Quadratic Functions

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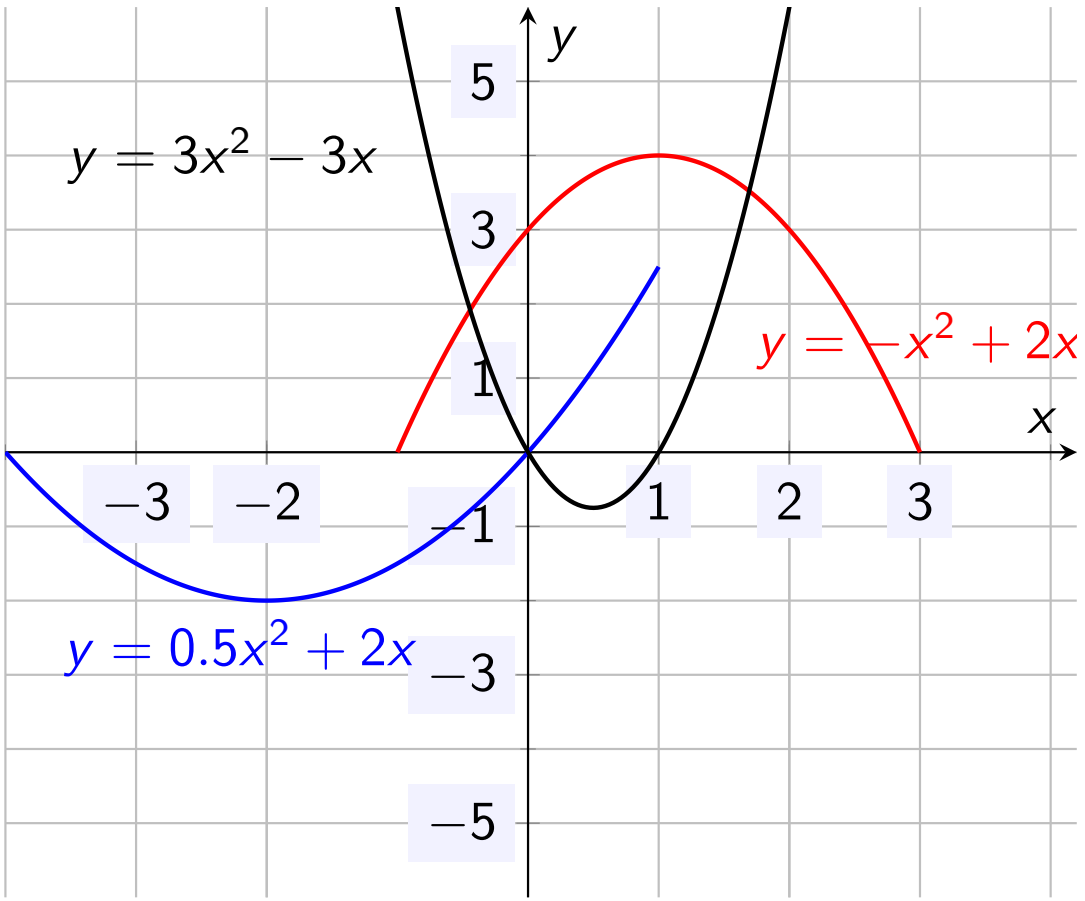
Quadratic Functions - Quick Review

- A quadratic function has the form $f(x) = ax^2 + bx + c$
  - The graph of a quadratic function is a parabola
  - The constant c is the y-intercept
  - If $a > 0$ the parabola "opens up"
  - If $a < 0$ the parabola "opens down"
  - If $a = 0$ the function is linear!
  - The *vertex* of the parabola occurs when $x = \dfrac{-b}{2a}$

Some parabolas



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- **What is the y-intercept?**
  - The graph crosses the y-axis at -1 since $f(0) = -1$. ($c = -1$ here)
- **What are the coordinates of the vertex of the parabola?** The x-coordinate is given by

$$x = \frac{-b}{2a} = \frac{-8}{2 \cdot 2} = -\frac{8}{4} = -2.$$

The y-coordinate is then

$$\begin{aligned} f(-2) &= 2 \cdot (-2)^2 + 8 \cdot (-2) - 1 \\ &= 2 \cdot 4 - 16 - 1 \\ &= 8 - 17 \\ &= -9. \end{aligned}$$

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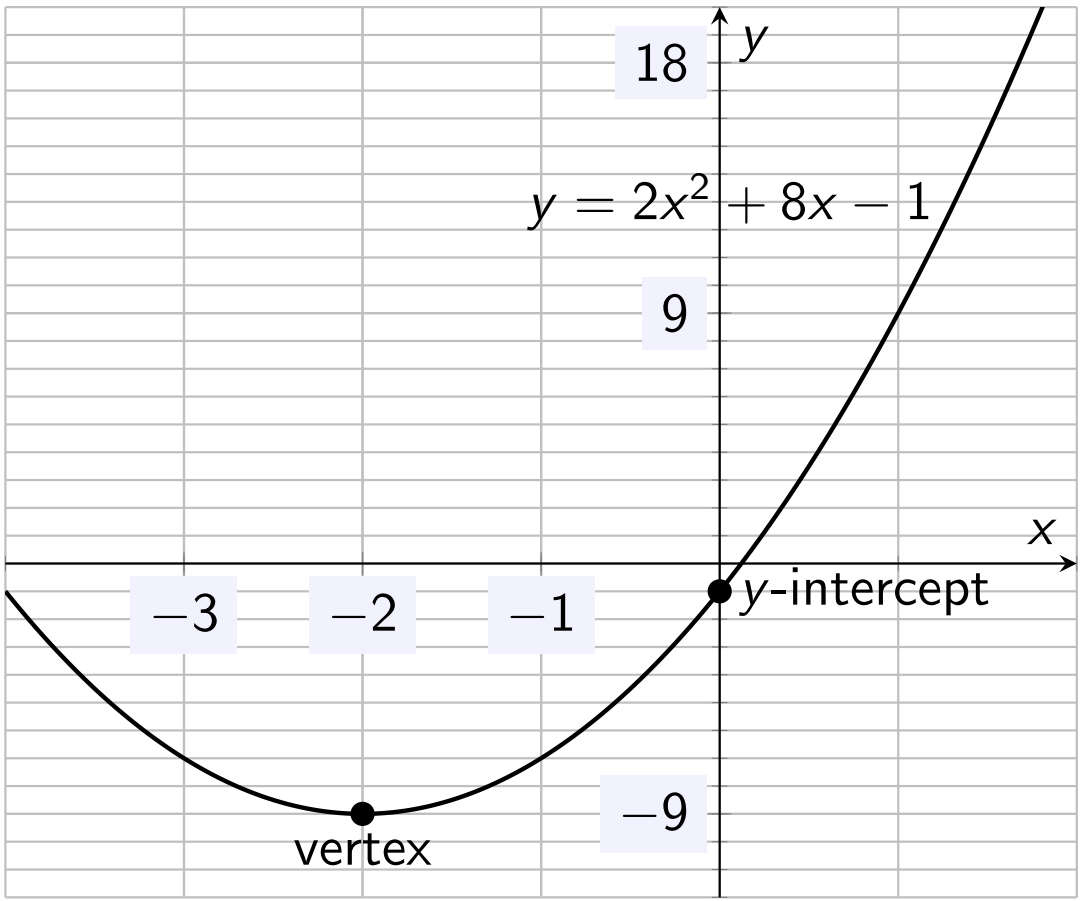
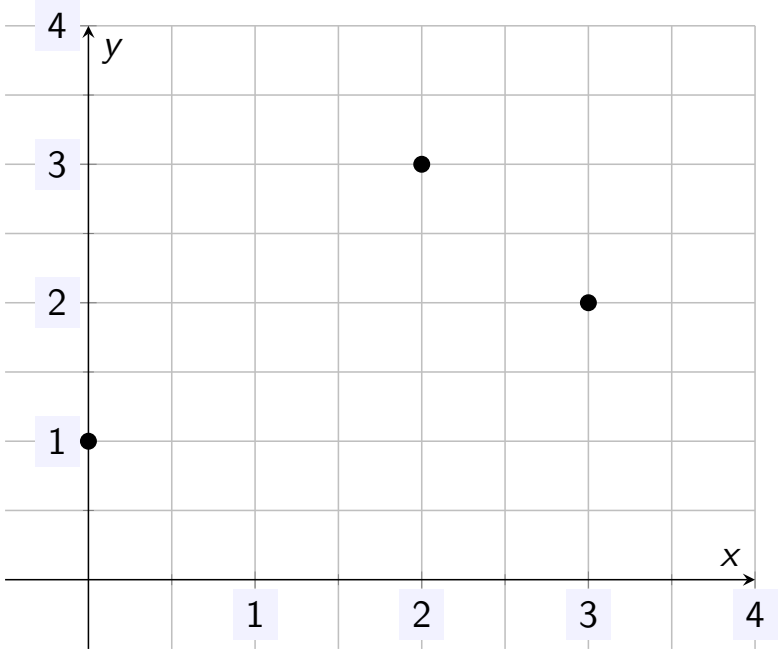


Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?

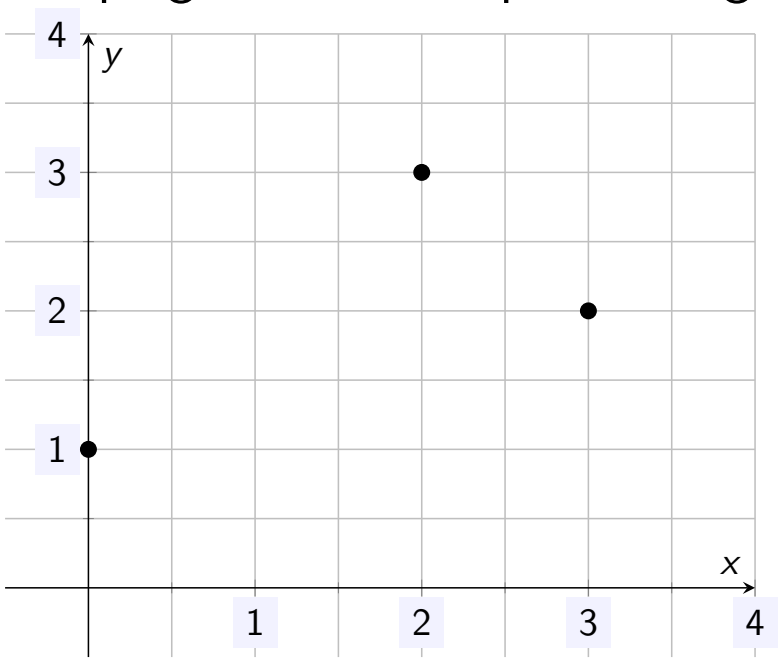
Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?
Hmm... let's start by plotting the points.



Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?
A solution looks like $f(x) = ax^2 + bx + c$ where a, b,, and c are constants.
Goal: Find values of the constants a, b, and c.
We plug in our three points to get three equations with three unknowns:



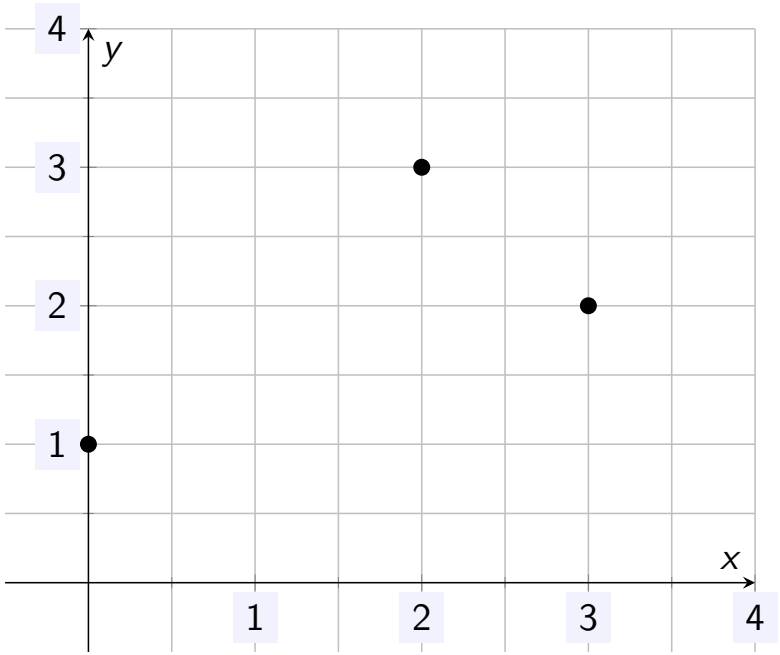
Point (0,1) $\rightarrow 1 = a(0)^2 + b(0) + c$

Point (2,3) $\rightarrow 3 = a(2)^2 + b(2) + c$

Point (3,2) $\rightarrow 2 = a(3)^2 + b(3) + c$

Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?
Simplifying the three equations:



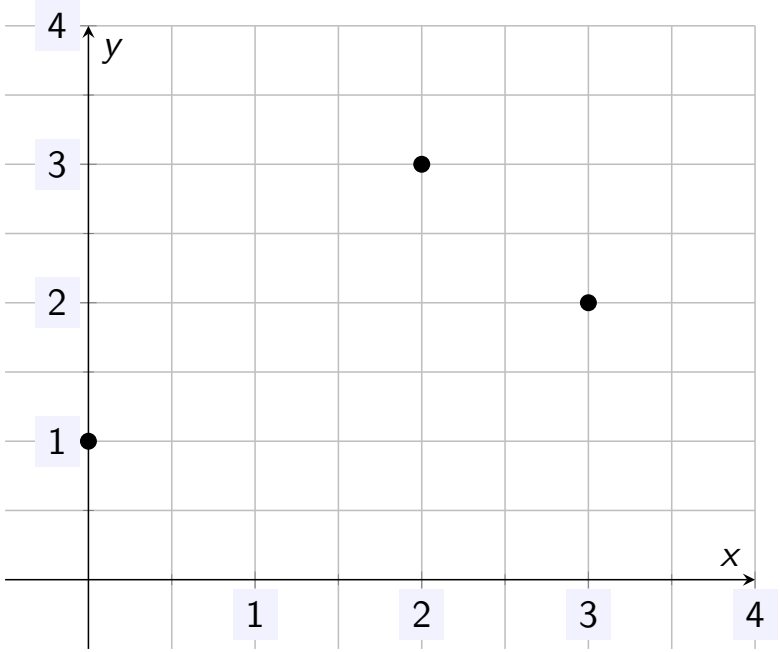
(1) $1 = c$

(2) $3 = 4a + 2b + c$

3) $2 = 9a + 3b + c$

Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?
Plug in $c = 1$ into the 2nd and 3rd equations:

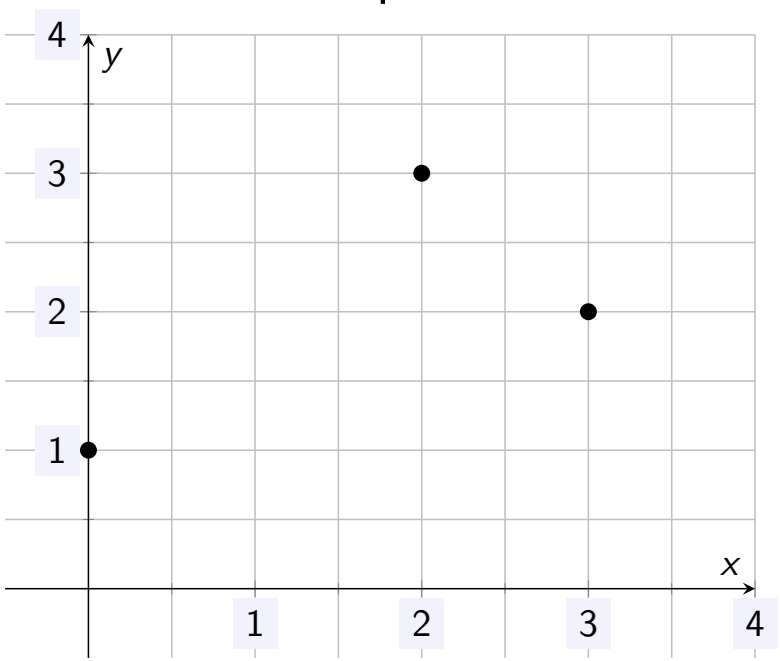


$$(2) \qquad 2 = 4a + 2b$$

$$(3) \qquad 1 = 9a + 3b$$

Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?
Solve the 2nd equation for b:



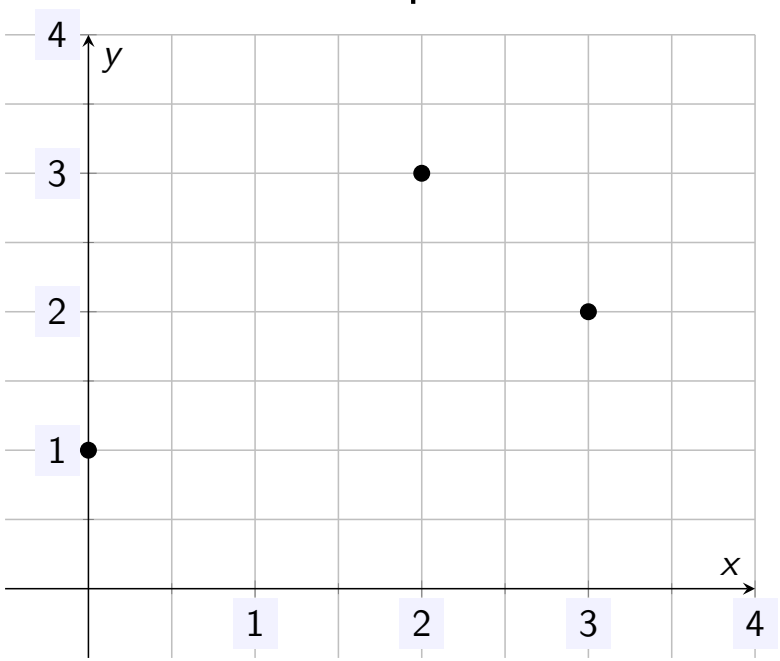
$$(2) \qquad 2 = 4a + 2b$$

$$(2) \qquad 1 = 2a + b$$

$$(2) \qquad b = 1 - 2a$$

Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?
Substitute this expression for b into Eqn 3, and solve for a:



$$(2) \quad b = 1 - 2a$$

$$(3) \quad 1 = 9a + 3b$$

$$(3) \quad 1 = 9a + 3(1 - 2a)$$

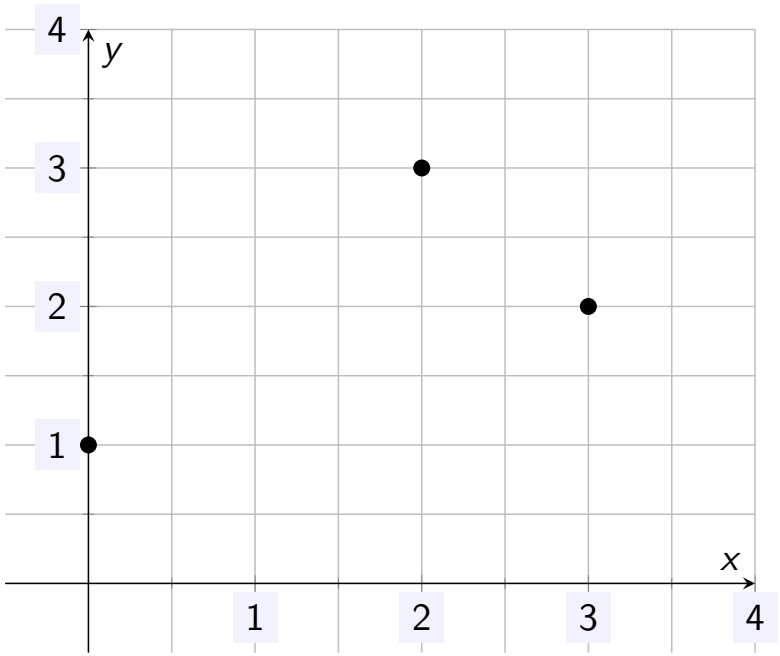
$$(3) \quad -2 = 3a$$

$$(3) \quad a = -2/3$$

Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?

Now that we know a, we solve for b:



$$(3) \qquad a = -2/3$$

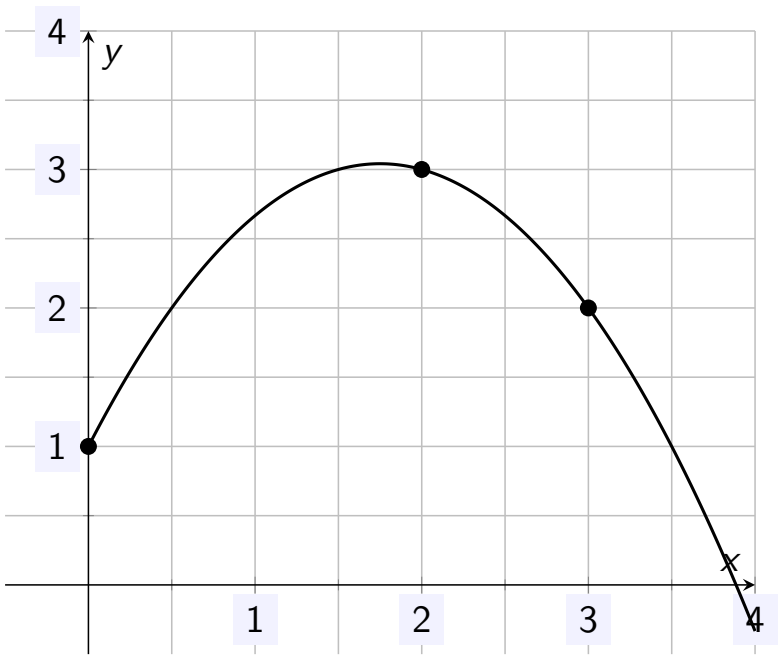
$$(2) \qquad b = 1 - 2a$$

$$(3) \qquad b = 7/3$$

Fitting a quadratic to three points

Question: Is there a quadratic function whose graph goes through the points (1,0), (2,3), and (3,1)?
Answer: Yes! It's

$$f(x) = -\frac{2}{3}x^2 + \frac{7}{3}x + 1.$$



Quadratic or not?

Are the 2nd differences constant (when generated from equally spaced x values)?

x	y
1	4
2	8
3	10
4	14
5	20

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Are the 2nd differences constant (when generated from equally spaced x values)?

x	y
1	4
2	8
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5	20

Yep! The slope between any two pairs of points is the same! y increases by 2 for each increase of x by 1!

Model?

x	y
1	2.0
2	4.0
3	6.0
4	7.9
5	10.1

Model?

x	y
1	2.0
2	4.0
3	6.0
4	7.9
5	10.1

As x increases by 1, y increases by about 2 each time - constant differences. So the relationship appears to be *linear*.

Model?

x	y
1	2.1
2	7.0
3	13.8
4	23.0
5	33.9

Model?

x	y
1	2.1
2	7.0
3	13.8
4	23.0
5	33.9

First differences are about 5, 7, 9, 11 - so not linear Second differences are 2, 2, 2 - constant! So *quadratic fit*.

Model?

x	y
1	2.3
2	1.1
3	0.4
4	0
5	0
6	0.4
7	1.2
8	2.5

Model?

x	y
1	2.3
2	1.1
3	0.4
4	0
5	0
6	0.4
7	1.2
8	2.5

1st diffs: -1.2, -0.7, -0.4, 0, 0.4, 0.8, 1.3

2nd diffs: 0.5, 0.3, 0.4, 0.4, 0.4, 0.5

Model?

x	y
1	2.3
2	1.1
3	0.4
4	0
5	0
6	0.4
7	1.2
8	2.5

1st diffs: -1.2, -0.7, -0.4, 0, 0.4, 0.8, 1.3

2nd diffs: 0.5, 0.3, 0.4, 0.4, 0.4, 0.5

Quadratic fit seems reasonable!

Model?

x	y
1	0.51
2	4.01
3	13.52
4	32.02
5	62.47
6	108.09
7	171.48
8	255.97

Model?

x	y
1	0.51
2	4.01
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4	32.02
5	62.47
6	108.09
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8	255.97

1st diffs: 3.50,9.51,18.50,30.45,45.62, 63.39, 84.49
2nd diffs: 6.01, 8.99, 11.95, 15.17, 17.77, 21.10
3rd diff: 2.98, 2.96, 3.22, 2.60, 3.33

Model?

x	y
1	0.51
2	4.01
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4	32.02
5	62.47
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1st diffs: 3.50,9.51,18.50,30.45,45.62, 63.39, 84.49
2nd diffs: 6.01, 8.99, 11.95, 15.17, 17.77, 21.10
3rd diff: 2.98, 2.96, 3.22, 2.60, 3.33
Hmm... 3rd differences nearly constant. *Cubic?*