MATH Academy Worksheet: Cup Stacking **Name**: ______________________________

Cup stacking is a thing. You see how quickly you can assemble and disassemble a triangular pyramid of cups from a single column of cups.

In this activity we investigate how high the single column of cups will be in terms of how many stories the assembled pyramid has.

1. To make sure we're all on the same page, the cup pyramid below has 3 stories. If we were to stack all 6 of these cups into a single column how high would the column be? Measure this with the cups you've been given (in cm, to one decimal place).

height of a single column stack of 6 cups: _________ cm

Primary Question: If we want to stack a triangular pyramid of cups with 1000 stories into a single column, how high will the column of cups be?

You can stop here and try to solve this question by your own method, or you can proceed by considering the following questions.

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2. How tall is a stack of 2 cups? How tall is a stack of 4 cups? 12 cups? Complete the table below (measure in cm to one decimal place), and plot the data in *Desmos*. Do you think the relationship between the height of a stack and the number of cups in the stack is linear? If so, use *Desmos* to determine a linear model. Record the linear model below.

cups (x)	2	4	6	12
height of column (y)				

linear model for height$\sim$cups: ______________________________

3. We also need to establish a relationship between how many cups x are used in a pyramid arrangement with n stories. Here we think of n as the input variable, and x as the output variable. Fill out the following table.

stories in a pyramid (n)	2	3	4	5	6
cups (x)					

4. Does this relationship look linear? Quadratic? Use *Desmos* to record the best linear model below, and the best quadratic model.

linear model for cups$\sim$stories: ______________________________

quadratic model cups$\sim$stories: ______________________________

5. Record two reasons why the quadratic model is the better model.

6. Use your quadratic model to determine how many cups would be needed to build a pyramid with 1000 stories.
   number of cups in a pyramid with 1000 stories: _________ cups.
7. Based on the linear model for the height of a cup column, how high would the resulting single column stack be:

   height of column: _________ cm.

   Convert that height to miles: _________ miles.
8. **Extrapolation**. We are extrapolating with our models in this activity, evaluating our linear model (height of stack $\sim$ cups) at a value well beyond the inputs used to establish the line. Is that a concern here? In particular, do you think the relationship between the number of cups and the height of a single column stack is truly linear? Are we adding to the height by the same amount with each cup we add to the column? Explain.

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9. We are also evaluating our quadratic model (cups $\sim$ stories) at a value well beyond those inputs used to establish the quadratic fit. But we can prove mathematically that the quadratic fit is truly the correct model. Here's the idea: The number of cups used to build a pyramid with n stories is found by adding the first n positive integers:

$$\text{number of cups used} = 1 + 2 + 3 + \cdots + n$$

since there is 1 cup on the top story, 2 on the story below that, 3 on the next, all the way down to n cups on the bottom story.

Let $S = 1 + 2 + 3 + \cdots + n$. Then, we have

$$\begin{aligned} S &= \quad 1 \quad + \quad 2 \quad + \quad 3 \quad + \cdots + (n-1) + \quad n \\ S &= \quad n \quad + (n-1) + (n-2) + \cdots + \quad 2 \quad + \quad 1 \\ 2S &= (n+1) + (n+1) + (n+1) + \cdots + (n+1) + (n+1) \\ 2S &= n(n+1) \end{aligned}$$

The third line above shows that 2 times S equals a sum of n $(n+1)$s, so solving the last line for S yields the result

$$S = \frac{n(n+1)}{2},$$

which can be rewritten as

$$S = \frac{1}{2}n^2 + \frac{1}{2}n$$

to match the quadratic model given by *Desmos*.