ESERCIZI POTENZA REGIME SINUSOIDALE

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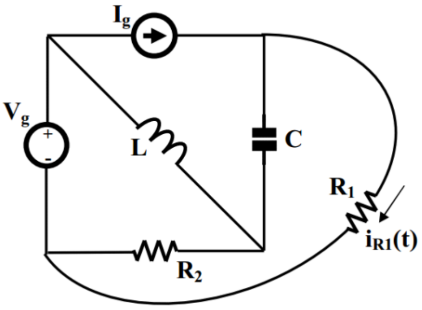

Il circuito in figura si trova a regime permanente sinusoidale.
Siano $V_g(t) = 12\cos(100t+\pi/2)$ V, $I_g(t) = 6\sqrt{2}\cos(100t-\pi/4)$ A, $R_1 = 0.5\ \Omega$, $R_2 = 0.5\ \Omega$, L = 2.5 mH, C = 20 mF.
Determinare:

- la potenza reattiva complessivamente scambiata dalla capacità C e dall'induttanza L;
- la corrente $i_{R1}(t)$ che scorre nel resistore R_1.

Calcolare anche il circuito equivalente di Thevenin visto dalla resistenza R1 e usatelo per calcolare la potenza complessa di R1 e verificate il valore ottenuto con la soluzione precedente

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1 Disegno circuito simbolico: calcolo fasori ed impedenze

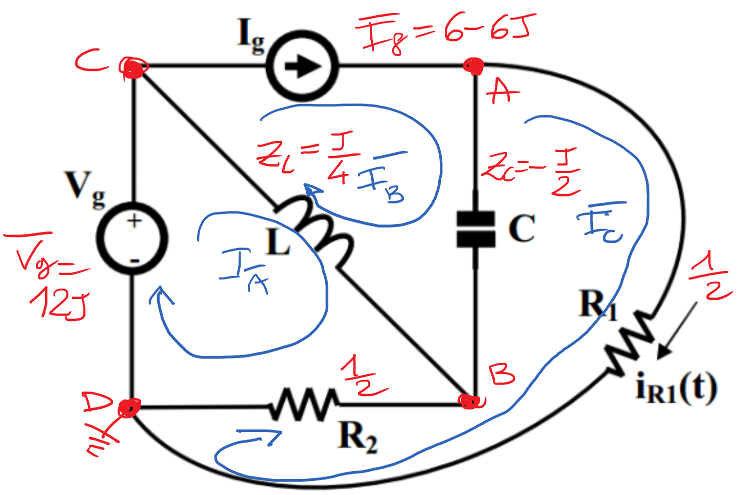
$$V_g(t) = 12\cos\left(100t+\frac{\pi}{2}\right)[V] \Longrightarrow \overline{V}_g = 12\,e^{J\frac{\pi}{2}} = 12J\ [V]$$

$$I_g(t) = 6\sqrt{2}\cos\left(100t-\frac{\pi}{4}\right)[A] \Longrightarrow \overline{I}_g = 6\sqrt{2}\left(\frac{1}{\sqrt{2}} - J\frac{1}{\sqrt{2}}\right) = 6-6J\ [A]$$

$$Z_{R_1} \triangleq R_1 = \frac{1}{2} \quad Z_{R_2} \triangleq R_2 = \frac{1}{2}\ ; \ Z_L = J\omega L = J10^2 \cdot 2.5\,10^{-3} = J\frac{1}{4}\ [\Omega]$$

$$Z_C = \frac{1}{J\omega C} = \frac{-J}{\omega C} = \frac{-J}{10^2\,2\cdot10^{-2}} = -J\frac{1}{2}\ [\Omega]$$

$$Y_{R_1} = G_1 = \frac{1}{R_1} = 2\,[S] \quad Y_{R_2} = 2\,[S] \quad Y_C = 2J\ [S] \quad Y_L = -J4\,[S]$$



NODI $\quad \overline{e}_D = 0\ ;\ \overline{e}_C = \overline{V}_g$

$\overline{e}_A, \overline{e}_B$ incogniti

ANELLI $\quad \overline{I}_B = \overline{I}_g$

$\overline{I}_A, \overline{I}_C$ incogniti

$$(A) \quad \frac{\overline{e}_A - \overline{e}_B}{Z_C} + \frac{\overline{e}_A - \overline{e}_D}{Z_{R1}} = \overline{I}_g$$

$$(\overline{e}_A - \overline{e}_B)Y_C + (\overline{e}_A - \overline{e}_D)G_1 = \overline{I}_g$$

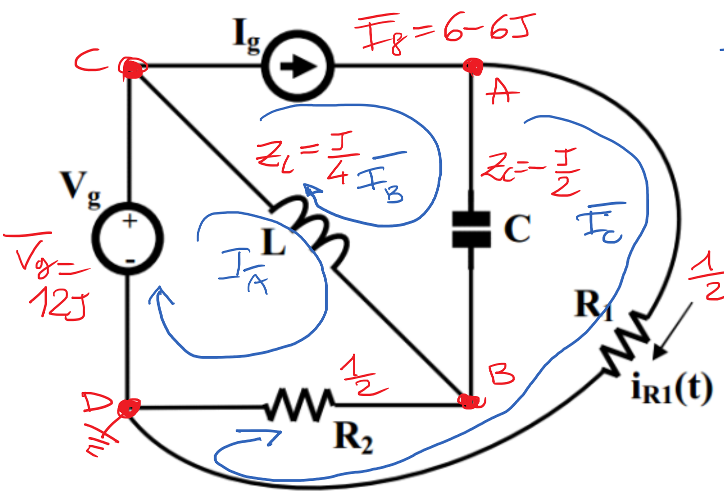
$$(B) \quad (\overline{e}_B - \overline{e}_A)Y_C + (\overline{e}_B - \overline{e}_C)Y_L + (\overline{e}_B - \overline{e}_D)G_2 = 0$$

$$\Longrightarrow \begin{bmatrix} Y_C + G_1 & -Y_C \\ -Y_C & Y_C + Y_L + G_2 \end{bmatrix} \begin{bmatrix} \overline{e}_A \\ \overline{e}_B \end{bmatrix} = \begin{bmatrix} \overline{I}_g \\ \overline{V}_g Y_L \end{bmatrix}$$

$$\begin{bmatrix} 2+2J & -2J \end{bmatrix} \begin{bmatrix} \overline{e}_A \end{bmatrix} \begin{bmatrix} 6-6J \end{bmatrix} \qquad \overline{e}_A = 6J$$

$$\begin{bmatrix} 2+2j & -2j \\ -2j & 2-2j \end{bmatrix} \begin{bmatrix} \bar{e}_A \\ \bar{e}_B \end{bmatrix} = \begin{bmatrix} 6-6j \\ 48 \end{bmatrix} \Longrightarrow \begin{matrix} \bar{e}_A = 6j \\ \bar{e}_B = 9+9j \end{matrix}$$

$\Delta = 12$



$\bar{I}_B = \bar{I}_g$

Ⓐ $-\bar{V}_g + (\bar{I}_A - \bar{I}_g) Z_L + (\bar{I}_A - \bar{I}_C) Z_{R_2} = 0$

Ⓒ $\bar{I}_C Z_{R1} + (\bar{I}_C - \bar{I}_A) Z_{R_2} + (\bar{I}_C - \bar{I}_g) Z_C = 0$

Ⓐ $-\bar{V}_g + (\bar{I}_A - \bar{I}_g) Z_L + (\bar{I}_A - \bar{I}_C) Z_{R_2} = 0$

Ⓒ $\bar{I}_C Z_{R1} + (\bar{I}_C - \bar{I}_A) Z_{R_2} + (\bar{I}_C - \bar{I}_g) Z_C = 0$

$$\begin{bmatrix} Z_L + Z_{R_2} & -Z_{R_2} \\ -Z_{R_2} & Z_{R_1} + Z_{R_2} + Z_C \end{bmatrix} \begin{bmatrix} \bar{I}_A \\ \bar{I}_C \end{bmatrix} = \begin{bmatrix} \bar{V}_g + \bar{I}_g Z_L \\ \bar{I}_g Z_C \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} + \frac{j}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 1 - \frac{j}{2} \end{bmatrix} \begin{bmatrix} \bar{I}_A \\ \bar{I}_C \end{bmatrix} = \begin{bmatrix} 12j + (6-6j)\frac{j}{4} = \frac{3}{2} + \frac{27}{2}j \\ (6-6j) \cdot -\frac{j}{2} = -3 - 3j \end{bmatrix}$$

$\Delta = 0.375 = \frac{3}{8}$

$\bar{I}_A = 18 + 30J \quad ; \quad \bar{I}_C = 12J$

$$\bar{S}_L = \frac{1}{2}\bar{V}_L \cdot \bar{I}_L^* = \boxed{\frac{1}{2}\frac{|V_L|^2}{Z_L^*}} = \frac{1}{2}Y_L^*|\bar{V}_L|^2 = \frac{1}{2}Z_L|\bar{I}_L|^2$$

$6\ V^2$

$$Q_L = Im\{\bar{S}_L\} = \frac{1}{2}|\bar{V}_g - \bar{e}_B|^2 \cdot Im\{J4\} = 2|\bar{V}_g - \bar{e}_B|^2 = 180\ VAR$$

$$= Im\{\bar{S}_L\} = \frac{1}{2}Im\{Z_L\}|\bar{I}_L|^2 = \frac{1}{2}\cdot\frac{1}{4}\cdot|\bar{I}_A - \bar{I}_B|^2 = 180\ VAR$$

$$\bar{S}_C = \frac{1}{2}\bar{V}_C \cdot \bar{I}_C^* = \frac{1}{2}Y_C^*|\bar{V}_C|^2 = \frac{1}{2}Z_C|\bar{I}_C|^2 =$$

$$Q_C = \frac{1}{2}Im\{-2J\}|\bar{e}_A - \bar{e}_B|^2 = -|e_A - e_B|^2 = -90\ [VAR]$$

$$Q_C = \frac{1}{2}Im\left\{-\frac{J}{2}\right\}|\bar{I}_{CAP}|^2 = -\frac{1}{4}|I_C - I_B|^2 = -90\ [VAR]$$

$i_{R1}(t) \qquad \bar{I}_{R1} = \bar{I}_C = 12J \Rightarrow$

$$i_{R1}(t) = 12\cos\left(\omega t + \frac{\pi}{2}\right) = -12\,\mathrm{sen}(\omega t)$$

$$i_{R1} \Rightarrow \quad \bar{I}_{R1} = \frac{\bar{e}_A}{Z_{R1}} = 2(\bar{e}_A) = 12J$$

$3 + 12J$

$3\cos(\omega t) - 12\,\mathrm{sen}(\omega t)$