MATH 140 **Name** _____ **Date:**

Worksheet: The Central Limit Theorem for Sample Means

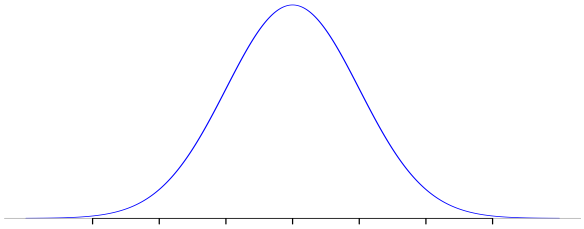
The Central Limit Theorem for Sample Means

When we collect a sufficiently large sample of n independent observations from a population with mean μ and standard deviation σ, the sampling distribution of $\overline{x}$ will be nearly normal with

$$\text{Mean} = \mu \text{ and Standard Error (SE)} = \frac{\sigma}{\sqrt{n}}.$$

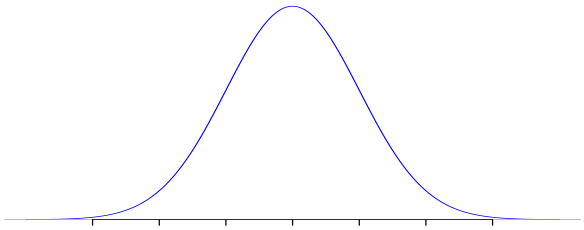
1. The actual weight of a certain candy bar, whose advertised weight is 2 ounces, varies according to a normal distribution with mean $\mu = 2$ ounces and standard deviation $\sigma = 0.04$ ounces.

   (a) Label the tick marks on the bell curve below so that it represents the distribution of weights of this type of candy bar.



   (b) What is the probability that an individual candy bar will weigh between 1.98 and 2.02 oz? Sketch the probability of interest as an area in the density curve above, convert the question to z-scores, and use a table or R to find your answer.

   (c) Suppose you plan to gather a simple random sample of 16 candy bars and calculate the sample mean weight, $\overline{x}$. What does the Central Limit Theorem say about the sampling distribution for $\bar{x}$? Label the tick marks on the bell curve below to represent the sampling distribution of $\bar{x}$ in this problem.



   (d) Shade in the region on the bell curve above that corresponds to the probability that the sample mean weight of these 16 candy bars will fall between 1.98 and 2.02 ounces. Determine this probability by converting to z-scores and using R.

2. Suppose we draw a simple random sample of size $n = 36$ from a population having $\mu = 55$ and $\sigma = 7.5$. Use the CLT to estimate the probability that the sample mean $\bar{X}$ is within 2 units of 55. That is, find the probability $P(53 < \bar{X} < 57)$.

3. Suppose we draw a simple random sample of size $n = 100$ from a population having $\mu = 55$ and $\sigma = 7.5$. Use the CLT to estimate the probability that the sample mean $\bar{X}$ is within 2 units of 55.

4. The weight of the eggs produced by a certain breed of hen is normally distributed with mean 65 grams and standard deviation 5 grams.

   (a) Determine the probability that a single randomly chosen egg weighs more than 72 grams.

   (b) Treat a carton of 12 eggs as a SRS of size 12 from the population. What is the probability that the weight of a carton falls between 750 and 825 grams? Hint: Convert this question to a question about the sample mean of 12 eggs.