

MATH 140 Check-Up

January 12, 2023

Question 1

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Answer

The first value has standardized value $Z = (12 - 10)/0.5 = 4$, while the second has standardized value $Z = (20 - 10)/5 = 2$, so the first value is more extreme in its distribution.

Question 2

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Answer

X = how many 4s you get when you roll a single die ten times. Y = the weight (in grams) of newborn chicks.

Question 3

Assume 60% of all ripe watermelons at a particular farm weigh more than 20 pounds. We randomly select 10 to sample and find 40% of them weigh more than 20 pounds. (a) Is 60% a parameter or a statistic? (b) Is 40% a parameter or a statistic?

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Answer

(a) parameter; (b) statistic

Question 4

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Answer

If X = the diameter of a randomly chosen ball-bearing, we want $P(1.18 < X < 1.22)$.

This probability equals $P(z_{low} < Z < z_{high})$ where

$$z_{low} = (1.18 - 1.2)/0.05 = -0.4, \text{ and}$$

$$z_{high} = (1.22 - 1.2)/0.05 = 0.4.$$

Finally, $P(-0.4 < Z < 0.4) = \text{pnorm}(0.4) - \text{pnorm}(-0.4) = .3108$

Question 5

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Answer

To use CLT, convert everything to proportions.

p = proportion of blue marbles in the box = $1800/5000 = 0.36$.

We gather a sample of size $n = 200$ and ask for this probability:

$P(\hat{p} > 90/200)$

By the CLT, \hat{p} lives in a normal distribution with mean = $p = .36$, and standard deviation $\sqrt{p(1-p)/n} = .03394$.

Now, $90/200 = 0.45$ and the Z-score for 0.45 is $Z = (0.45 - 0.36)/.03394 = 2.65$.

So we want $P(Z > 2.65) = 1 - \text{pnorm}(2.65) = .0040$.