Proofs are Programs

Summer 2023 @ Ruhr Uni Bochum

Lecturers: Cătălin Hriţcu and Clara Schneidewind

TAs: Sebastian Holler and Maxi Wuttke

Max Planck Institute for Security and Privacy (MPI-SP)

This course in a nutshell

1. Logic and proofs
2. Functional programming
3. Program verification

- **Using the Coq proof assistant**
- **Curry-Howard correspondence**
  - –proofs = programs
  - –bridge between logic and computer science

["Le coq mécanisé" picture by Lilia Anisimova]

Logics and proofs

Q: How do we know something is true?

A: We prove it

Q: How do we know that we have a **proof**?

A: We need to know what it means for something to be a proof.
First cut: A proof is a "logical" sequence of arguments,
starting from some initial assumptions

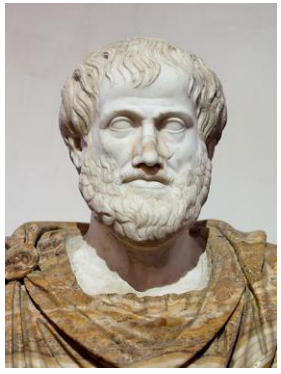
Q: How do we agree on what is a **valid** sequence of arguments?
Can any sequence be a proof? E.g.

All humans are mortal

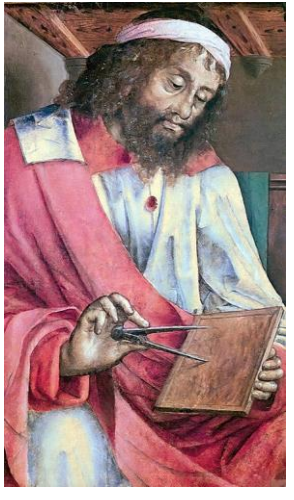
All Greeks are human

Therefore I am a Greek!

A: No, no, no! We need to think harder about **valid** ways of reasoning...



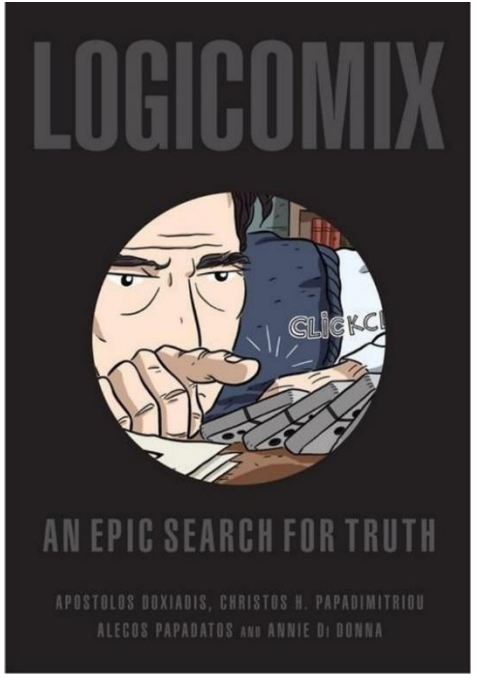
Aristotle
384 – 322 BC



Euclid
~300 BC

The story of logics is quite fascinating

- **David Hilbert** (1862 – 1943)
- **Gottlob Frege** (1848-1925)
  - first-order logic (∀x. P x /\ R x x)
  - derive the laws of arithmetic from first principles (Hilbert's 2nd problem)
- **Bertrand Russell** (1872 - 1970)
  - paradox, inconsistency in Frege's logical system, tried to fix it
  - Principia Mathematica
- **Kurt Gödel** (1906 - 1978)
  - incompleteness theorems



- **Gerhard Gentzen** (1909-1945) -- consistency of arithmetic
- **Alonzo Church** (1903 - 1995) -- lambda calculus, simple theory of types
- **Alan Turing** (1912 - 1954) -- undecidability of arithmetic, halting problem
- ...

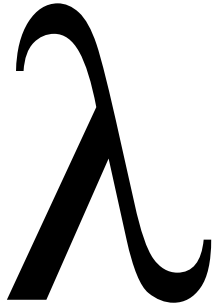
Logics and proofs

- Foundation of **mathematics** and **computer science**
  - **formal proofs** with respect to **valid inference steps/rules**
- **This course: typed constructive higher-order logic**
  - **higher-order**
    - can quantify not only over individuals (∀x:nat. P x)
    - but also over propositions (∀P:Prop. P), predicates (∀Q:nat->Prop. Q x), relations (∀R:nat->nat->Prop. R x y), functions (∀f:nat->bool, ∀g:(nat->bool)->bool), ...
  - **constructive**, aka **intuitionistic logic**:
    - a proposition is valid if one can construct a proof
    - philosophically rejects excluded middle (P ∨ ¬P, classical logic)
  - **typed** (dependently typed)

Logics and computer science

- **Logics and CS greatly influenced on each other**, e.g.:
  - **automated theorem provers** (e.g., SAT and SMT solvers)
  - **model checkers** (the course right after this one!)
  - **proof assistants**: Coq, Isabelle, HOL family, F*, ACL2, etc.
    - interactively constructed, machine-checked proofs
    - addictive, gamification of proofs
- This course: **Coq proof assistant**
  - developed at Inria since 1983 (in OCaml)
  - Curry-Howard: proofs = typed purely functional programs

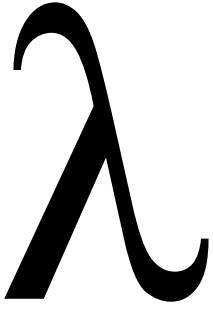
Functional programming



- **Try to write computations as pure functions**
  - **without side-effects**, such as mutating the heap
    - sorting a list in place (imperative) vs producing a new list (functional)
  - **Coq is purely functional = zero side-effects**
    - all computations are mathematical functions (in particular terminating)
  - **Functional programming languages** like OCaml, Haskell, ...
    - try to reduce and/or control side-effects
    - make it easy to write pure functions

Functional programming in practice

- **Functional languages have some practical success**
  - **Facebook** (OCaml, Haskell), **Microsoft** (F# and F*), **Twitter** (Scala)
  - **Financial industry**: Jane Street (OCaml), various banks (Haskell, ...)
  - **Blockchains**: Cardano (Haskell, Rust), Tezos (OCaml), ...
- **Not yet fully mainstream, but ...**
  - **Functional programmers earn more** (Stack Overflow survey)
  - **Many ideas already adopted by mainstream languages**: lambdas, generics in Java/C#, Google's Map-Reduce, ...
  - **Makes program verification and informal reasoning easier**

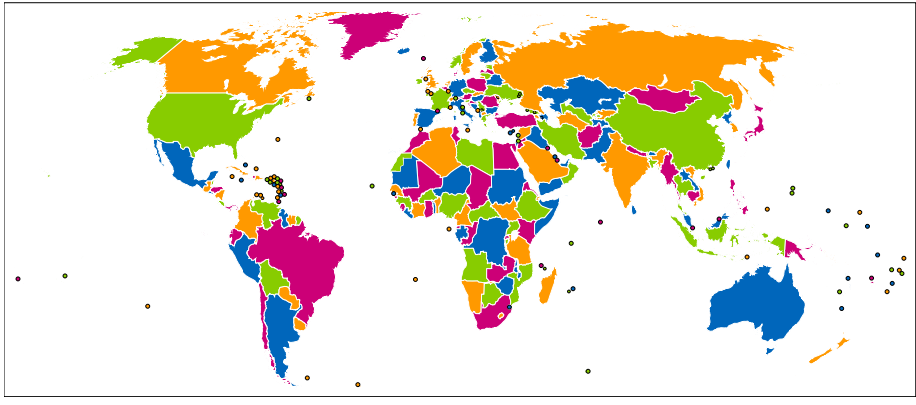


Program verification in proof assistants

Proving mathematically that a **program** satisfies a **specification**

- **verification of smart contracts** (Coq, F*, ...) -- Clara working on this hot topic
- **the CompCert C compiler** (Coq) -- Catalin working on a more secure version
- **the seL4 OS microkernel** (Isabelle/HOL), **the CertiKOS hypervisor** (Coq)
- **EverCrypt cryptographic library** (F*) -- shipping in Firefox and the Linux kernel
- **EverParse3D verified binary parsers** (F*) -- shipping in Windows kernel
  - Catalin working on design of F* proof assistant ("Coq + SMT automation")
- **Libjade high-assurance post-quantum crypto library** (EasyCrypt)
  - Peter Schwabe and Gilles Barthe from MPI-SP

Machine-checked proofs of math theorems



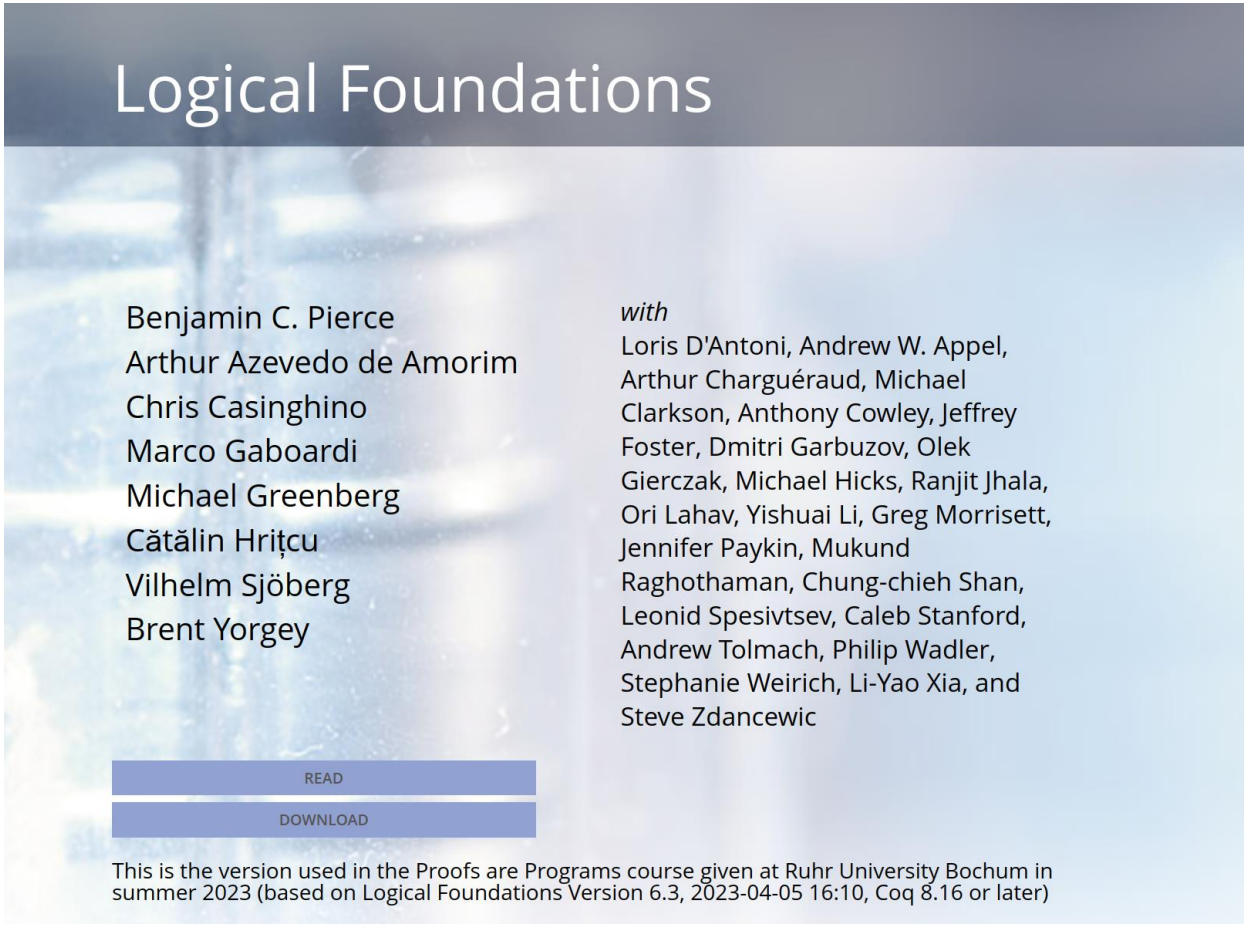
[World map from Wikipedia]

- **The 4-color theorem**
  - proved in 1976, simplified in 1996
  - proofs rely on a computer
  - infeasible for a human to check
  - initially not accepted by all mathematicians
  - mechanized in Coq by Georges Gonthier in 2005
- **Odd order theorem (Feit-Thompson)**
  - Coq proof from 2012 introduced the Mathematical Components library
- **Construction of perfectoid spaces** (Peter Scholze, Lean)
  - Mathlib math library for Lean has over 100,000 theorems, over 1,000,000 lines of code, and an active community including many mathematicians

This course

- **Write purely functional programs in Coq**
  - natural numbers, lists, maps, trees, program syntax
- **Verify these programs by proving theorems about them**
  - case analysis, induction, inversion, proof automation, ...
- **Curry-Howard correspondence**
  - proofs = typed purely functional programs

- **Advice: focus, ask questions, interact**



Textbook written in Coq; our version at:

https://mpi-sp-pap-2023.github.io/book

Lecture logistics

- 12 lectures: first 6 by Clara, last 6 by Catalin
- Public holidays, so no lecture on 18 May and 8 June
- Vacation 29 May to 2 June, so no lecture, no tutorials
- ~~Starting next week only in presence~~
  - Since some participants asked we decided to still record and stream the following lectures (announcement will follow)

Exercises

- **This is a very hands on course**
  - Coq turns proofs & logic into programming, and it's fun!
- **New exercise sheet will be released after most courses**
  - 1(-2) Coq file(s) with holes you have to fill (basically 1-2 book chapters)
  - so there will be 8-10 exercise sheets in total
- You have to **turn in your solution before next course**
  - up to Wednesday at 23:59
  - Easter exception: this week's exercise sheet is due in 2 weeks
- **We'll be using Moodle** (probably for everything)
- **Exercises count for 40% of final grade** (+5% in bonus points)
  - not required to do the optional exercises; they don't count for grade
- **Exercises are individual, please don't share your solutions in any way!**
- **Exercises highly recommended even if you're not taking this for credit**

Tutorials: Q&A about the exercise sheets

- **TAs:** Sebastian Holler and Maxi Wuttke
- **Monday ~~12:00-14:00~~** and **Tuesday ~~12:00-14:00~~**
  - **new times: 12:15-13:45**
- You can come to any/both and ask questions
- You can also work on your own during tutorials
  - and ask questions as they arise
- If you manage to solve all the exercises and prefer to not come, no problem, you are not forced to come
  - can sometimes still get good advice though
  - can also ask about old assignments (but solutions anyway on Moodle)
- Seems okay to bring your lunch / arrive late / leave early

Exams

- **Midterm exam** (optional)
  - practice for the final exam
  - also written, on paper
  - duration: 60 minutes
  - bonus points: up to 10%
  - date: 23 May, ~~13:00-14:00~~
    - **new time: 12:45-13:45**

- **Final exam**
  - written, on paper
    - so we will also teach you how to write down proofs informally
  - duration: 120 minutes
  - 60% of the grade
  - date: 3 August
  - re-exam: 19 September

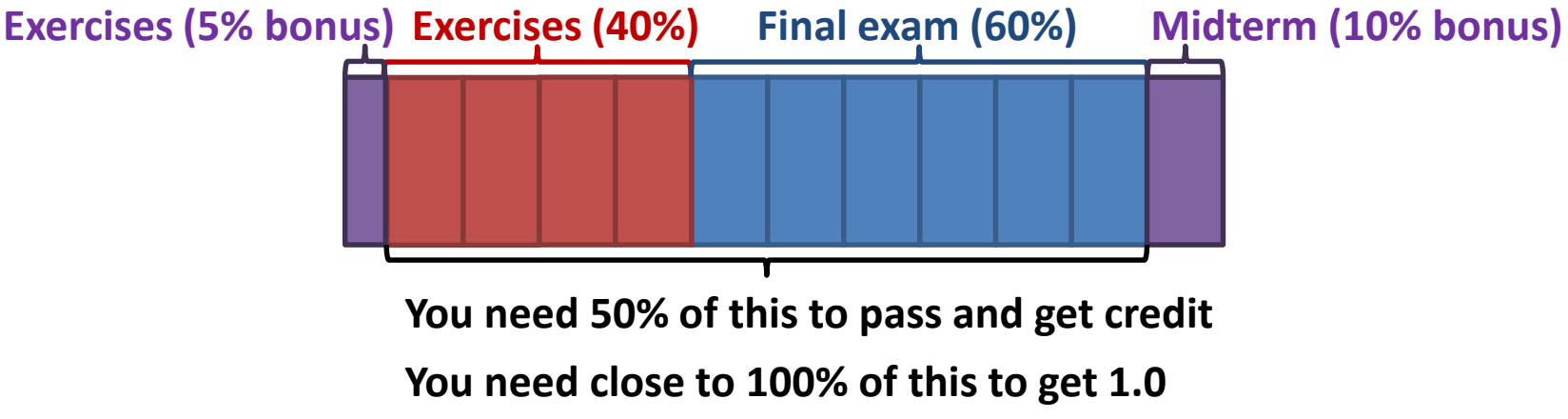
Final grade

CASA PhD students

– attendance requirement is to receive 50% of the exercises score

You can get credit if you study

- Computer Science BSc (soon MSc too)
- IT Security BSc and MSc
- Mathematics MSc



Already considering follow up course (next semester?)

Foundations of Programming Languages, Verification, and Security