Notes on the function,
gsw_latentheat_melting(SA,p)

This function, **gsw_latentheat_melting**, finds the "latent heat of melting", which is also called the "enthalpy of melting" or "melting enthalpy". The output of this function is in units of $J\ kg^{-1}$ while the input variables are Absolute Salinity S_A (in $g\ kg^{-1}$) and sea pressure p (dbar).

The latent heat of melting of sea ice, $L_p^{SI}(S_A, p)$, is given in terms of the specific enthalpy of seawater, h, and the specific enthalpy of ice Ih, h^{Ih}, by Eqn. (3.34.6) of the TEOS-10 Manual (IOC *et al.* (2010)), repeated here,

$$L_p^{SI}(S_A, p) = h - S_A \left.\frac{\partial h}{\partial S_A}\right|_{T,p} - h^{Ih}, \tag{3.34.6}$$

where S_A is the Absolute Salinity of the seawater into which the ice is melting (Feistel *et al.* (2010)). The present function **gsw_latentheat_melting** is a polynomial fit to data obtained from the SIA (Seawater-Ice-Air) code of TEOS-10. This **gsw_latentheat_melting** function is applicable up to an Absolute Salinity of $42\ g\ kg^{-1}$ and up to 10 000 dbar, and it fits the SIA values of $L_p^{SI}(S_A, p)$ within $1\ J\ kg^{-1}$, which is considerably better than the accuracy of $200\ J\ kg^{-1}$ of the melting enthalpy data which underlies TEOS-10.

References

Feistel, R., D. G. Wright, D. R. Jackett, K. Miyagawa, J. H. Reissmann, W. Wagner, U. Overhoff, C. Guder, A. Feistel and G. M. Marion, 2010: Numerical implementation and oceanographic application of the thermodynamic potentials of liquid water, water vapour, ice, seawater and humid air - Part 1: Background and equations. *Ocean Science*, **6**, 633-677. http://www.ocean-sci.net/6/633/2010/os-6-633-2010.pdf and http://www.ocean-sci.net/6/633/2010/os-6-633-2010-supplement.pdf

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Here follows sections 3.33 and 3.34 of the TEOS-10 Manual (IOC *et al.* (2010)).

3.33 Freezing temperature

Freezing occurs at the temperature t_f at which the chemical potential of water in seawater μ^W equals the chemical potential of ice μ^{Ih}. Thus, t_f is found by solving the implicit equation

$$\mu^W\left(S_A, t_f, p\right) = \mu^{Ih}\left(t_f, p\right) \tag{3.33.1}$$

or equivalently, in terms of the two Gibbs functions,

$$g\left(S_A, t_f, p\right) - S_A\, g_{S_A}\left(S_A, t_f, p\right) = g^{Ih}\left(t_f, p\right). \tag{3.33.2}$$

The Gibbs function for ice Ih, $g^{Ih}\left(t, p\right)$, is defined by IAPWS-06 (IAPWS (2009a)) and Feistel and Wagner (2006) and is summarized in appendix I below. In the special case of zero salinity, the chemical potential of water in seawater reduces to the Gibbs function of pure water, $\mu^W\left(0, t, p\right) = g^W\left(t, p\right)$. A simple correlation function for the melting pressure as a function of temperature is available from IAPWS (2008b) and has been implemented in the SIA library.

At the ocean surface, $p = 0$ dbar, from Eqn. (3.33.1) the TEOS-10 freezing point of pure water is $t_f\left(0\,\text{g kg}^{-1}, 0\,\text{dbar}\right) = 0.002\,519$ °C with an uncertainty of only 2 μK, noting that the triple point temperature of water is exactly 273.16 K by definition of the ITS-90 temperature scale. The freezing temperature of the standard ocean is $t_f\left(S_{SO}, 0\,\text{dbar}\right) = -1.919$ °C with an uncertainty of 2 mK. Note that Eqn. (3.33.1) is valid for air-free water/seawater. Dissolution of air in water lowers the freezing point slightly; saturation with air lowers the freezing temperatures by about 2 mK.

To estimate the effects of small changes in the pressure or salinity on the freezing temperature, it is convenient to consider a power series expansion of (3.33.1). The result in the limit of an infinitesimal pressure change at fixed salinity gives the pressure coefficient of freezing point lowering, as (Clausius-Clapeyron equation, Feistel *et al.* (2010a)),

$$\left.\frac{\partial t_f}{\partial p}\right|_{S_A} = \chi_p\left(S_A, p\right) = -\frac{g_p - S_A g_{S_A p} - g_p^{Ih}}{g_T - S_A g_{S_A T} - g_T^{Ih}}. \tag{3.33.3}$$

Its values, evaluated from TEOS-10, vary only weakly with salinity between $\chi_p\left(0\,\text{g kg}^{-1}, 0\,\text{dbar}\right) = -0.7429$ mK/dbar for pure water and $\chi_p\left(S_{SO}, 0\,\text{dbar}\right) = -0.7483$ mK/dbar for the standard ocean. TEOS-10 is consistent with the most accurate measurement of χ_p and its experimental uncertainty of 0.0015 mK/dbar (Feistel and Wagner (2005), (2006)). Since the value of $\left|\chi_p\right|$ always exceeds that of the adiabatic lapse rate $\left|\Gamma\right|$, cold seawater may freeze and decompose into ice and brine during adiabatic uplift but this can never happen to a sinking parcel.

In the limit of infinitesimal changes in Absolute Salinity at fixed pressure, we obtain the saline coefficient of freezing point lowering, as (Raoult's law),

$$\left.\frac{\partial t_f}{\partial S_A}\right|_p = \chi_S\left(S_A, p\right) = \frac{S_A g_{S_A S_A}}{g_T - S_A g_{S_A T} - g_T^{Ih}}. \tag{3.33.4}$$

Typical numerical values are $\chi_S\left(0\,\text{g kg}^{-1}, 0\,\text{dbar}\right) = -59.2$ mK/(g kg^{-1}) for pure water and $\chi_S\left(S_{SO}, 0\,\text{dbar}\right) = -56.9$ mK/(g kg^{-1}) for seawater.

As a raw practical estimate, Eqn. (3.33.4) can be expanded into powers of salinity, using only the leading term of the TEOS-10 saline Gibbs function, $g^S \approx R_S T S_A \ln S_A$, which stems from Planck's ideal-solution theory (Planck (1888)). Here, $R_S = R/M_S = 264.7599$ J kg^{-1} K^{-1} is the specific gas constant of sea salt, R is the universal molar gas constant, and $M_S = 31.403\,82$ g mol^{-1} is the molar mass of sea salt with Reference Composition. The denominator of Eqn. (3.33.4) is proportional to the melting heat L_p^{SI}, Eqn. (3.34.7). The convenient result obtained with these simplifications is

$$\left.\frac{\partial t_f}{\partial S_A}\right|_p \approx -\frac{R_S}{L_p^{SI}}\left(T_0 + t_f\right)^2 \approx -59 \text{ mK/(g kg}^{-1}). \tag{3.33.5}$$

where we have used $t_f = -2\,^{\circ}\text{C}$ and $L_p^{SI} = 330$ J kg^{-1} as approximations that are appropriate for the standard ocean. This simple result is only weakly dependent on these choices and is in reasonable agreement with the exact values from Eqn. (3.33.4) and with Millero and Leung (1976). The freezing temperature of seawater is always lower than that of pure water.

When sea-ice is formed, it often contains remnants of seawater included in brine pockets. At equilibrium, the salinity in these pockets depends only on temperature and pressure, rather than, for example, on the pocket volume, and can be computed in the functional form $S_A(t, p)$ as an implicit solution of Eqn. (3.33.1). Measured values for the brine salinity of Antarctic sea ice agree very well with those computed of Eqn. (3.33.1) up to the saturation concentration of about 110 g kg^{-1} at surface pressure (Feistel *et al.* (2010b)). At high pressures, the validity of the Gibbs function of seawater, and therefore of the computed freezing point or brine salinity, too, is limited to only 50 g kg^{-1}.

We note that in the first approximation, as inferred from Planck's theory of ideal solutions, the above properties depend on the number of dissolved particles regardless of the particle sizes, masses or charges. In other words, they depend mainly on the molar rather than on the mass density of the solute, in contrast to properties such as the density of seawater and properties derived from it. The properties considered in the remainder of this section (3.33-3.42) which share this attribute are referred to as the colligative properties of seawater.

3.34 Latent heat of melting

The melting process of ice in pure water can be conducted by supplying heat at constant pressure. If this is done slowly enough that equilibrium is maintained, then the temperature will also remain constant. The heat required per mass of molten ice is the latent heat, or enthalpy, of melting, L_p^{WI}. It is found as the difference between the specific enthalpy of water, h^W, and the specific enthalpy of ice, h^{Ih}, (Kirchhoff's law, Curry and Webster (1999)):

$$L_p^{WI}(p) = h^W(t_f, p) - h^{Ih}(t_f, p). \tag{3.34.1}$$

Here, $t_f(p)$ is the freezing temperature of water, section 3.33. The enthalpies h^W and h^{Ih} are available from IAPWS-95 (IAPWS (2009b)) and IAPWS-06 (IAPWS (2009a)), respectively.

In the case of seawater, the melt water will additionally mix with the ambient brine, thus changing the salinity and the freezing temperature of the seawater. Consequently, the enthalpy related to this phase transition will depend on the particular conditions under which the melting occurs.

Here, we define the latent heat of melting as the enthalpy increase per infinitesimal mass of molten ice of a composite system consisting of ice and seawater, when the temperature is increased at constant pressure and at constant total masses of water and salt, in excess of the heat needed to warm up the seawater and ice phases individually (Feistel and Hagen (1998), Feistel *et al.* (2010b)). Mass conservation of both water and salt during this thermodynamic process is essential to ensure the independence of the latent heat formula from the unknown absolute enthalpies of salt and water that otherwise would accompany any mass exchange.

The enthalpy of sea ice, h^{SI}, is additive with respect to its constituents ice, h^{Ih}, with the mass fraction w^{Ih}, and seawater, h, with the liquid mass fraction $(1 - w^{Ih})$:

$$h^{\mathrm{SI}} = \left(1 - w^{\mathrm{Ih}}\right) h\left(S_{\mathrm{A}}, t, p\right) + w^{\mathrm{Ih}} h^{\mathrm{Ih}}\left(t, p\right). \tag{3.34.2}$$

Upon warming, the mass of melt water changes the ice fraction w^{Ih} and the brine salinity S_{A}. The related temperature derivative of Eqn. (3.34.2) is

$$\left.\frac{\partial h^{\mathrm{SI}}}{\partial T}\right|_{p} = \left(1 - w^{\mathrm{Ih}}\right)\left.\frac{\partial h}{\partial T}\right|_{S_{\mathrm{A}}, p} + \left(1 - w^{\mathrm{Ih}}\right)\left.\frac{\partial h}{\partial S_{\mathrm{A}}}\right|_{T, p}\left.\frac{\partial S_{\mathrm{A}}}{\partial T}\right|_{p} + w^{\mathrm{Ih}}\left.\frac{\partial h^{\mathrm{Ih}}}{\partial T}\right|_{p} + \left(h^{\mathrm{Ih}} - h\right)\left.\frac{\partial w^{\mathrm{Ih}}}{\partial T}\right|_{p}. \tag{3.34.3}$$

The rate of brine salinity change with temperature is given by the reciprocal of Eqn. (3.33.4) and is related to the isobaric melting rate, $-\partial w^{\mathrm{Ih}} / \partial T\big|_{p}$, by the conservation of the total salt, $\left(1 - w^{\mathrm{Ih}}\right) S_{\mathrm{A}} = \mathrm{const}$, in the form

$$\left.\frac{\partial S_{\mathrm{A}}}{\partial T}\right|_{p} = \frac{S_{\mathrm{A}}}{1 - w^{\mathrm{Ih}}} \left.\frac{\partial w^{\mathrm{Ih}}}{\partial T}\right|_{p}. \tag{3.34.4}$$

Using this relation, Eqn. (3.34.3) takes the simplified form

$$\left.\frac{\partial h^{\mathrm{SI}}}{\partial T}\right|_{p} = \left(1 - w^{\mathrm{Ih}}\right) c_{p} + w^{\mathrm{Ih}} c_{p}^{\mathrm{Ih}} - L_{p}^{\mathrm{SI}} \left.\frac{\partial w^{\mathrm{Ih}}}{\partial T}\right|_{p}. \tag{3.34.5}$$

The coefficient in front of the melting rate,

$$L_{p}^{\mathrm{SI}}\left(S_{\mathrm{A}}, p\right) = h - S_{\mathrm{A}} \left.\frac{\partial h}{\partial S_{\mathrm{A}}}\right|_{T, p} - h^{\mathrm{Ih}}, \tag{3.34.6}$$

provides the desired expression for isobaric melting enthalpy, namely the difference between the partial specific enthalpies of water in seawater and of ice. As is physically required for any measurable thermodynamic quantity, the arbitrary absolute enthalpies of ice, water and salt cancel in the formula (3.34.6), provided that the reference state conditions for the ice and seawater formulations are chosen consistently (Feistel *et al.* (2008a)). Note that because of $h = g + \left(T_{0} + t\right) \eta$ and Eqn. (3.33.2), the latent heat can also be written in terms of entropies η rather than enthalpies h, in the form

$$L_{p}^{\mathrm{SI}}\left(S_{\mathrm{A}}, p\right) = \left(T_{0} + t_{\mathrm{f}}\right) \times \left(\eta - S_{\mathrm{A}} \left.\frac{\partial \eta}{\partial S_{\mathrm{A}}}\right|_{T, p} - \eta^{\mathrm{Ih}}\right). \tag{3.34.7}$$

Again the result is independent of unknown (and unknowable) constants.

The latent heat of melting depends only weakly on salinity and on pressure. At the surface pressure, the computed value is $L_{p}^{\mathrm{SI}}\left(0, 0\right) = L_{p}^{\mathrm{WI}}\left(0\right) = 333\,426.5\ \mathrm{J\ kg^{-1}}$ for pure water, and $L_{p}^{\mathrm{SI}}\left(S_{\mathrm{SO}}, 0\right) = 329\,928.5\ \mathrm{J\ kg^{-1}}$ for the standard ocean, with a difference of about 1% due to the dissolved salt. At a pressure of 1000 dbar, these values reduce by 0.6% to $L_{p}^{\mathrm{SI}}\left(0, 1000\,\mathrm{dbar}\right) = L_{p}^{\mathrm{WI}}\left(1000\,\mathrm{dbar}\right) = 331\,528\ \mathrm{J\ kg^{-1}}$ and $L_{p}^{\mathrm{SI}}\left(S_{\mathrm{SO}}, 1000\,\mathrm{dbar}\right) = 328\,034\ \mathrm{J\ kg^{-1}}$. TEOS-10 is consistent with the most accurate measurements of L_{p}^{WI} and their experimental uncertainties of 200 J kg⁻¹, or 0.06% (Feistel and Wagner (2005), (2006)).