Building a Mathematical Model

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Guiding Principles

- Occam's Razor
- Beware Extrapolation

Occam's Razor

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- If several explanations are compatible with a set of observations, Occam's razor advises us to buy the simplest.
- 'A theory with mathematical beauty is more likely to be correct than an ugly one that fits some experimental data.' - Paul Dirac

Extrapolation

- We extrapolate when we evaluate a model at inputs well outside the range of those inputs used to build the model.
- If we extrapolate, we are making an unreliable bet that the approximate relationship we've established will be valid in places where it has not been analyzed.

Models in this workshop

- Linear
$$y = mx + b$$
- Quadratic
$$y = ax^2 + bx + c$$
- Power
$$y = ax^b$$
- Exponential
$$y = ab^x$$

A linear model

Is the tip you leave a function of your bill?

bill	tip
70.29	12.7
43.58	8.7
106.27	18.60
49.72	8.8
88.01	16.2
32.98	6.2

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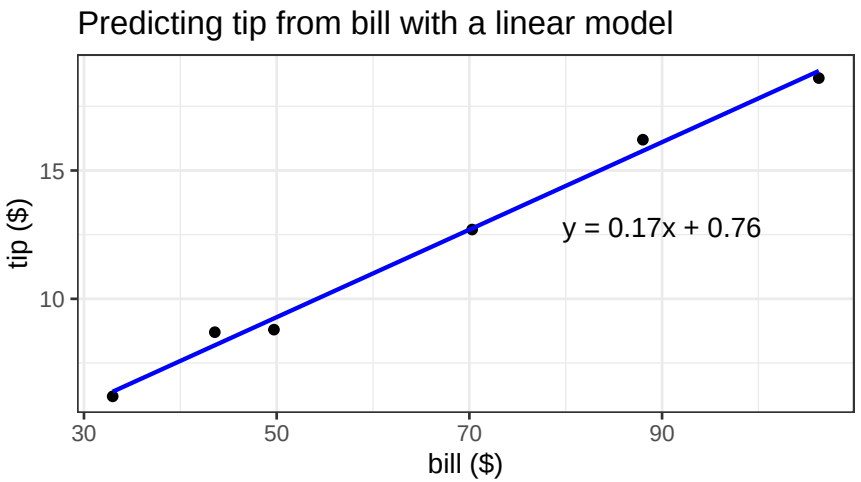
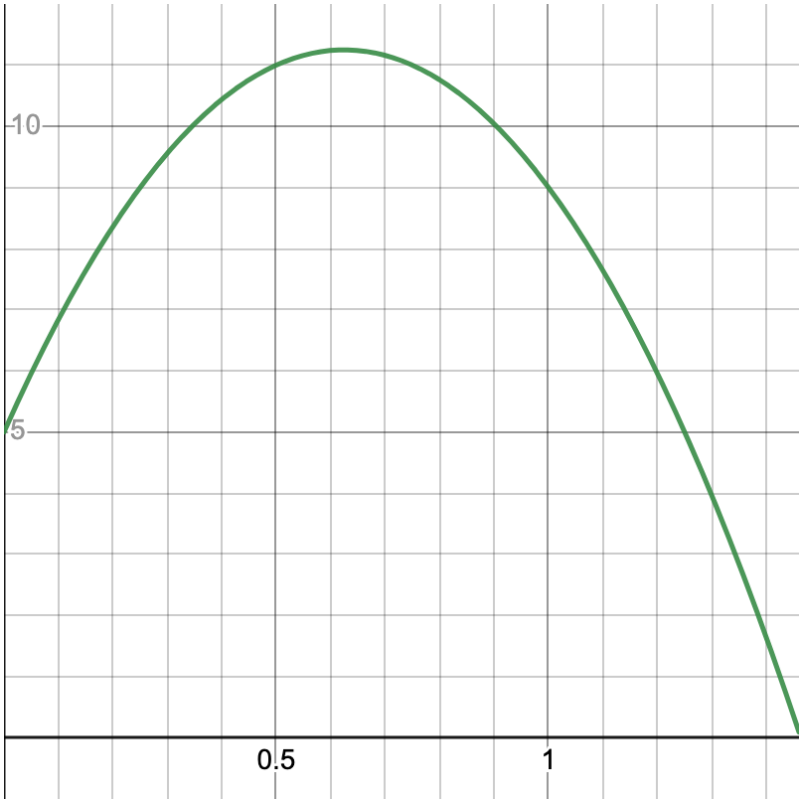


A quadratic model

Twinkie Tossing: What is the twinkie's height t seconds after you toss it?



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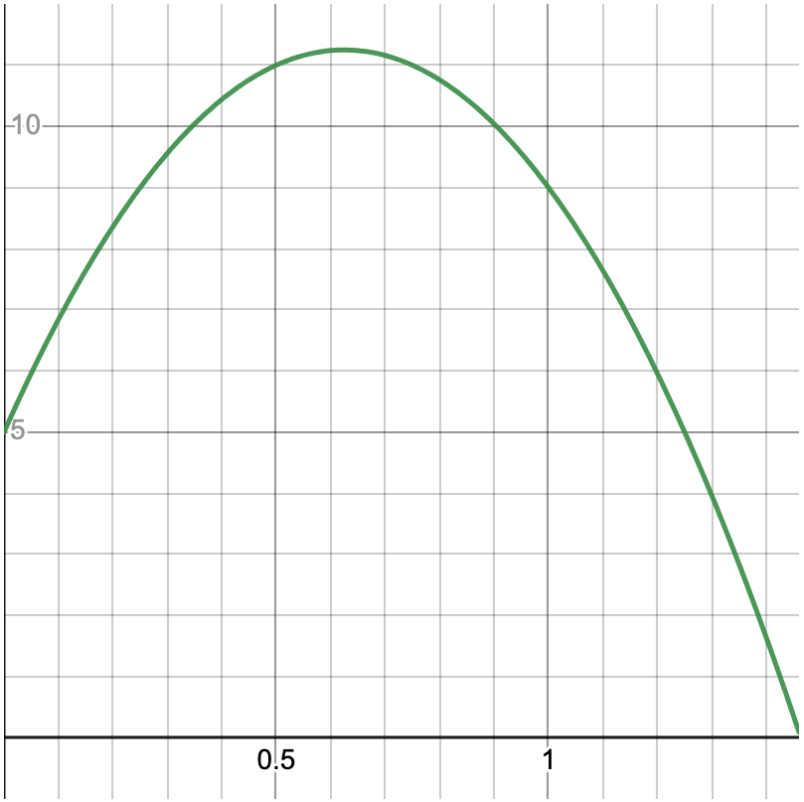

$$h(t) = -16t^2 + v_0 t + h_0$$

A power model

Planets orbiting the Sun: Is the time it takes a planet to make a revolution around the Sun dependent on how far away the planet is?

distance (x)	period (y)	
Mercury	36	88
Venus	67	225
Earth	93	365
Mars	142	687
Jupiter	484	4332
Saturn	887	10760
Uranus	1765	30684
Neptune	2791	60188
Pluto	3654	90467

units:
x - milions of miles from Sun;
y - Earth days

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power model

$$y = 0.4x^{1.5}$$

units:
x - milions of miles from Sun;
y - Earth days

An exponential model

Cooling Coffee: Can we predict the temperature of a cup of coffee x minutes after pouring it?

time (x)	temperature (y)
0	72
10	59
20	52
30	46
40	40
50	37
60	34

units:

x - minutes;

y - $^\circ C$

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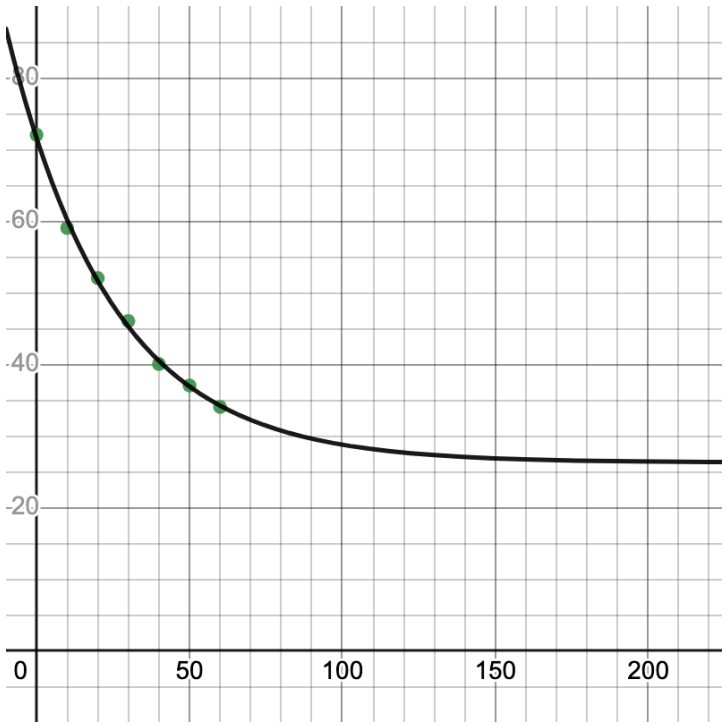

$$y = 45.3 \cdot e^{-.029x} + 26.3$$

(Newton's Law of Cooling)

Fitting Models in *Desmos*

desmos.com - A great online graphing calculator

Building models in *Desmos*

- Enter data in table (click the + symbol to select table)
- Note: It is also possible to copy and paste a table into Desmos from a spreadhseet!
- Notice the table variables are x_1 and y_1 - keep those as is!
- Enter a desired model expression
  - Linear model: $y_1 \sim mx_1 + b$
  - Quadratic model: $y_1 \sim ax_1^2 + bx_1 + c$
  - Power model: $y_1 \sim ax_1^b$
  - Exponential, e.g., $y_1 \sim ab^{x_1}$ or $y_1 \sim ae^{bx_1} + c$

Linear Association?

- Research Question: the bigger the car, the worse the mileage?

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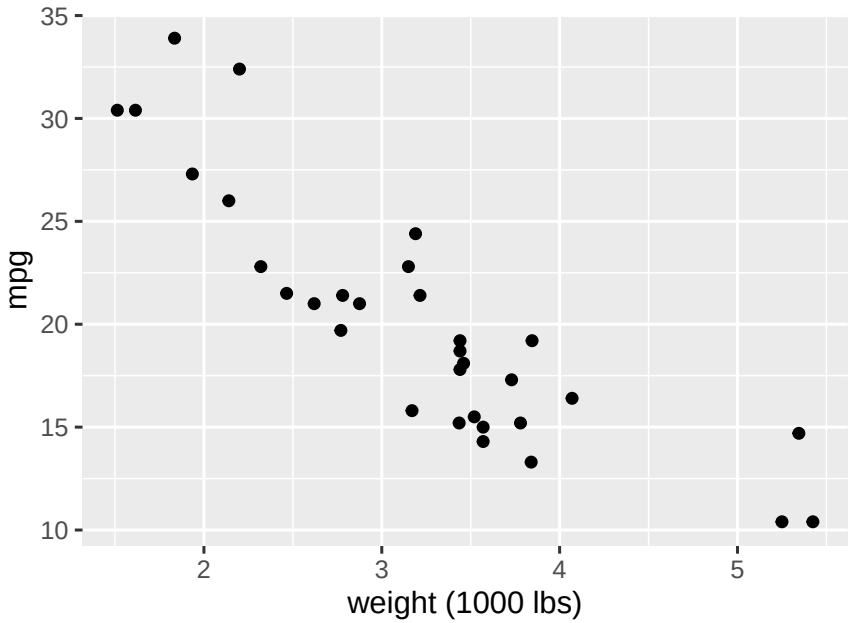
- An *explanatory variable* (x) is a variable that attempts to explain observed outcomes.
- A *response variable* (y) measures an outcome of a study.
- *Linear regression* is the statistical method for fitting a line to data where the relationship between two variables, x and y, can be modeled by a straight line with some error:

$$y = mx + b + \epsilon.$$

car	mpg	wt
Mazda RX4	21.00	2.62
Mazda RX4 Wag	21.00	2.88
Datsun 710	22.80	2.32
Hornet 4 Drive	21.40	3.21
Hornet Sportabout	18.70	3.44
⋮	⋮	⋮
Porsche 914-2	26.00	2.14
Lotus Europa	30.40	1.51
Ford Pantera L	15.80	3.17
Ferrari Dino	19.70	2.77
Maserati Bora	15.00	3.57
Volvo 142E	21.40	2.78



Underlying Functions

As we consider each type of model, we also review their underlying functions.
We start with the simplest of the models today, the linear model, and we review linear functions.