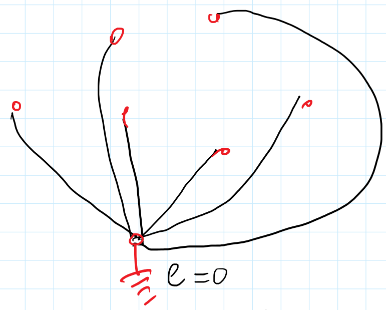
ALBERO	CO-ALBERO
N-1 RAMI	R-N+1 RAMI

R = n° rami TOT
N = n° nodi

N-1 tensioni rami albero $\{v_\alpha\}$ | R-N+1 correnti rami co-albero



N-1 potenziali dei nodi rispetto al nodo di riferimento

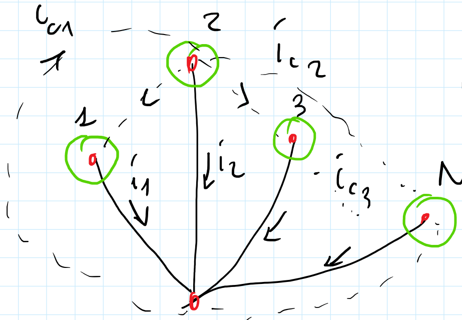
$$\{v_a\} \equiv \{e_a\} = \{e_n\}$$

N-1 incognite

$\Downarrow$

N-1 eq indipendenti

N-1 KCL ai nodi (escluso il riferimento)



$\{i_k\}$ correnti rami albero $k=1, N-1$

$\{i_{c_k}\}$ $c_k = 1, R-N+1$ correnti coalbero

R-N+1

$$\text{KCL NODO 1} = i_1 + \sum_{c_k=1} \alpha_{c_k} i_{c_k} = 0$$

$$\text{KCL NODO 2} = i_2 + \sum \beta_{c_k} i_{c_k} = 0$$

$$\vdots$$

$$\text{KCL NODO N-1} = i_{N-1} + \sum \gamma_{c_k} i_{c_k} = 0$$

N-1 eq ind.

$$\alpha x_1 + \beta x_2 = 0$$

$$\gamma x_1 + \delta x_2 = 0$$

$$\varepsilon x_1 + \xi x_2 = 0 = A(\alpha x_1 + \beta x_2) + B(\gamma x_1 + \delta x_2)$$

ALGORITMO

1 INDIVIDUO NODO DI RIFERIMENTO

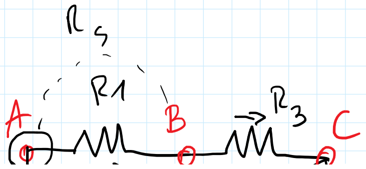
(1bis) DISEGNO UN ALBERO T/C I RAMI SIANO TUTTI CONNESSI AL NODO DI RIFERIMENTO

2 SCRIVO LE N-1 KCL AI NODI (escluso il riferimento)

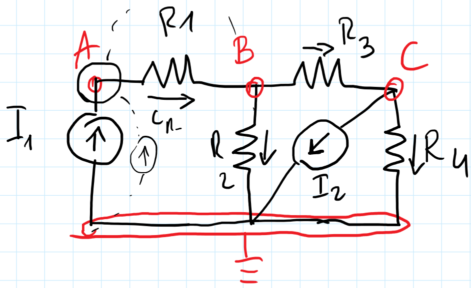
3 SOSTITUISCO LE CORRENTI IN FUNZIONE DEI POTENZIALI TRAMITE LE relazioni costitutive

$$N-1 \begin{bmatrix} \hat{G} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{N-1} \end{bmatrix} = \begin{bmatrix} \text{TERMINI NOTI} \end{bmatrix} = \begin{bmatrix} \text{GEN. CORRENTE} \end{bmatrix}$$

$$\hat{G}\vec{e} = \vec{i} \quad \Big| \quad \hat{R}\vec{i} = \vec{v}$$



NODO A : $-I_1 + i_{R1} = 0$



NODO A : $-I_1 + i_{R_1} = 0$

NODO B : $-i_{R_1} + i_{R_2} + i_{R_3} = 0$

NODO C : $-i_{R_3} + i_{R_4} + I_2 = 0$

$i_{R_1} = G_1 (e_a - e_b)$ $\qquad$ $i_{R_2} = G_2 (e_b - 0) = G_2 e_b$

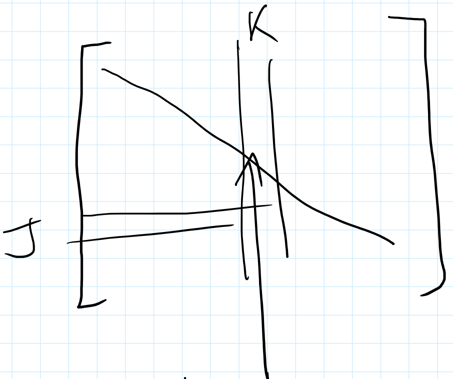
$\overset{\circ}{A}_{+} \longrightarrow \overset{\circ}{B}_{-}$ $\qquad$ $i_{R_3} = G_3 (e_b - e_c)$

$i_{R_4} = G_4 e_c$

$$
\begin{cases}
+ G_1 (e_a - e_b) = I_1 \\
-G_1 (e_a - e_b) + G_2 e_b + G_3 (e_b - e_c) = 0 \\
-G_3 (e_b - e_c) + G_4 e_c = -I_2
\end{cases}
$$

$$
\underbrace{\begin{bmatrix} G_1 & -G_1 & 0 \\ -G_1 & G_1+G_2+G_3 & -G_3 \\ 0 & -G_3 & G_3+G_4 \end{bmatrix}}_{\hat{G}}
\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}
=
\begin{bmatrix} I_1 \\ 0 \\ -I_2 \end{bmatrix}
$$

$$
e_a = \frac{\begin{vmatrix} I_1 & -G_1 & 0 \\ 0 & G_1+G_2+G_3 & -G_3 \\ -I_2 & -G_3 & G_3+G_4 \end{vmatrix}}{|\hat{G}|}
$$



$G_{K,K} = \text{K-esimo potenziale} = \sum_{\alpha} G_\alpha = \{G_\alpha\}$ comune al nodo $K = \sum_\alpha \frac{1}{R_\alpha}$

$G_{K,J} = G_{J,K} = -\left(\sum_\beta G_\beta\right)$ $\{G_\beta\}$ insieme delle conduttanze comune tra e_K e e_J

n-esima

n-esimo termine noto

$\sum_\alpha (\pm) I_\alpha$ $\{I_\alpha\}$ gen. corrente convergenti al nodo

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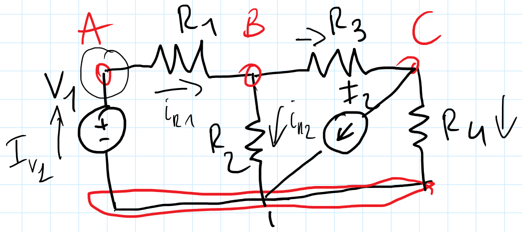


$D \Rightarrow e_D = 0$

$-i_{V_1} + i_{R_1} = 0$ (A)

$-i_{R_1} + i_{R_2} + i_{R_3} = 0$

$-i_{R_3} + i_{R_4} + I_2 = 0$

$+i_{V_1}$ → un'incognita aggiuntiva

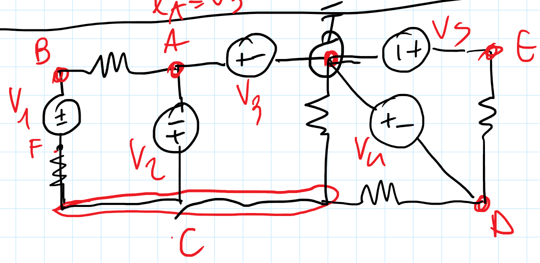
$i_{R_1} = G_1 (e_A - e_B)$ $\quad i_{R_4} = G_4 e_C$

$i_{R_2} = G_2 e_B$ $\quad i_{R_3} = G_3 (e_B - e_C)$

$e_A = V_1$ $\quad i_{V_1} = i_{R_1} = (e_A - e_B) G_1 = (V_1 - e_B) G_1$

$e_A = V_1$ $\quad e_C = -V_2$

$e_A = V_3$



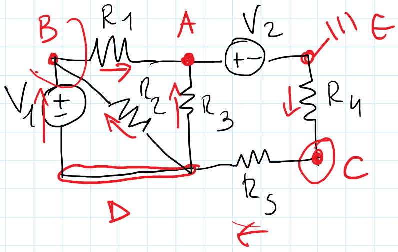
$e_E = V_5$

$e_D = -V_4$

$e_C - e_A = V_2$

$e_C = V_2 + V_3$

$e_B - e_F = V_1$



$e_E = 0$

5 NODI

⇓

4 potenziali incogniti

+

2 gen. Tensione

D R_5

$+$

2 gen Tensione

$\Downarrow$

sistema

2 eq

$e_A = V_2$

$e_B - e_D = V_1$

$$\begin{cases} e_C : i_{R_5} - i_{R_4} = 0 \\ e_D : -i_{R_5} + i_{R_2} + i_{R_3} + i_{V_1} = 0 \end{cases} = -i_{R_5} + i_{R_3} + i_{R_1}$$

$i_{R_5} = (e_C - e_D) G_5 = \dfrac{(e_C - e_D)}{R_5}$ $\qquad i_{R_4} = -e_C G_4 = -\dfrac{e_C}{R_4}$

$i_{R_2} = G_2 (e_D - e_B)$ $\qquad i_{R_3} = G_3 (e_D - e_A)$

$i_{R_1} = G_1 (e_B - e_A)$ $\qquad -i_{V_1} - i_{R_2} + i_{R_1} = 0 \qquad i_{V_1} = i_{R_1} - i_{R}$

$$G_5 (e_C - e_D) + e_C G_4 = 0$$

$$-G_5 (e_C - e_D) + G_3 (e_D - \underset{V_2}{\underset{\parallel}{e_A}}) + G_1 (\underset{e_D + V_1}{\underset{\parallel}{e_B}} - \underset{V_2}{\underset{\parallel}{e_A}}) = 0$$

$$\begin{cases} e_C (G_5 + G_4) - e_D G_5 = 0 \\ -e_C G_5 + e_D (G_5 + G_3 + G_1) \underbrace{- G_3 V_2 + G_1 V_1 - G_1 V_2}_{} = \\ -e_C G_5 + e_D (G_5 + G_3 + G_1) = G_3 V_2 - G_1 V_1 + G_1 V_2 \end{cases}$$

$$\begin{bmatrix} G_4 + G_5 & -G_5 \\ -G_5 & G_5 + G_3 + G_1 \end{bmatrix} \begin{bmatrix} e_C \\ e_D \end{bmatrix} = \begin{bmatrix} 0 \\ G_3 V_2 - G_1 V_1 + G_1 V_2 \end{bmatrix}$$

7

G