Notes on the GSW function gsw_pot_enthalpy_from_pt

This function **gsw_pot_enthalpy_from_pt** calculates potential enthalpy h^0 for given input values of Absolute Salinity S_A ($g\ kg^{-1}$) and potential temperature θ ($°C$, ITS-90), this being the potential temperature referenced to the reference sea pressure $p_r = 0\ dbar$. Potential enthalpy h^0 is defined by Eqn. (3.2.2) of the TEOS-10 manual, repeated here,

$$h^0\left(S_A, t, p\right) = h\left(S_A, \theta, 0\right) = g\left(S_A, \theta, 0\right) - \left(T_0 + \theta\right) g_T\left(S_A, \theta, 0\right). \tag{3.2.2}$$

The present GSW function **gsw_pot_enthalpy_from_pt** could have evaluated potential enthalpy via two calls to the library function **gsw_gibbs**; one call to evaluate $g\left(S_A, \theta, 0\right)$ and the other to evaluate $g_T\left(S_A, \theta, 0\right)$.

Instead, we have collected like powers of the non-dimensional variables representing S_A and θ in $g\left(S_A, \theta, 0\right) - \left(T_0 + \theta\right) g_T\left(S_A, \theta, 0\right)$ so that **gsw_pot_enthalpy_from_pt** is a simple polynomial in terms of these non-dimensional variables. There are three reasons for doing this. First, since pressure is zero, we can avoid the several unnecessary multiplications involving various powers of zero. Second, the act of collecting together the like powers of the two remaining variables effectively halves the number of multiplications that are needed, so making the function more computationally efficient. Third, both $g\left(S_A, \theta, 0\right)$ and $g_T\left(S_A, \theta, 0\right)$ contain terms in the logarithm of S_A which exactly cancel from the combination $g\left(S_A, \theta, 0\right) - \left(T_0 + \theta\right) g_T\left(S_A, \theta, 0\right)$. Not having to calculate these logarithm terms saves computer time and also avoids numerical issues that would otherwise occur as S_A approaches zero (actually these terms are four terms in $S_A \ln\left(\sqrt{S_A}\right)$ which do individually go to zero as S_A vanishes).

Here follows section 3.2 of the TEOS-10 manual (IOC *et al*. (2010)).

3.2 Potential enthalpy

Potential enthalpy h^0 is the enthalpy that a fluid parcel would have if its pressure were changed to a fixed reference pressure p_r in an isentropic and isohaline manner. Because heat fluxes into and out of the ocean occur mostly near the sea surface, the reference pressure for potential enthalpy is always taken to be $p_r = 0\ dbar$ (that is, at zero sea pressure). Potential enthalpy can be expressed as the pressure integral of specific volume as (from McDougall (2003) and see the discussion below Eqn. (2.8.2))

$$
\begin{aligned}
h^0\left(S_A, t, p\right) = h\left(S_A, \theta, 0\right) = \tilde{h}^0\left(S_A, \theta\right) = {} & h\left(S_A, t, p\right) - \int_{P_0}^{P} v\left(S_A, \theta\left[S_A, t, p, p'\right], p'\right) dP' \\
= {} & h\left(S_A, t, p\right) - \int_{P_0}^{P} \hat{v}\left(S_A, \eta, p'\right) dP' \\
= {} & h\left(S_A, t, p\right) - \int_{P_0}^{P} \tilde{v}\left(S_A, \theta, p'\right) dP' \\
= {} & h\left(S_A, t, p\right) - \int_{P_0}^{P} \hat{v}\left(S_A, \Theta, p'\right) dP',
\end{aligned}
\tag{3.2.1}
$$

and we emphasize that the pressure integrals here must be done with respect to pressure expressed in Pa rather than $dbar$. In terms of the Gibbs function, potential enthalpy h^0 is evaluated as

$$h^0\left(S_A, t, p\right) = h\left(S_A, \theta, 0\right) = g\left(S_A, \theta, 0\right) - \left(T_0 + \theta\right) g_T\left(S_A, \theta, 0\right). \tag{3.2.2}$$

Also, note Figure A.17.1 below (from IOC *et al.* (2010)) showing the difference between potential temperature and Conservative Temperature (which is proportional to potential enthalpy).
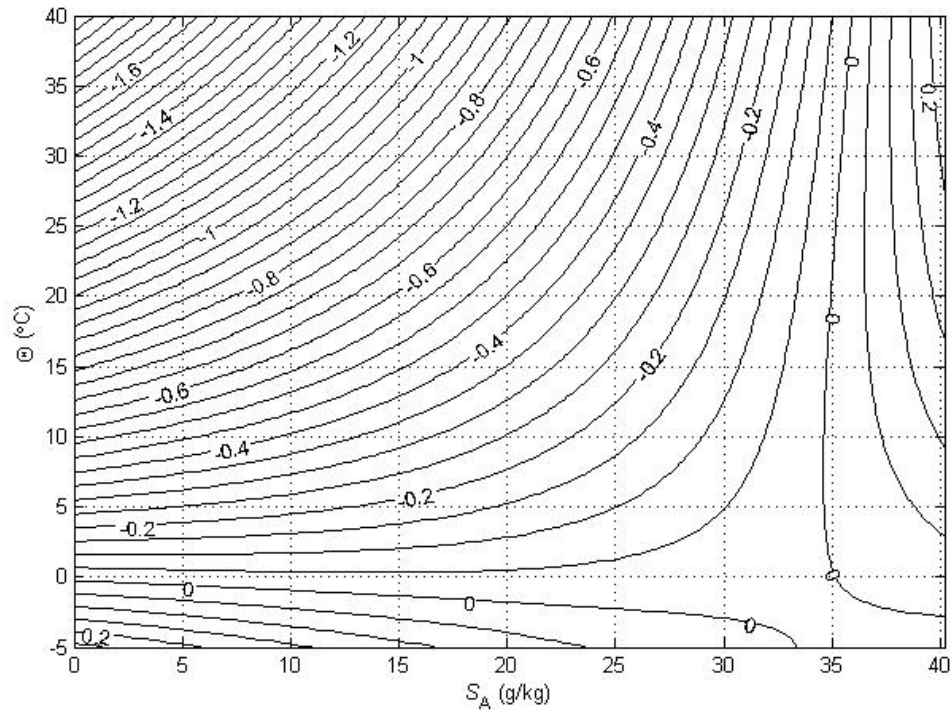
Figure A.17.1. Contours (in °C) of the difference between potential temperature and Conservative Temperature $\theta - \Theta$. This plot illustrates the non-conservative production of potential temperature θ in the ocean.