1-sided test of Significance for p ("less than" variety)

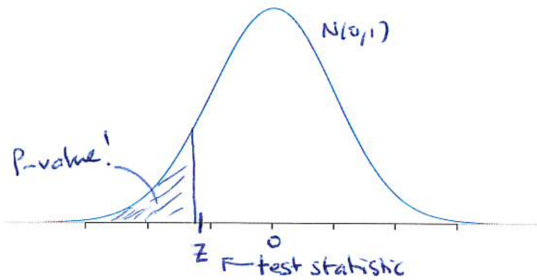
Hypotheses: $H_o : p = p_0$ vs $H_a : p < p_0$

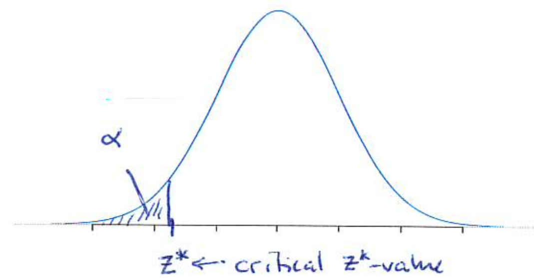
Summary Statistics: n = sample size; $\hat{p}$ = sample proportion.

Test statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

P-value: The proportion of the N(0,1) dist'n **less than** the test stat z. Found in R with `pnorm(`z`)`

Critical z-score associated to significance level α: The critical z-score is $z^* =$ `qnorm(`α`)`. That is, z^* is the z-score **below** which one finds area α under the bell curve. The Rejection Region (RR) is the set of z-scores **less than** z^*. If our test statistic is **less than** z^* we reject H_o in favor of H_a. Alternatively, we reject H_o in favor of H_a if the P-value of our test is less than α.





Example ("less than" alternative): Do fewer than 1/2 of *all* voters favor a particular issue? Conduct a test of significance at the $\alpha = .05$ level.

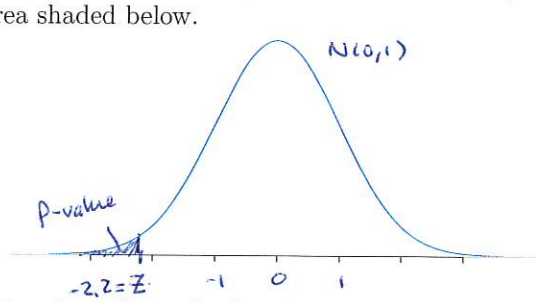
Hypotheses: $H_o : p = 0.5$ vs $H_a : p < 0.5$.

Sample: In an independent sample of 100 voters, 39 say "yes".
This means that $n = 100$, and $\hat{p} = 39/100 = 0.39$.

Test Statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \dfrac{0.39 - 0.50}{\sqrt{\frac{0.5\cdot(1-0.5)}{100}}} = \dfrac{-0.11}{0.05} = -2.2.$

P-value of the test: `pnorm(-2.2)`=.0139.

Interpretation of the P-value: If 1/2 of all voters favor the issue (in other words, if the null hypothesis is true), the probability of obtaining a sample proportion less than or equal to $\hat{p} = .39$ in a sample of 100 voters, is about 0.0139, the area shaded below.



Conclusion based on the P-value: Since the P-value is less than $\alpha = .05$, we have enough evidence to reject H_o in favor of the alternative, and we conclude from the sample and our analysis that fewer than 1/2 of *all* voters favor this issue.

Conclusion based on critical z^*-value Testing at the $\alpha = .05$ level with a "less than" alternative hypothesis, the critical z^*-value is $z^* =$ `qnorm(.05)` $= -1.645$. Since our test statistic of $z = -2.2$ is less than $z^* = -1.645$ we reject H_o in favor of H_a at the 5% level.

1-sided test of Significance for p ("greater than" variety)

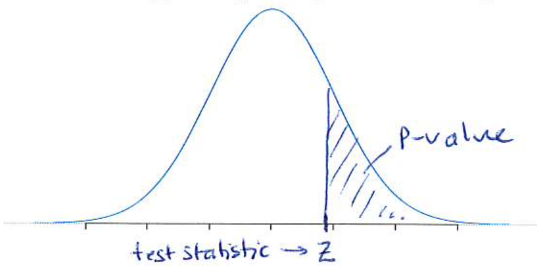
Hypotheses: $H_o : p = p_0$ vs $H_a : p > p_0$

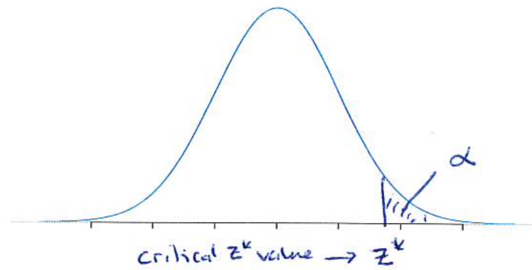
Summary Statistics: n = sample size; $\hat{p}$ = sample proportion.

Test statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

P-value: The proportion of the $N(0,1)$ dist'n **greater than** the test stat z. Found in R with $1-\texttt{pnorm}(z)$

Critical z-score associated to significance level α: The critical z-score is $z^* = \texttt{qnorm}(1-\alpha)$. That is, z^* is the z-score **above** which one finds area α under the bell curve. The Rejection Region (RR) is the set of z-scores **greater than** z^*. If our test statistic is **greater than** z^* we reject H_o in favor of H_a. Alternatively, we reject H_o in favor of H_a if the P-value of our test is less than α.





Example ("greater than" alternative: Do more than 30% of all high school students participate in music or theatre programs before graduation? Conduct a test of significance at the $\alpha = .01$ level.

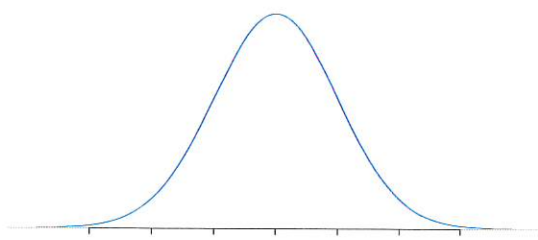
Hypotheses: $H_o : p = 0.3$ vs $H_a : p > 0.3$.

Sample: In an independent sample of 250 students, 80 say "yes".
This means that $n = 250$, and $\hat{p} = 80/250 = 0.32$.

Test Statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \dfrac{0.32 - 0.30}{\sqrt{\frac{0.3\cdot(1-0.3)}{250}}} = \dfrac{0.02}{0.05} = 0.69.$

P-value of the test: `1-pnorm(0.69)`=.2451.

Interpretation of the P-value: If 30% of all high school students participate in music or theatre before graduation, the probability of obtaining a sample proportion greater than or equal to $\hat{p} = .32$ in a sample of 250 students, is about 0.2451 the area shaded below.



Conclusion based on the P-value: Since the P-value is not less than $\alpha = .01$, we do not have enough evidence to reject H_o in favor of the alternative. These data do not provide statistically significant evidence that more than 30% of high school students participate in music or theatre programs prior to graduation.

Conclusion based on critical z^***-value** Testing at the $\alpha = .01$ level with a "greater than" alternative hypothesis, the critical z^*-value is $z^* = \texttt{qnorm}(.99) = 2.326$. Since our test statistic of $z = 0.69$ is not greater than $z^* = 2.326$ we fail to reject H_o in favor of H_a at the 1% level.

2-sided test of Significance for p ("not equal to" variety)

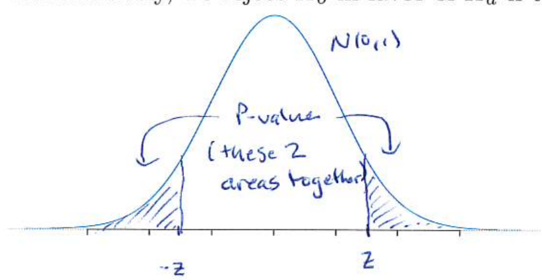
Hypotheses: $H_o : p = p_0$ vs $H_a : p \neq p_0$

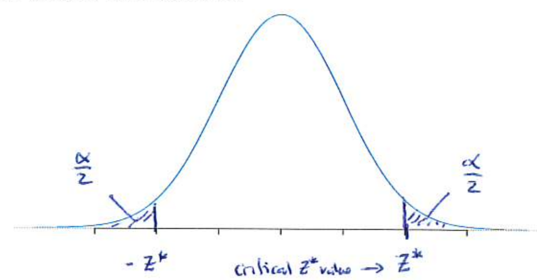
Summary Statistics: n = sample size; $\hat{p}$ = sample proportion.

Test statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

P-value: The proportion of the $N(0,1)$ dist'n further away from 0 than the test stat z. Found in R with $2 \cdot$ `pnorm(-abs(z))`.

Critical z-score associated to significance level α: The critical z-score is $z^* = |\texttt{qnorm}(\alpha/2)|$. That is, z^* is the z-score above which one finds area $\alpha/2$ under the bell curve. Then, a total area of α is found above z^* and below $-z^*$. If our test statistic z is such that $|z| > z^*$ then we reject H_o in favor of H_a. Alternatively, we reject H_o in favor of H_a if the P-value of our test is less than α.





Example ("not equal to" alternative: Do 2/3 of Americans support a certain constitutional amendment? Test at the $\alpha = .05$ level.

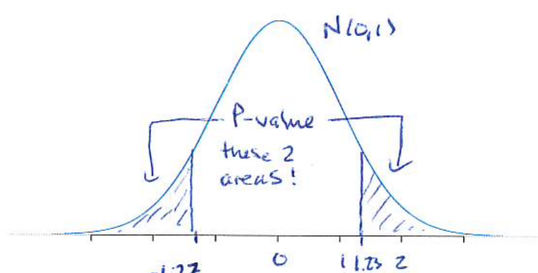
Hypotheses: We choose the two-sided alternative: $H_o : p = 2/3$ vs $H_a : p \neq 2/3$.

Sample: In an independent sample of 1000 Americans, 685 say "yes".
This means that $n = 1000$, and $\hat{p} = 0.685$.

Test Statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \dfrac{0.62 - 2/3}{\sqrt{\frac{2/3 \cdot (1-2/3)}{1000}}} = \dfrac{0.02}{0.05} = 1.23.$

P-value of the test: $2 \cdot$`pnorm(-1.23)`$=.2187$.

Interpretation of the P-value: If 2/3 of all Americans supported the amendment, the probability of obtaining a sample proportion further away from 2/3 than $\hat{p} = .685$ in a sample of size 1000 is about 0.2187 the area shaded below.



Conclusion based on the P-value: Since the P-value is not less than $\alpha = .05$, we do not have enough evidence to reject H_o in favor of the alternative. These data do not provide statistically significant evidence that the percentage of Americans supporting this amendment is different than 2/3.

Conclusion based on critical z^***-value** Testing at the $\alpha = .05$ level with a "not equal to" alternative hypothesis, the critical z^*-value is $z^* = |\texttt{qnorm}(.025)| = 1.96$. Since the absolute value of our test statistic of $z = 1.23$ is not greater than $z^* = 1.96$ we fail to reject H_o in favor of H_a at the 5% level.