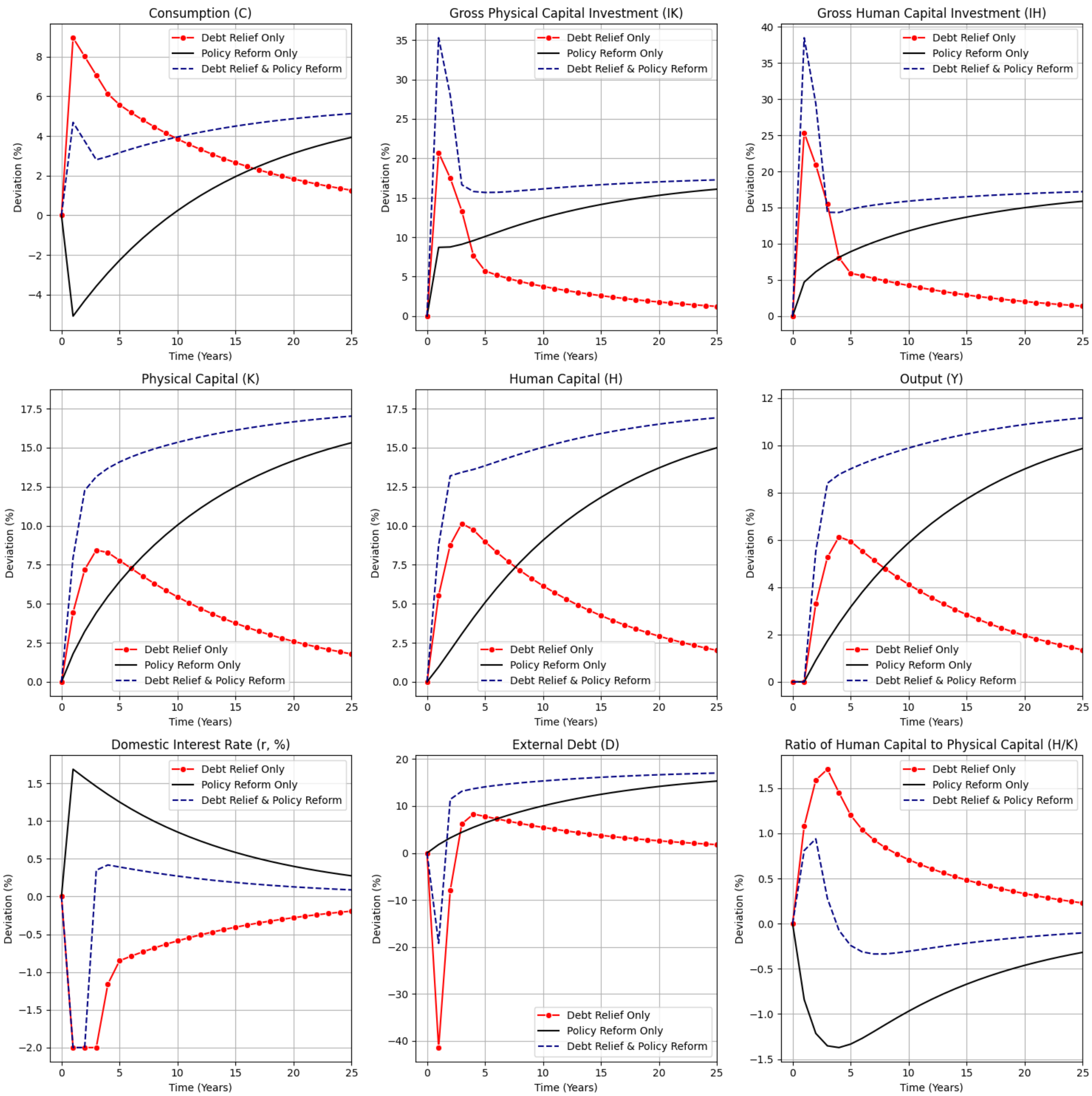

Figure 7: Policy reform with and without debt relief, with partly collateralizable capital

Non-stochastic simulations using Dynare (debt_relief_may_2024.mod). The policy reform is a cut in the tax rate on GDP from 20% to 15%. Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. There is no human capital externality. The figure above is produced by plots.ipynb.

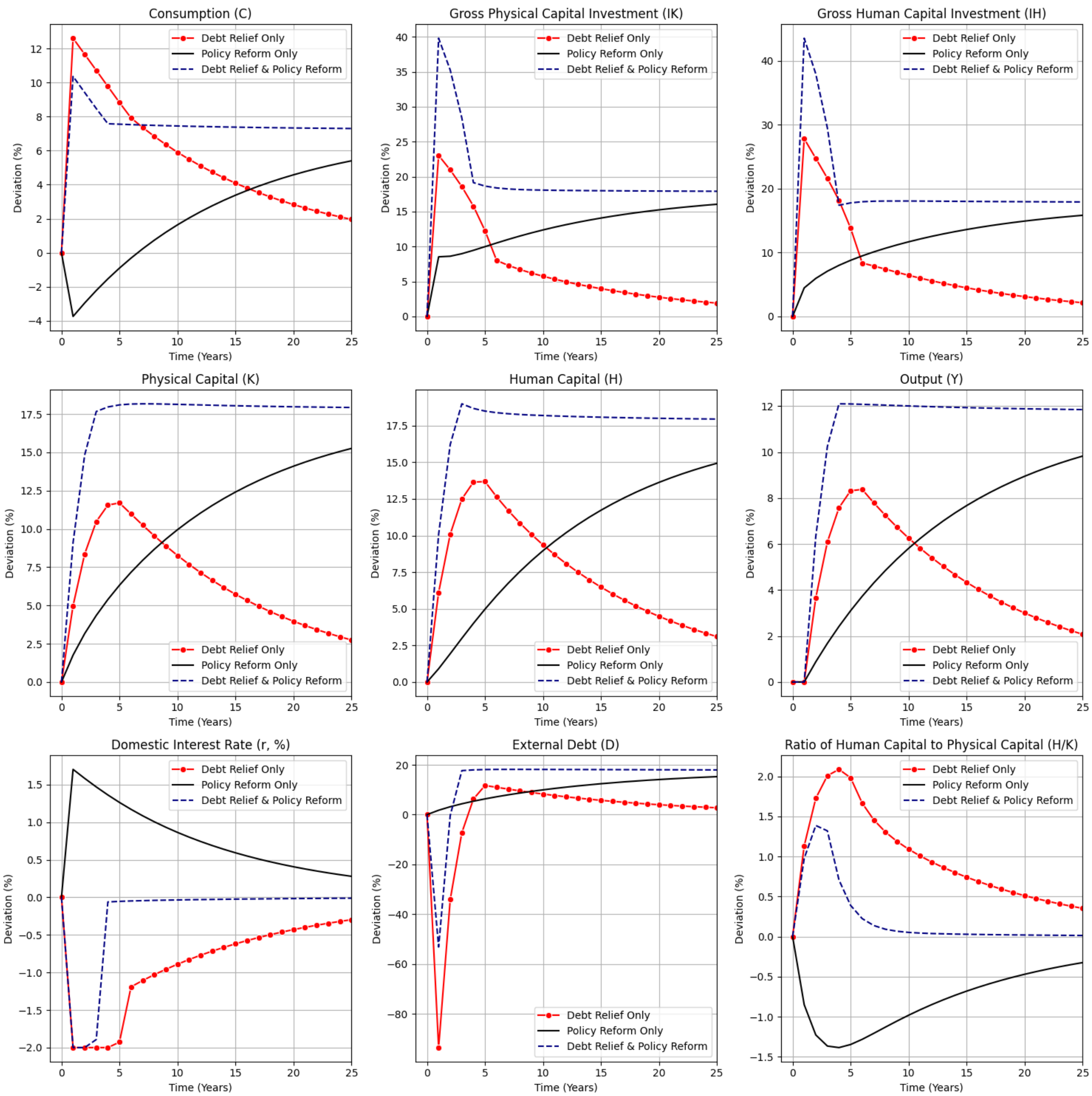

Figure 8: Policy reform with and without debt relief, with partly collateralizable capital

Non-stochastic simulations using Dynare (debt_relief_may_2024.mod). The policy reform is a cut in the tax rate on GDP from 20% to 15%. Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. The human capital externality parameter is set to 0.1. The figure above is produced by plots.ipynb.

7.4 Appendix 5: A reduction in the discount rate

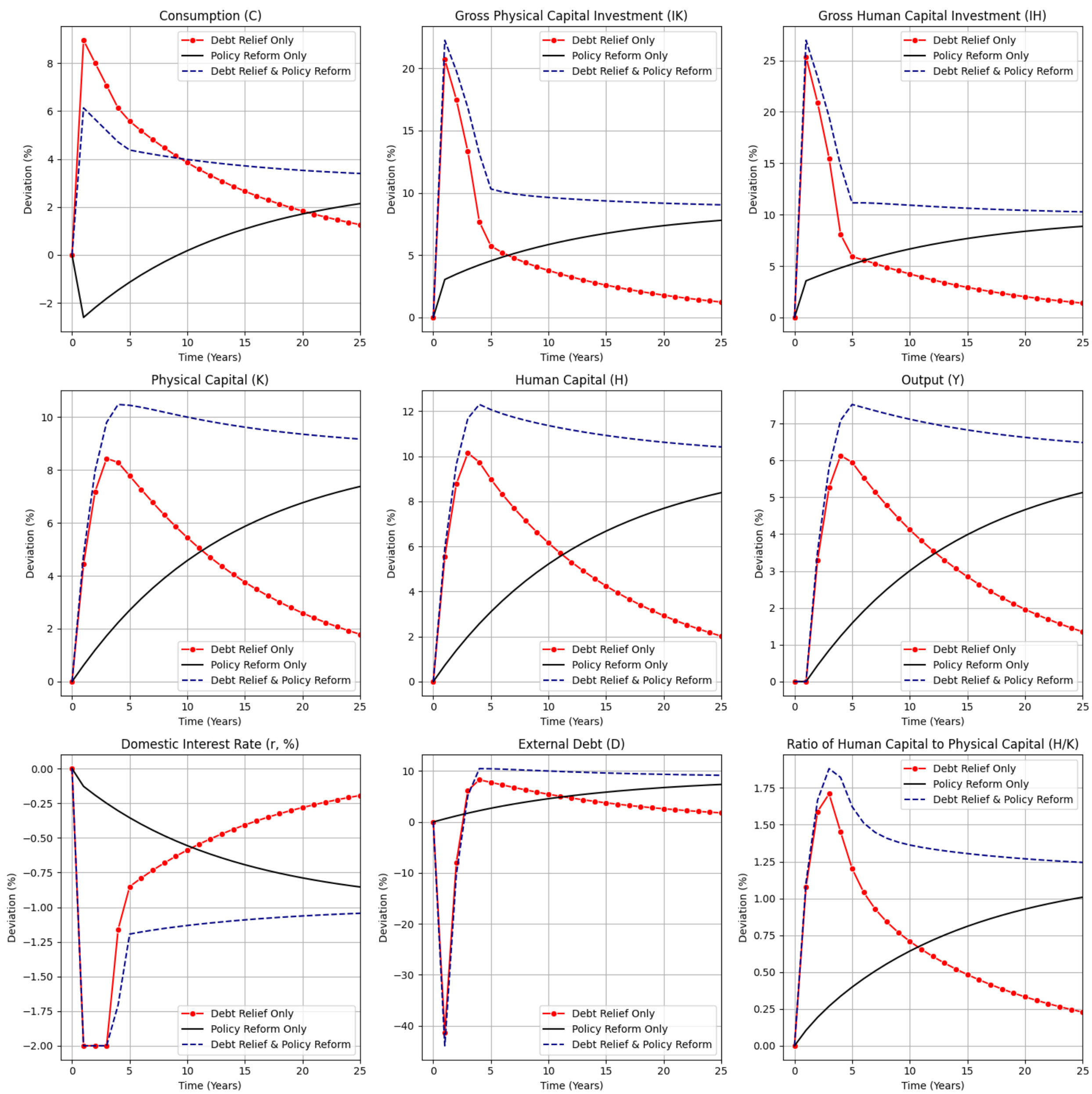

Figure 9: Policy reform with and without debt relief, with partly collateralizable capital

Non-stochastic simulations using Dynare (debt_relief_may_2024.mod). The policy reform is a cut in the discount rate from 7% to 6%. Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. There is no human capital externality. The figure above is produced by plots.ipynb.

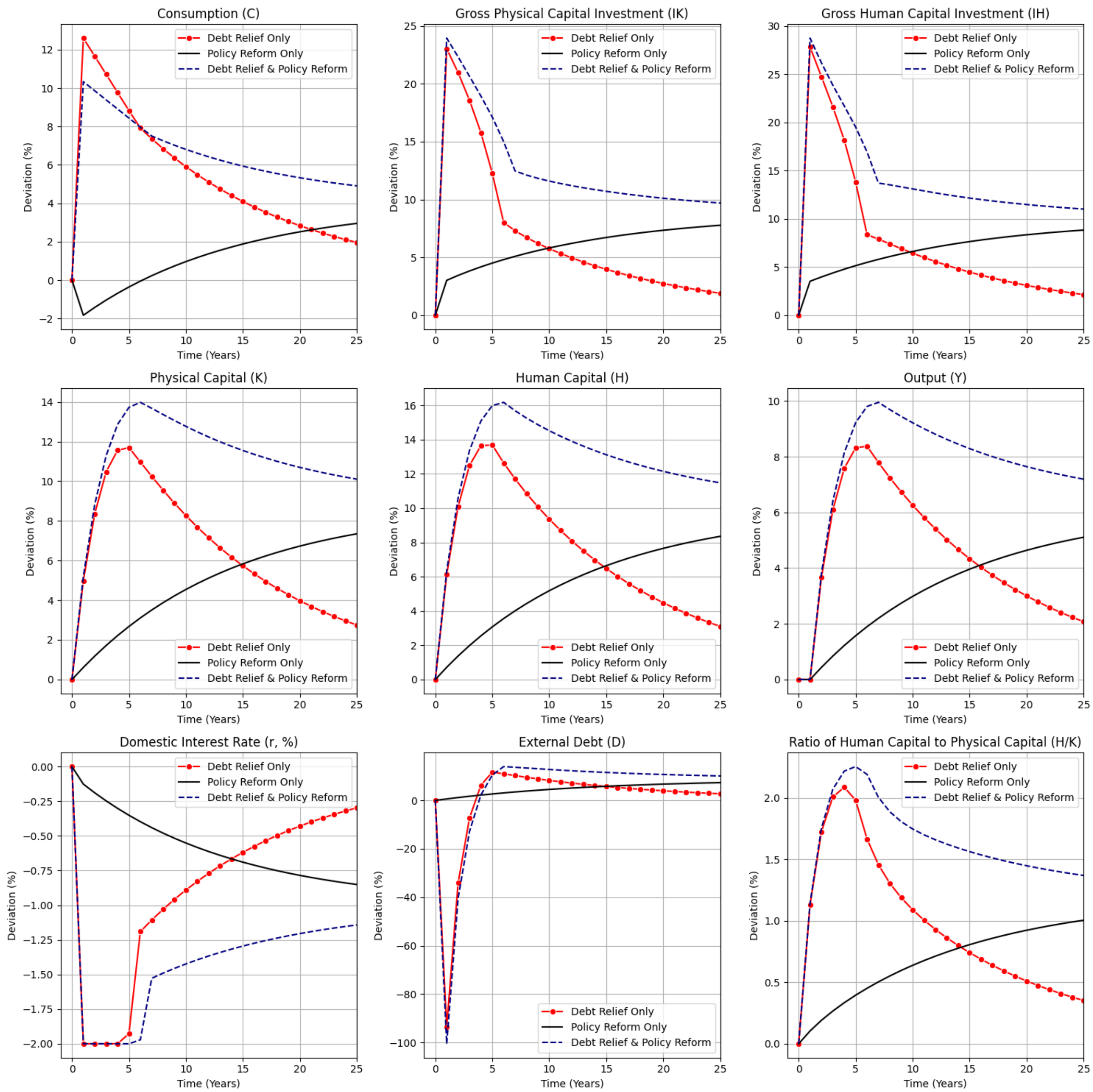

Figure 10: Policy reform with and without debt relief, with partly collateralizable capital

Non-stochastic simulations using Dynare (debt_relief_may_2024.mod). The policy reform is a cut in the discount rate from 7% to 6%. Debt relief takes the form of a one-time cancellation that reduces outstanding debt from 40% to 15% of baseline GDP. THe human capital externality parameter is set to 0.1. The figure above is produced by plots.ipynb.