MATH 140 **Name** ________________________ **Date:**

Worksheet: The Central Limit Theorem for Sample Proportions

The Central Limit Theorem for Sample Proportions

When we collect a sufficiently large sample of n independent observations from a population with population proportion of p, the sampling distribution of $\hat{p}$ will be nearly normal:

$$N\left(\mu = p, \sigma = \sqrt{\frac{p(1-p)}{n}}\right).$$

The sample size is typically considered sufficiently large when the success-failure condition has been met: $np \geq 10$ and $n(1-p) \geq 10$, but n does not exceed 10% of the population.

1. Suppose that a widget manufacturer finds that one-third of all widgets coming off its assembly line are defective.

   (a) Is the number one-third a parameter or a statistic? Explain.

   (b) Let $\hat{p}$ denote the sample proportion of defective widgets in a random sample of $n = 80$ widgets. According to the CLT, what is the approximate sampling distribution for $\hat{p}$?

   (c) Use your answer to (b) to find the probability that in a sample of 80 widgets we see a sample proportion of defective widgets less than 0.25.

   (d) Now suppose engineers have designed some modifications for the manufacturing process that they believe will decrease this proportion of defective widgets, and they are excited to test whether this is the case! They sample and examine 80 widgets and find 20 defective. If, in fact, their modifications had no effect on the quality of the widgets produced, what is the probability that they happen to obtain a sample with 20 or fewer defective widgets?

   (e) Based on your answer to (d), do you think this sample provides good evidence that their modifications are helpful, widget-wise?