Proofs are Programs

Basics - A small introduction to functional Programming and simple Proofs

Imperative vs. Functional Programming

```
int n = …  ;
int res = 1;
for (int i = 1, i < n, i++){
  res = res * i;
}
```

$$factorial \in \mathbb{N} \to \mathbb{N}$$

$$factorial(n) = \begin{cases} 1 & n = 0 \\ n * factorial(n-1) & n > 0 \end{cases}$$

- Program = sequence of instructions
- Stateful (variables)

Imperative vs. Functional Programming

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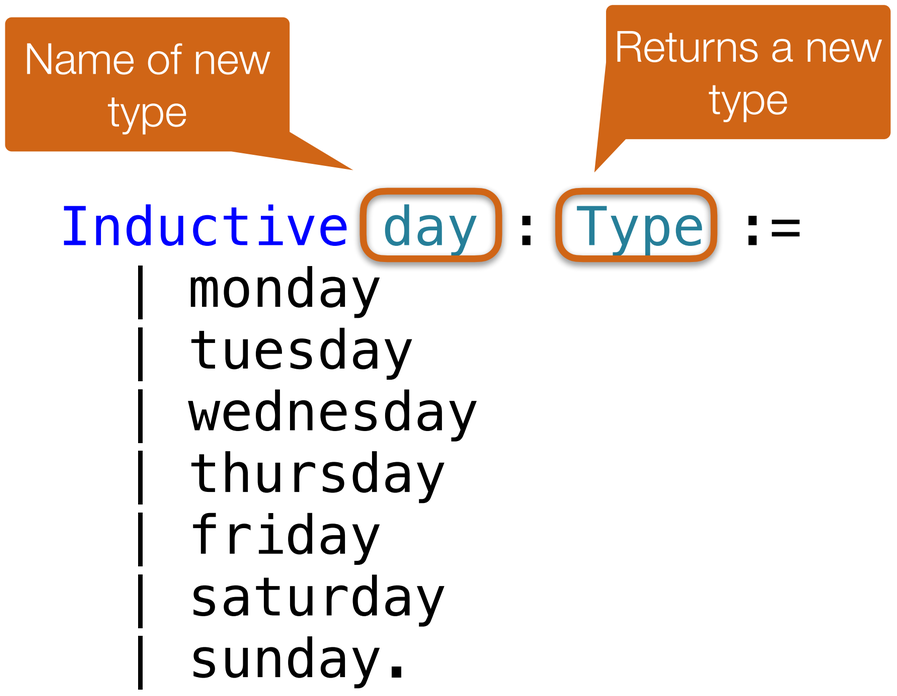
- Function = Mapping from input to output values
- No state
- No side effects

Enumerated Types

```
Inductive day : Type :=
  | monday
  | tuesday
  | wednesday
  | thursday
  | friday
  | saturday
  | sunday.
```

Enumerated Types



Enumerated Types

Name of new type

Returns a new type

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Inductive day : Type :=
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```

```
Definition next_weekday (d:day) : day :=
  match d with
  | monday    => tuesday
  | tuesday   => wednesday
  | wednesday => thursday
  | thursday  => friday
  | friday    => monday
  | saturday  => monday
  | sunday    => monday
  end.
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Evaluating Functions

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Compute (next_weekday friday).
(* ==> monday : day *)
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Evaluates a function

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Evaluates a function

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Compute (next_weekday friday).
(* ==> monday : day *)
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```
Compute (next_weekday (next_weekday saturday)).
(* ==> tuesday : day *)
```

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```

```
Definition andb (b1:bool) (b2:bool) : bool :=
  match b1 with
  | true => b2
  | false => false
  end.
```

Cascading Function Definitions

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Definition orb (b1:bool) (b2:bool) : bool :=
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```

```
Compute (orb true false).
(* ==> true : bool *)
```

```
Compute (orb false false).
(* ==> false : bool *)
```

Notations (for convenience)

```
Notation "x && y" := (andb x y).

Notation "x || y" := (orb x y).
```

```
Compute (false || false || true).
(* ==> true: bool *)
```

Conditionals

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  | black
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Definition invert (x: bw) : bw :=
  if x then white
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  else black.
```

```
Compute (invert black).
(* ==> white: bw *)
```

```
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(* ==> black: bw *)
```