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UNIVERSITY OF MALAWI

SCHOOL OF NATURAL & APPLIED SCIENCES

Mathematical Sciences Department

TEST 1: Introduction to MATLAB & Programming with MATLAB

(*For 2nd year Science students taking MAT 213*)

4:55 – 6:35

Wednesday, 22nd November 2023 **Time: 2 hours (from 16:30hrs)**

Instructions

(1) ***This is a closed book test*** where you are expected to do the test alone without any assistance from some other person(s) or some other form of notes or communication.

(2) ***Non-programmable calculators may be used***. However, mobile phones are not allowed. If accidentally brought in, they should be switched off and packed away.

(3) ***Show your method or explanation clearly***. Most marks shown in square brackets at the end of each part are allocated to the method.

(4) Attempt ***ALL questions***.

Question 1:[43 marks]

(a) Name ***four main*** features or windows of MATLAB and provide a brief description of each? **[12]**

(b) For each of the following commands or symbols as used in MATLAB, describe their uses.

Command/symbol	**Description**
help	
;	
who, whos	
%	
clear **Y**	
:	
What	
&	
clc	
~	
dir or Is	
~ =	
pwd	
…	
NaN	

[15]

(c) Define any 4×4 matrix A and extract a submatrix B consisting of rows 2 and 3 and columns 1 and 2 of the matrix A. **[3]**

(d) From a matrix A=[1 2 3;4 5 6;7 8 9], define a concatenated matrix B=[A 10*A;-A eye(3)]. **[3]**

(e) Suppose that a matrix $H_{n\times n}$ is said to be a Hadamard matrix if and only if $H = (a_{ij})_{n\times n}$ such that $HH^T = nI_n$ where $a_{ij} = -1$ or 1. Given that

$$H = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

i. Demonstrate that H is a Hadamard matrix. **[3]**

ii. Compute the inverse of the matrix H. ~~**[2]**~~ 3

iii. Find the eigenvalues of the matrix H. ~~**[3]**~~ 6

iv. Hence, show that $H^{-1}H = HH^{-1} = I$ **[2]**

Question 2:[30 marks]

(a) Given the following ***while…loop*** MATLAB function file, provide comment on each line, and hence describe the output. **[3.5]**

```
function trial_3
x = 1;
while x <= 10
    x = 3*x;
    x
end
```

(b) Consider the following quadratic equation $y = 2x^2 + 3x - 4$.

- **i.** Express it in the form of $ax^2 + bx + c = 0$ and deduce the values of a, b and c. **[2]** 1.5
- **ii.** Write a MATLAB function file using an ***if…else*** statement to classify the type of quadratic roots of the given equation using the discriminant term $b^2 - 4ac$, where roots are imaginary, repeated, or non-repeated when $b^2 - 4ac < 0$, $b^2 - 4ac = 0$, or $b^2 - 4ac > 0$, respectively. **[5.5]**
- **iii.** Hence, describe the output of your program for the above given equation. **[1.0]**

(c) Consider the following system of linear equations

$$\begin{cases} x + 2y + 3z = 1 \\ 3x + 3y + 4z = 1 \\ 2x + 3y + 3z = 1 \end{cases}$$

- **i.** Derive an expression of the form $A\boldsymbol{x} = \boldsymbol{b}$. **[3]** 1.5
- **ii.** Hence, write a Matlab function file to solve for $\boldsymbol{x}$ using the LU factorization method. **[5]**

(d) The square root of 2, i.e., $\sqrt{2}$ can be estimated using the recurrence equation

$$y_t = \frac{1}{2}(y_{t-1} + \frac{2}{y_{t-1}})$$

for $t = 1,2,3,\cdots n$, when $y_0 = 3$.

$[L\ U] = lu(A)$

$Ux = z$

$Lz = B$

$z = L^{-1}B$

$x = U^{-1} z$

- **i.** Write down a pseudocode of a ***for loop*** function to estimate $\sqrt{2}$ when t=[1,⋯,7]. **[5]**
- **ii.** Hence, use your above written pseudocode to write a MATLAB function/program file that estimates $\sqrt{2}$. **[5]**

~~~~~ooooOOOO **End of Test 1 Questions** OOOOoooo~~~~~
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MA 213 — SOLUTION GUIDE

Q.1

QUESTION No.................... Examination No....................

(a) We have

1) Command window – used to enter variables and to run functions and M-files. Again used to display immediate computational results. 1 2

2) Command History – used for storing of all statements or commands that we enter in the command window. 1 2

3) Work space – lists all the variables in use or used before so long as we have our MATLAB running 1 2

4) Current Directory – Also called the working folder which lists down all M-files during the programming process. 1 2

Q.1

(b) help - provides us with general MATLAB features and different items to explore 1

; - used as a separator of rows in a defined matrix or as supressor of output in a Matlab program file. 1

who, whos - provides list of all variables and their value types 1

% - used for commenting in a Matlab program file 1

Clear Y - clears a specified variable Y or item Y in the workspace 1

: - used as an incrementor of consecutive numbers or array in a sequence 1

what - lists all M-files in the working folder 1

& - logical operator for combining two expressions or statements 1

clc - Clearing the Command window 1

~ - negation symbol 1

dir - lists all files in the current working directory 1

~= - Not equal to 1

Pwd - Shows current working directory 1

QUESTION No. 4 Examination No.

(b) Contd

... — shows a continuation of statement (1)

NaN — represents an empty value (1)

(c) Let the 4x4 matrix A be

>> A=[a,b,c,d;e,f,g,h;i,j,k,l;m,n,o,p] (1)

$$>> A = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

(1)

>> B = (i=2:3, j=1:2) or B=(2:3,1:2)

$$>> B = \begin{pmatrix} e & f \\ i & j \end{pmatrix}$$ (1)

(d) Given A=[1,2,3;4,5,6;7,8,9]

$$>> A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

$$>> 10*A = \begin{pmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{pmatrix}$$ (1)

$$>> -A = \begin{pmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{pmatrix} \quad 1$$

$$>> eye(3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad 1$$

Hence

B= [A 10*A ;-A eye(3)]

$$= \begin{pmatrix} 1 & 2 & 3 & 10 & 20 & 30 \\ 4 & 5 & 6 & 40 & 50 & 60 \\ 7 & 8 & 9 & 70 & 80 & 90 \\ -1 & -2 & -3 & 1 & 0 & 0 \\ -4 & -5 & -6 & 0 & 1 & 0 \\ -7 & -8 & -9 & 0 & 0 & 1 \end{pmatrix} \quad 1$$

QUESTION No. 1 Examination No.

(e) Since matrix $H_{n\times n}$ is Hadmard iff

(i) $HH^T = nI_n$

Given $H = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$, then $H^T = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ $\frac{1}{2}$

So that

$$HH^T = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} * \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad 1$$

$$= 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \frac{1}{2}$$

$\frac{1}{2}$

$\Rightarrow HH^T = 2I_2$ $\frac{1}{2}$, H is indeed Hadamard matrix since $H_{2\times 2}$.

(ii) $H^{-1} = \frac{1}{\det(H)} \times \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ 1

but $\det(H) = (1)(1) - (-1)(1) = 2$ 1

$\therefore H^{-1} = \frac{1}{2} \times \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ $\frac{1}{2}$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \frac{1}{2}$$

Q1

(e) (iii) the eigenvalues of H are formed from the solutions of the equation $\frac{1}{2}$

$$\det(\lambda I - H) = 0$$

Thus $\lambda I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$ $\frac{1}{2}$

Hence $\lambda I - H \Rightarrow \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} \lambda - 1 & -1 \\ 1 & \lambda - 1 \end{pmatrix} \quad 1$$

So that

$$\det(\lambda I - H) = \begin{vmatrix} \lambda - 1 & -1 \\ 1 & \lambda - 1 \end{vmatrix} = 0$$

$\frac{1}{2}$

$\Rightarrow$

$$(\lambda - 1)(\lambda - 1) - (1)(-1) = 0$$

$$\left.\begin{aligned} \lambda^2 - \lambda - \lambda + 1 + 1 &= 0 \\ \lambda^2 - 2\lambda + 2 &= 0 \end{aligned}\right\} \frac{1}{2}$$

Thus $\lambda = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$ 1

$$\frac{1}{2} = \frac{2 \pm \sqrt{4 - 8}}{2} = \cancel{\frac{2 \pm 1}{2}}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 + \sqrt{-4}}{2} \quad \frac{1}{2} \quad \text{or} \quad \frac{2 - \sqrt{-4}}{2}$$

1

$\therefore \lambda_1 = 1 + 2i$ and $\lambda_2 = 1 - 2i$

QUESTION No. 1 Examination No.

(e) (iv) Since $H^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ and $H = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$$H^{-1}H = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad \frac{1}{2}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \frac{1}{2}$$

and $HH^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\frac{1}{2}$

$\frac{1}{2}$

$\therefore H^{-1}H = HH^{-1}$

QUESTION No. Q-2 Examination No.

(a) For the program

```
1. function trial_3 % defines function name
2. x=1; % initialises the variable x=1
3. while x<=10 % sets a condition for the while
4.       x=3*x; % computes new sequence values of x, as long as the while condition remains TRUE
5. end % closes the while loop.
```

½ ½ ½ ½ ½

The output is thus 1

1, 3, 9, 27

(b) From the given equation $y = 2x^2 + 3x - 4$

(i) $2x^2 + 3x - 4 = 0$

Hence $a=2$, $b=3$, $c=-4$ } (½, ½, ½) for each correct value

(ii) For the Matlab program file, we have

```
function quadratic_roots(a,b,c)        1
discr = b*b - 4*a*c;                   ½
if discr<0                             ½
    disp('roots are imaginary')        ½
elseif discr==0                        ½
    disp('roots are repeated')         ½
else                                   ½
    disp('roots are real')             ½
end                                    ½
end                                    ½
```

Q-2

(b) (iii) from the values $a=2$, $b=3$, and $c=-4$ (1)

$$b^2-4ac \equiv (3)^2-4(2)(-4) = 9+32 = 41 > 0 \quad \left(\tfrac{1}{2}\right)$$

Hence, roots are real. ($\tfrac{1}{2}$)

(c) from the system

$$x+2y+3z=1$$
$$3x+3y+4z=1$$
$$2x+3y+3z=1$$

(i)
$$A=\begin{pmatrix}1&2&3\\3&3&4\\2&3&3\end{pmatrix}\left(\tfrac{1}{2}\right),\quad x=\begin{pmatrix}x\\y\\z\end{pmatrix}\left(\tfrac{1}{2}\right),\quad b=\begin{pmatrix}1\\1\\1\end{pmatrix}\left(\tfrac{1}{2}\right)$$

(ii) For Matlab function

```
function factorization_method        (1)
    A=[1 2 3; 3 3 4; 2 3 3];          (1/2)
    b=[1 1 1]^T or [1 1 1]';          (1/2)
    [L U] = lu(A);                    (1)
    z = inv(L)*b;                     (1)
    x = inv(U)*z;                     (1)
    x                                 (1/2)
end
```

QUESTION No. Q.2 Examination No.

(d) If the function $y_t = \frac{1}{2}\left(y_{t-1} + \frac{2}{y_{t-1}}\right)$ approximate $\sqrt{2}$ for $t = 1, 2, 3, \ldots, n$; $y_0 = 3$

(i) Pseudo Code

1 Input : $n > 0$, Y
1 Output : $(n+1)$-terms of y_t for $t = 0:n$

$y = 1 \times n+1$ row vector $y(0), y(1), \ldots, y(n)$
$\frac{1}{2}$ $y_0 = Y = 3$;
1 for $t = 1$ to n, do
$Y = 0.5 * (Y + 2/Y)$ 1
$\frac{1}{2}$ end

(ii) MATLAB Program

```
function [y] = root_two (Y)      1
    n=7;                         1/2
    for t =1 :n ;                1/2
        y = 0.5 * (Y + 2/(Y));   1
        Y = y;                   1/2
    end                          1/2
    disp (y)                     1/2
end                              1/2
```