

# Supplementary material for “Detecting Patterns of Attacks to Network Security in Urban Air Mobility with Answer Set Programming”

**Abstract.** This document provides the complete ASP encoding for the contrast sequential pattern mining task along with the experimental results in tabular form.

## 1 ASP encoding

### 1.1 CSPM

The full ASP encoding for CSPM is reported below in Listing 1. It encompasses two phases; The first aims at the discovery of frequent sequential patterns, the latter checks which among the discovered patterns are of contrast with some class.

**Listing 1.** ASP encoding for finding contrast sequential patterns

```
1 item(I) :- seq(_, _, I).
2
3 % FREQUENT SEQUENTIAL PATTERN
4 %% sequential pattern generation
5 patpos(1).
6 { patpos(X+1) } :- patpos(X), X < maxlen.
7 patlen(L) :- patpos(L), not patpos(L+1).
8 :- patlen(L), L < minlen.
9
10 1 {pat(X, I): item(I)} 1 :- patpos(X).
11
12 %% pattern embeddings
13 occ(T, 1, P) :- seq(T, P, I), pat(1, I).
14 occ(T, L, P) :- occ(T, L, P-1), seq(T, P, _).
15 occ(T, L, P) :- occ(T, L-1, P-1), seq(T, P, C),
16 pat(L, C).
17
18 %% frequency constraint
19 seqlen(T, L) :- seq(T, L, _), not seq(T, L+1, _).
20 support(T) :- occ(T, L, LS), patlen(L),
21 seqlen(T, LS).
22 :- { support(T) } < minsup.
23
24 %% how many sequences contains pattern
25 len_support(N) :- N = #count{T : support(T)}.
26
27 % CONTRAST SEQUENTIAL PATTERN
28 % D1 cardinality
29 card(Card, c1) :- Card = #count{T : c1(T, c1)}.
30
31 % D2 cardinality
32 card(Card, c2) :- Card = #count{T : c1(T, c2)}.
33
34 % C1 support
35 sup(Sup, c1) :- Sup = #count{T : support(T),
36 seq(T, _, _), c1(T, c1)}.
37
38 % C2 support
39 sup(Sup, c2) :- Sup = #count{T : support(T),
40 seq(T, _, _), c1(T, c2)}.
41
42 % GR_c1(X):
43 % inf if sup(X, c2)=0 and sup(X, c1)!=0
```

```
44 gr_rate("inf", c1) :- sup(Sup1, c1),
45 Sup1 != 0, sup(0, c2).
46
47 % (sup(X, c1)/|D1|)/(sup(X, c2)/|D2|)
48 % otherwise
49 gr_rate(@gr(Sup1, Card1, Sup2, Card2), c1) :-
50 sup(Sup1, c1), card(Card1, c1), sup(Sup2, c2),
51 card(Card2, c2), Sup1 > 0, Sup2 > 0.
52
53 % GR_c2(X)
54 % "inf" if sup(X, c1)=0 and sup(X, c1)!=0
55 gr_rate(inf, c2) :- sup(Sup2, c2),
56 Sup2 != 0, sup(0, c1)
57
58 % (sup(X, c2)/|D2|)/(sup(X, c1)/|D1|)
59 % otherwise
60 gr_rate(@gr(Sup2, Card2, Sup1, Card1), c2) :-
61 sup(Sup1, c1), card(Card1, c1), sup(Sup2, c2),
62 card(Card2, c2), Sup1 > 0, Sup2 > 0.
63
64 contr_pat(yes, Class) :- gr_rate(inf, Class).
65 contr_pat(@csp(Cr, mincr), Class) :- gr_rate(Cr,
66 Class), Cr != inf
67
68 % delete a pattern if it does not contrast
69 % with any class
70 :- contr_pat(no, c1), contr_pat(no, c2).
```

### 1.2 CSPM constraints

Below the constraints described in the article to be replaced in the pattern embedding section of the ASP encoding for CSPM reported in Listing 1 (Lines 12-16).

**Listing 2.** ASP encoding of the span constraint.

```
1 occS(T,1,P,P) :- seq(T,P,I), pat(1,I).
2 occS(T,L,P,IP) :- occS(T,L,P-1,IP), seq(T,P,_).
3 occS(T,L,P,IP) :- occS(T,L-1,P-1,IP), seq(T,P,C),
4 pat(L,C), P-IP+1>=minspan, P-IP+1<=maxspan.
```

**Listing 3.** ASP encoding of the gap constraint.

```
1 occG(T,1,P) :- seq(T,P,I), pat(1,I).
2 occG(T,L,P) :- occG(T,L,P-1), seq(T,P,_).
3 occG(T,L,P) :- occG(T,L-1,P-1), seq(T,P,C), pat(L,C),
4 pat(L-1,C1), seq(T,P2,C1), P-P2-1<=mingap,
5 P-P2-1<=maxgap.
```

**captioncaption**

```
1 occSG(T,1,P,P) :- seq(T,P,I), pat(1,I).
2 occSG(T,L,P,IP) :- occSG(T,L,P-1,IP), seq(T,P,_).
3 occSG(T,L,P,IP) :- occSG(T,L-1,P-1,IP), seq(T,P,C),
4 pat(L,C), pat(L-1,C1), seq(T,P2,C1),
5 P-P2-1>=mingap, P-P2-1<=maxgap,
6 P-IP+1>=minspan, P-IP+1<=maxspan.
```

minlen=2 & maxlen=2				
minsup	mincr	#pat	time	memory
10%	1	162	22.58	415.91
20%	1	97	19.81	415.91
30%	1	72	21.34	415.91
40%	1	63	15.44	415.91
50%	1	60	14.78	415.91

minlen=2 & maxlen=6				
minsup	mincr	#pat	time	memory
10%	1	50,596	T.O	667.14
20%	1	83,962	T.O	594.66
30%	1	63,152	T.O	598.47
40%	1	34,853	2,372.92	552.66
50%	1	29,080	1,785.24	475.62

**Table 1.** Number of patterns, runtime (seconds), grounder time (seconds), solver time (seconds) and memory consumption (MB) on Auth\_failure\_1000. *minsup* was varied and *mincr* was left unchanged. T.O. means time-out

minlen=2 & maxlen=2				
mincr	minsup	#pat	time	memory
1	30%	72	21.49	415.91
2	30%	19	35.91	416.06
3	30%	19	2000.52	452.64
4	30%	19	310.45	417.48
6	30%	19	58.097	422.5

minlen=2 & maxlen=6				
mincr	minsup	#pat	time	memory
1	30%	63,142	T.O	599.25
2	30%	19,197	T.O	751.66
3	30%	10,826	T.O	997.11
4	30%	11,067	T.O	657.53
5	30%	3,289	T.O	563.07

**Table 2.** Number of patterns, runtime (seconds), grounder time (seconds), solver time (seconds) and memory consumption (MB) on Auth\_failure\_1000. *mincr* was varied and *minsup* was left unchanged. T.O. means time-out

(a)				
minsup	mincr	#pat	time	memory
10%	1	121	41.09	508.9
20%	1	64	46.92	508.9
30%	1	53	112.49	508.99
40%	1	44	44.73	508.89
50%	1	39	33.35	508.89

(b)				
mincr	minsup	#pat	time	memory
1	30%	53	112.60	508.89
2	30%	23	1,247.88	560.62
3	30%	23	461.04	541.97
4	30%	21	T.O	660.19
5	30%	21	T.O	628.59

**Table 3.** Number of patterns, runtime (seconds), and memory consumption (MB) on Auth\_failure\_1000 with *mingap*=0, *maxgap*=0, *minlen*=2, and *maxlen*=6. (a) by varying *minsup* and (b) by varying *mincr*. T.O. means time-out

(a)				
minsup	mincr	#pat	time	memory
10%	1	6,204	T.O	1,750.6
20%	1	5,630	2,219.28	2,291.09
30%	1	2,556	1,985.18	1,696.91
40%	1	2,126	1,754.92	2,122.04
50%	1	1,385	1,105.34	1,746.43

(b)				
mincr	minsup	#pat	time	memory
1	30%	2,256	1,993.68	1,696.91
2	30%	357	T.O	1,487.53
3	30%	488	T.O	1,543.18
4	30%	322	T.O	1,465.94
5	30%	297	T.O	1,464.66

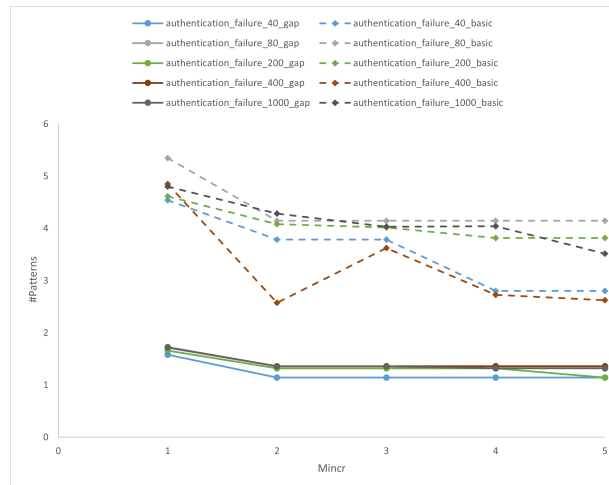
**Table 4.** Number of patterns, runtime (seconds), and memory consumption (MB) on Auth\_failure\_1000 with  $minspan=1$ ,  $maxspan=10$ ,  $minlen=2$ , and  $maxlen=6$ . (a) by varying minsup and (b) by varying mincr. T.O. means time-out

(a)				
minsup	mincr	#pat	time	memory
10%	1	98	137.85	1565.71
20%	1	52	89.58	1565.71
30%	1	44	85.686	1563.64
40%	1	44	91.66	1542.93
50%	1	39	87.28	1560.89

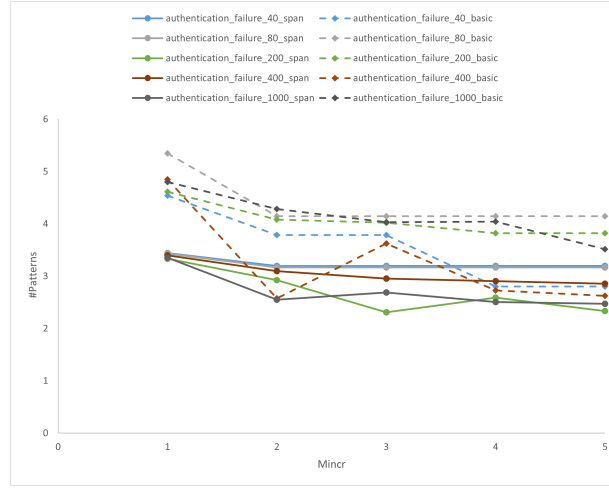
  

(b)				
mincr	minsup	#pat	time	memory
1	30%	44	81.88	1552.45
2	30%	13	T.O	1544.85
3	30%	14	T.O	1566.44
4	30%	14	T.O	1566.79
5	30%	12	T.O	1567.12

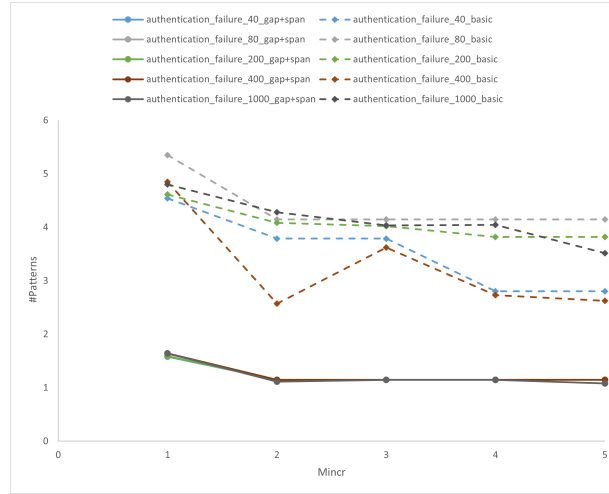
**Table 5.** Number of patterns, runtime (seconds), and memory consumption (MB) on Auth\_failure\_1000 with  $mingap=0$ ,  $maxgap=0$ ,  $minspan=1$ ,  $maxspan=10$ ,  $minlen=2$ , and  $maxlen=6$ . (a) by varying minsup and (b) by varying mincr. T.O. means time-out



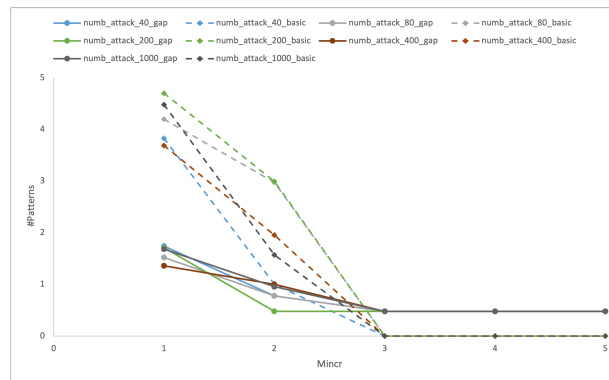
**Figure 1.** Comparison between basic encoding and gap constraint on extracted patterns  $mingap=0$ ,  $maxgap=0$ ,  $minlen=2$ , and  $maxlen=6$ . The number of patterns is in logarithmic scale to facilitate viewing and comparison



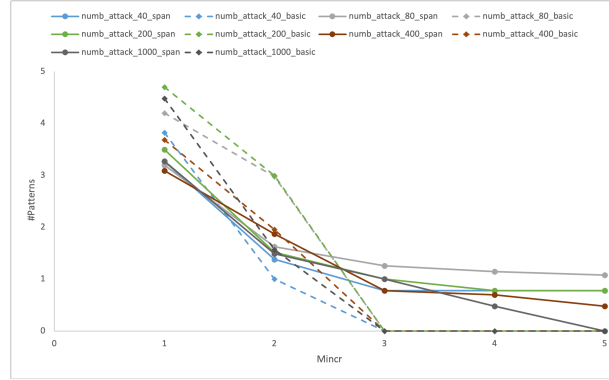
**Figure 2.** Comparison between basic encoding and span constraint on extracted patterns with  $minspan=1$ ,  $maxspan=10$ ,  $minlen=2$ , and  $maxlen=6$ . The number of patterns is in logarithmic scale to facilitate viewing and comparison



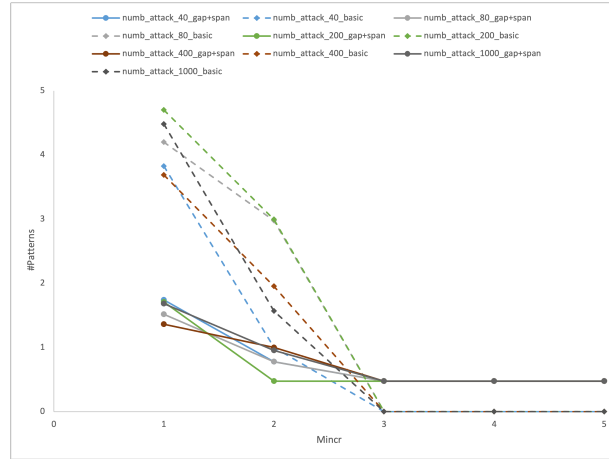
**Figure 3.** Comparison between basic encoding and gap+span constraint on extracted patterns with  $mingap=0$ ,  $maxgap=0$ ,  $minspan=1$ ,  $maxspan=10$ ,  $minlen=2$ , and  $maxlen=6$ . The number of patterns is in logarithmic scale to facilitate viewing and comparison



**Figure 4.** Comparison between basic encoding and gap constraint on extracted patterns  $mingap=0$ ,  $maxgap=0$ ,  $minlen=2$ , and  $maxlen=6$ . The number of patterns is in logarithmic scale to facilitate viewing and comparison



**Figure 5.** Comparison between basic encoding and span constraint on extracted patterns with  $minspan=1$ ,  $maxspan=10$ ,  $minlen=2$ , and  $maxlen=6$ . The number of patterns is in logarithmic scale to facilitate viewing and comparison



**Figure 6.** Comparison between basic encoding and gap+span constraint on extracted patterns with  $mingap=0$ ,  $maxgap=0$ ,  $minspan=1$ ,  $maxspan=10$ ,  $minlen=2$ , and  $maxlen=6$ . The number of patterns is in logarithmic scale to facilitate viewing and comparison