UNIVERSITY OF MALAWI

SCHOOL OF NATURAL & APPLIED SCIENCES

Mathematical Sciences Department

TEST 2: MATLAB Functions and Plots

(*For 2nd year Science students taking MAT 213*)

Tuesday, 19th December 2023 **Time: 2 hours (from 17:30hrs)**

Instructions

(1) ***This is a closed book test*** where you are expected to do the test alone without any assistance from some other person(s) or some other form of notes or communication.

(2) ***Non-programmable calculators may be used***. However, mobile phones are not allowed. If accidentally brought in, they should be switched off and packed away.

(3) ***Show your method or explanation clearly***. Most marks shown in square brackets at the end of each part are allocated to the method.

(4) Attempt ***ALL questions***.

Question 1:[42.5 marks]

(a) In order to compute the (N+1)-term Taylor series approximation to e^x where $e^x \approx \sum_{k=0}^{N} \frac{x^N}{k!}$,

(i) Write a pseudo-code for implementing this in Matlab. **[6]**

Pseudo code

Input: $x, N > 0$

Output: $(N+1)$-term Taylor series approximation to e^x

```
taylor=1; factorial=1; xpowk=1;
for k=1 to N, do
        factorial = factorial * 1;
        xpowk=xpowk*x;
        taylor=taylor + (xpowk/factorial);
end
```

(ii) Hence, write an error free Matlab script file based on the pseudo-code written in Q1(a)(i) to display results. **[6]**

```
function [taylor]=Taylorestimate(x,N)

taylor=1;
factorial=1;
xpowk=1;

for k=1:N
    factorial=factorial*k;
    xpowk=xpowk*x;
    taylor=taylor+(xpowk/factorial);
end
disp('The (N+1)-term Taylor series approx to
e^x is:')
end
```

(b) Matlab uses various line markers, styles and colours to distinguish the data sets plotted in the given graph as per table options.

Symbol	**Color**	Symbol	**Line style**	Symbol	**Marker**
k	Black	–	Solid	+	Plus sign
r	Red	– –	Dashed	o	Circle
b	Blue	:	Dotted	*	Asterisk
g	Green	–.	Dash-dot	.	Point
c	Cyan	none	No line	×	Cross
m	Magenta			s	Square
y	Yellow			d	Diamond

Provide a description of each line style, color and symbol in the table above. **[9.5]**

(c) Write an error free Matlab plot function which compares on the same graph two function graphs $f = \text{sine}(-2\theta)$ and $g = cosine(-2\theta)$ for $\theta \in [-2\pi: 2\pi]$ with the help of any two different colors. **[6]**

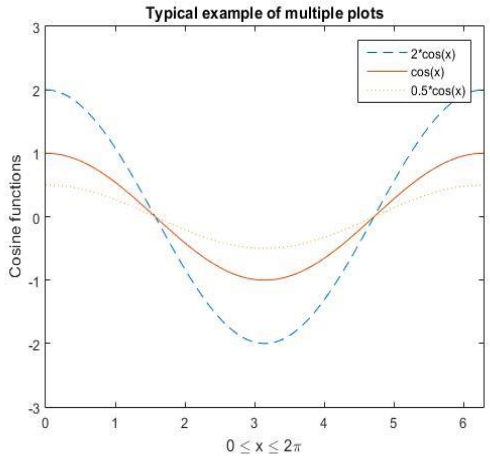
```
function Comparison

theta= -2*pi:2*pi;
f_theta=sin(-2*theta);
g_theta=cos(-2*theta);
plot(theta,f_theta,'r-');
hold on
plot(theta,g_theta,'k-');
hold off

end
```

(d) Consider the following Matlab code (left) and the output (right).

```
>> x = 0:pi/100:2*pi;
>> y1 = 2*cos(x);
>> y2 = cos(x);
>> y3 = 0.5*cos(x);
>> plot(x,y1,'--',x,y2,'-',x,y3,':')
>> xlabel('0 \leq x \leq 2\pi')
>> ylabel('Cosine functions')
>> legend('2*cos(x)','cos(x)','0.5*cos(x)')
>> title('Typical example of multiple plots')
>> axis([0 2*pi -3 3])
```



Provide comment on each line of the code, and hence describe the output. **[15]**

- Define array of variable x, from 0 to 2π
- Defining a function y1 of 2*cosine(x)
- Defining a second function y2 of cosine(x)
- Defining a third function y3 of 0.5*cos(x)
- Use the 2D plot function to plot y1, y2, and y3 on the same graph
- Define x-axis label as "0 to 2π"
- Define the y-axis label calling it "cosine functions"
- Specifying names of the plotted function y1, y2, and y3
- Providing title of the plot graph
- Specifying the interval values of x (0 to 2π) and y (-3 3) axes.

Question 2:[18.5 marks]

(a) Let *M(t)* be the amount of a product that decreases with time *t* and the rate of decrease is proportional to the amount *M* as follows *dM/dt=-kM* where *dM/dt* is the first derivative of *M*, $k > 0$ and *t* is the time.

(i) Using method of separation of variables (SOV), solve for *M* in the given differential equation. **[3.5]**

Since *dM/dt=-kM* can be separated as

$$dM/M = -k\,dt$$

Integrating both sides of equation above,

$$\int \frac{dM}{M} = \int -k dt$$

results in

$$\ln M = -kt + C$$

Now, if we let

$$M = e^{-kt+C}$$

we obtain

$$M(t) = A\,e^{-kt}$$

where A is non zero constant. If we further assume that $M = M_0$ at $t = 0$, then

$$M_0 = A\,e^{0}$$

which gives $A = M_0$. Thus, the solution is

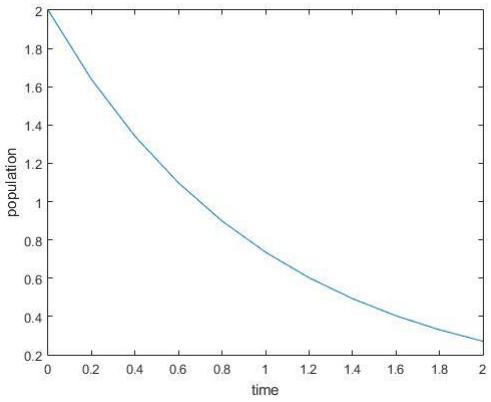
$$M(t) = M_0 e^{-kt}$$

(ii) Write a Matlab function file called "***exponential_decay***" to demonstrate implementation and graphical display of the solution found in Q2(a)(ii) above. **[8.5]**

```
function exponential_decay

k=1;
t=0;
N=10;
M(1)=2;
for i=1:N
    t(i+1)=t(i)+0.2;
    M =M(1)*exp(-k*t);
end
plot(t,M)
 xlabel('time');
 ylabel('population')

end
```



(b) Write the Matlab commands for plotting the function

$$f(x,y) = -\left(\frac{x}{5}\right)^2 - \left(\frac{y}{2}\right)^2 - 16$$

for $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$ in 3D using the mesh plot function and label the axis and give title. **[6.5]**

```
>> [X,Y] = meshgrid(-5:0.25:5,-5:0.25:5);
>> f=-(X/5).^2 - (Y/2).^2 - 16;
>> mesh(X,Y,f);
>> xlabel('x-axis')
>> ylabel('y-axis')
>> zlabel('z-axis')
>> title('a 3D plot function')
>> hidden off
```

```
>> x=-5:0.25:5;
>> y=-5:0.25:5;
>> [X,Y] = meshgrid(x,y);
>> f=-(X/5).^2 - (Y/2).^2 - 16;
>> mesh(X,Y,f);
>> xlabel('x-axis')
>> ylabel('y-axis')
>> zlabel('z-axis')
>> title('a 3D plot function')
>> hidden off
```

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