MATH 140 **Name** ______________________ **Date:**

Worksheet: Confidence Intervals, P-values, Hypothesis Tests

1. Eye Color at Linfield.

   (a) You know, 21 of the 50 people in our class survey reported having brown eyes. Treating this as a SRS of all Linfield students, determine a 95% confidence interval for the proportion of all Linfield students having brown eyes.

   (b) Notice that the difference between our sample proportion $\hat{p} = 0.42$ and the US proportion is 0.03. If, in fact, the proportion of *all* Linfield students with brown eyes, p, matches the US proportion of 0.45, what is the probability of obtaining a sample proportion $\hat{p}$ more than 0.03 away from 0.45?

2. How many hours a night to Linfield students sleep? Here are the summary statistics from our class survey of $n = 50$ students: $\overline{x} = 7.37$ hours, $s = 1.059$ hours.

   (a) Based on our class survey, which we assume is a random sample of all Linfield students, determine a 95% confidence interval for μ_L, the population mean hours of sleep per night for Linfield students.

   (b) According to the CDC[1], a *Healthy People 2020* report recommends that adults get 7 or more hours of sleep each day. Based on your confidence interval, do you have reason to believe that the population mean μ_L is at least 7 hours?

   (c) If, in fact, the true population mean hours of sleep for Linfield students is $\mu_L = 7$, use t-scores to estimate the probability of obtaining a sample mean $\overline{x}$ greater than the one we actually observed, when the sample size is $n = 50$.

3. Hypothesis test on one proportion. We want to test, based on our survey sample, whether we have evidence to conclude the population proportion of all Linfield students with brown eyes, p, is different than 0.45, the national proportion.

   (a) Clearly state the null and alternative hypotheses for this test. You should express these hypotheses using the symbol p.

   (b) From the summary statistics, calculate the relevant test statistic for the test.

   (c) Determine the P-value for this test.

   (d) Do you have statistically significant evidence at the $\alpha = 0.05$ level to reject the null hypothesis in favor of the alternative? With a sentence, state your conclusion in the context of the problem.

4. Hypothesis test on one mean. We want to test, based on our survey sample, whether we have evidence to conclude the population mean hours of sleep of all Linfield students, μ_L, is more than 7 hours, the minimum hours of sleep per night recommended by Healthy People 2020.

   (a) Clearly state the null and alternative hypotheses for this test. You should express these hypotheses using the symbol μ_L.

   (b) From the summary statistics, calculate the relevant test statistic for the test. Hint: This test statistic should be a t-score.

   (c) Determine the P-value for this test.

   (d) Do you have statistically significant evidence at the $\alpha = 0.05$ level to reject the null hypothesis in favor of the alternative? With a sentence, state your conclusion in the context of the problem.

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[1] `https://www.cdc.gov/niosh/index.htm`

CI, P-values, Hypothesis Tests Solutions

1) a) Here $n = 50$, $\hat{p} = \frac{21}{50} = 0.42$, and $z^* = 1.96$ for 95% confidence.

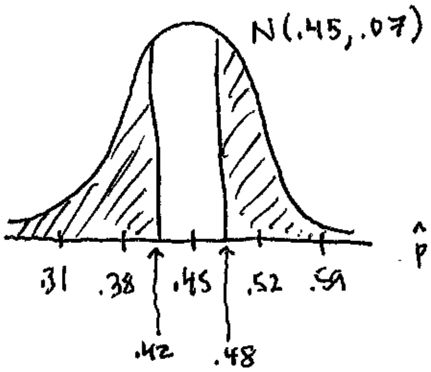
Using CI formula for a proportion $\hat{p} \pm z^* \sqrt{\hat{p}(1-\hat{p})/n}$ yields

$\boxed{0.42 \pm 0.137}$ OR $\boxed{.283 \text{ to } .557}$

b) If the pop'n proportion is $p = 0.45$, then $\hat{p}$ has a $N(0.45, \underbrace{0.07}_{\sqrt{(.45)(.55)/50}})$ dist'n

and the question asks for this shaded area

(the chance of being more than 0.03 away from .45)



We convert to Z-scores

$\hat{p} = .42$ has $Z = \frac{.42 - .45}{.07} = -0.429$, so pnorm(-.429) = .3340 gives the lower tail area.

By symmetry, the upper tail area is also .3340

so the total shaded area = .6680.

If $p = .45$, and we sample $n = 50$, the probability that $\hat{p}$ is more than .03 away from .45 equals about .6680.

2) a) Here $n = 50$, $\bar{x} = 7.37$, $s = 1.059$, and $t^* = qt(.975, 49) = 2.01$

Using CI formula for mean when σ is unknown, $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ yields

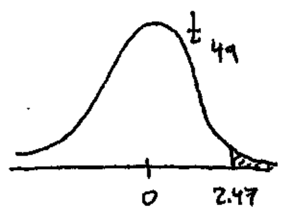
$\boxed{7.37 \pm 0.30}$ OR $\boxed{7.07 \text{ to } 7.67 \text{ hours}}$

b) Since the entire confidence interval is above 7, and we believe our interval captures μ_L, then yes, we believe $\mu_L > 7$.

2] c) If $\mu_L = 7$ then $t = \dfrac{\bar{x} - 7}{s/\sqrt{n}}$ lives in a t distribution with $(n-1)$ degrees of freedom

Then $P(\bar{x} > 7.37) = P\left(t > \dfrac{7.37 - 7}{1.059/\sqrt{50}}\right)$

$= P(t > 2.47)$



$= 1 - pt(2.47, 49)$

$= 0.0085$

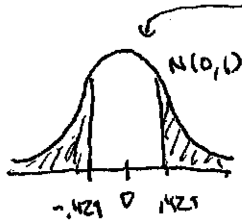
3] Let p = proportion of all Linfield students having brown eyes.

a) $H_0: p = .45$

$H_a: p \neq .45$

b) Relevant test statistic: $Z = \dfrac{\hat{p} - .45}{\sqrt{\dfrac{(.45)(.55)}{50}}} = \dfrac{.42 - .45}{.07} = -0.429$ (as found in 1(b))

c) Since test is 2-sided, P-value = $2 *$ pnorm(-0.429) = .6680 (as found in 1(b))



d) Since our P-value is large, larger than $\alpha = .05$, we do not have statistically significant evidence that p is different than 0.45. ~~We have~~ Our sample gives no real reason to rule out $p = .45$.

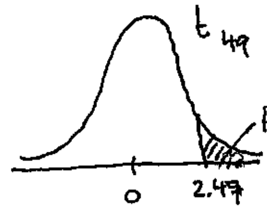
4) Let μ_L = population mean hours of sleep per night for all Linfield students

a) $H_0: \mu_L = 7$ Summary statistics: $n = 50$, $\bar{x} = 7.37$, $s = 1.059$

$H_a: \mu_L > 7$

b) Assuming $\mu_L = 7$, test statistic is $t = \dfrac{\bar{x} - 7}{s/\sqrt{n}} = 2.47$ (as found in 2(c))

and t lives in a t_{49} distribution.



c) P-value, gives H_a is "greater than" is

P-value = $1 - pt(2.47, 49) = .0085$ (as found in 2(c))

d) Yes, we have strong evidence against H_0 in favor of H_a since the P-value is so small. If $\mu_L = 7$, we would see a sample mean as extreme as we did with probability .0085, which is less than $\alpha = .05$. Since we did see that sample mean we reject H_0 in favor of H_a