# Multiplayer Elo-Ranking System for BB&B

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**Abstract**

This note documents the implementation of the Elo-ranking system for the BB&B board game club. It is a straight forward implementation of the well-known Elo-ranking system, developed to rank chess players. We apply the standard trick of permuting all participants in multi-player settings.

The new rank $R'$ for player $A$ is calculated as

$$R'_A = R_A + K(S_A - E_A) \tag{1}$$

where $R_A$ is player $A$'s previous rank, $K$ is a weighting factor, $S_A$ is the score of the player, and $E_A$ is the predicted probability of player $A$ winning the game. The score, $S_A$ depends on the number of participants in the game. Calculate the combinations as

$$\rho = \frac{n(n-1)}{2}, \tag{2}$$

which is the total number of one-on-one match ups in the game. $S_A$ is then given by

$$S_A = \frac{1}{\rho}(n - \alpha_A), \tag{3}$$

where $\alpha$ is the score of player $A$ in the game (i.e. *first, second, third...*). Subtract this score by the expected score of the player, given by

$$E_A = \frac{1}{\rho} \sum_{1 \geq i \geq n, i \neq A} \left[ 1 + b^{(R_i - R_A)/c} \right]^{-1}. \tag{4}$$

Finally, the $K$ factor is a function of the relative size of the game in terms of participants and the number of games played by player $A$

$$K = \frac{a}{m_A^{1/2} + (N - n)^2}, \tag{5}$$

where $m$ is the total number of previous games played by player $A$, $N$ is the maximum game size, and $n$ is once again the number of players in the specific game. $a$, $b$ and $c$ are constants to be calibrated.