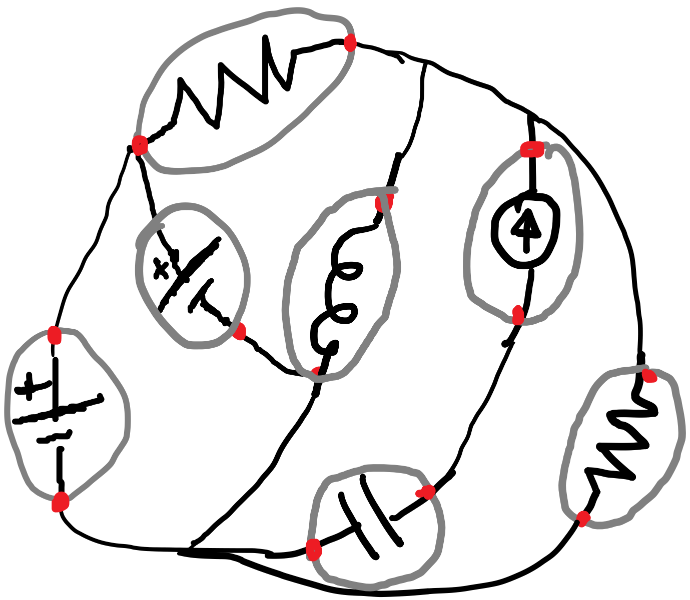
Modello circuitale: Vincoli topologici



- Tensione e corrente variano solo dentro i componenti;
- Ogni componente ha una caratteristica unica;
- I collegamenti sono considerati conduttori perfetti (conducibilità infinita)→ niente caduta di potenziale lungo i conduttori né dissipazione di energia;
- Nello spazio che circonda i componenti non ci sono cariche, il campo elettrico è conservativo, il campo magnetico è nullo

Topologia di un circuito

A lumped circuit is composed of lumped elements (sources, resistors, capacitors, inductors) and conductors (wires).

All the elements are assumed to be lumped, i.e., the entire circuit is of negligible dimensions.

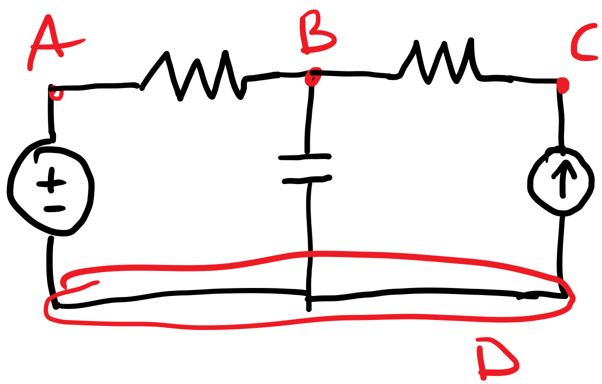
All conductors are perfect.

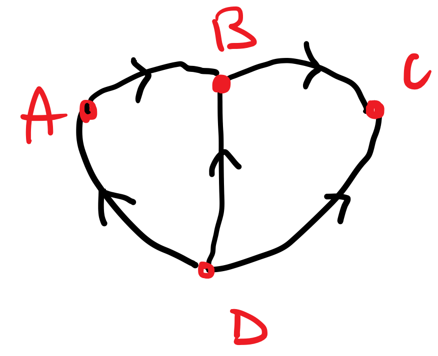
Topology of Circuits

A ***schematic diagram*** is an electrical representation of a circuit.

The location of a circuit element in a schematic may have no relationship to its physical location.

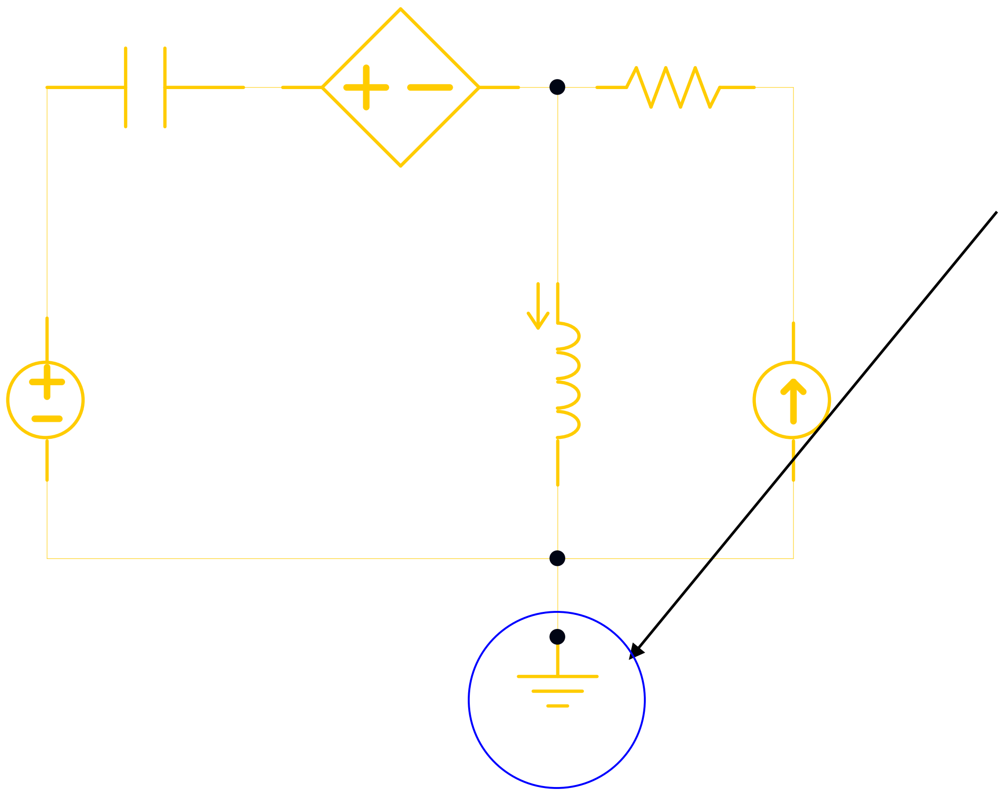
We can rearrange the schematic and have the same circuit as long as the connections between elements remain the same.





Topology of Circuits

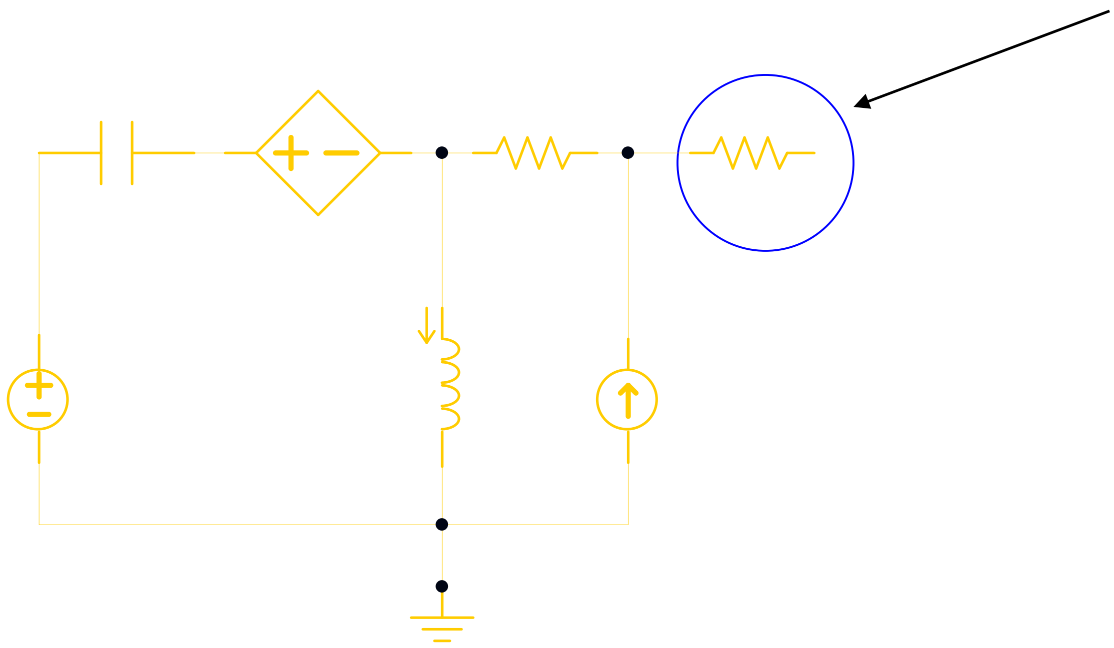
Example: Schematic of a circuit:



"***Ground***": a reference point where the voltage (or potential) is assumed to be zero.

Topology of Circuits

Only circuit elements that are in closed loops (i.e., where a current path exists) contribute to the functionality of a circuit.



This circuit element can be removed without affecting functionality. This circuit behaves identically to the previous one.

Topology of Circuits

A ***node*** is an *equipotential point* in a circuit. It is a topological concept – in other words, even if the circuit elements change values, the *node* remains an *equipotential point*.

To find a *node*, start at a point in the circuit. From this point, everywhere you can travel by moving only along perfect conductors is part of a single *node*.

Topology of Circuits

A ***loop*** is any closed path through a circuit in which no node is encountered more than once.

To find a loop, start at a node in the circuit. From this node, travel along a path back to the same node ensuring that you do not encounter any node more than once.

A ***mesh*** is a loop that has no other loops inside of it.

NOTA: Nel libro Alexander-Sadiku loop e mesh sono invertiti: loop = maglia=qualsiasi percorso chiuso; mesh= anello=maglia che non contiene altre maglie all’interno

Topology of Circuits

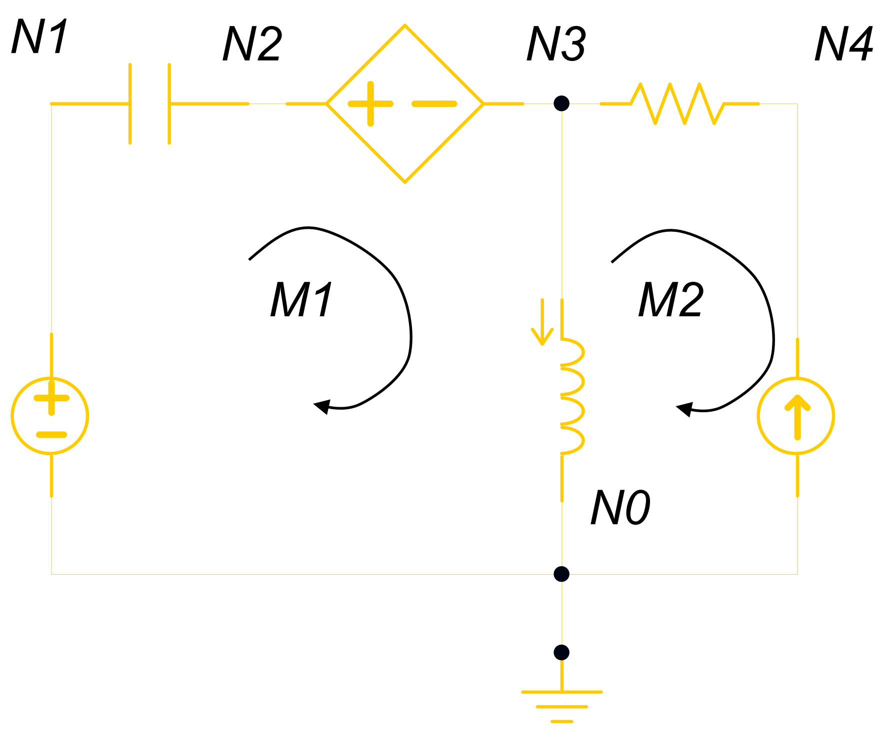
If we know the voltage at every node of a circuit relative to a reference node (*ground*), then we know everything about the circuit – i.e., we can determine any other voltage or current in the circuit.

The same is true if we know every mesh current.

$$\Downarrow$$

Nodes' potentials and meshes' currents are two sets of independent variables. By their knowledge, all the other values of voltages and currents for all the branches can be determined.

Topology of Circuits



- In this example there are 5 nodes and 2 meshes.
- In addition to the meshes, there is one additional loop (following the outer perimeter of the circuit).

KCL and KVL

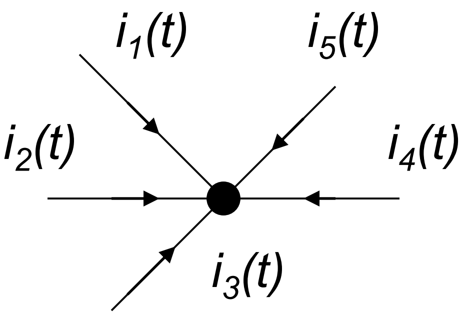
Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL) are the fundamental laws of circuit analysis.

- KCL is the basis of nodal analysis – in which the unknowns are the voltages at each of the nodes of the circuit.
- KVL is the basis of mesh analysis – in which the unknowns are the currents flowing in each of the meshes of the circuit.

KCL and KVL

KCL

- The sum of all currents entering a node is zero, or
- The sum of currents entering node is equal to sum of currents leaving node.



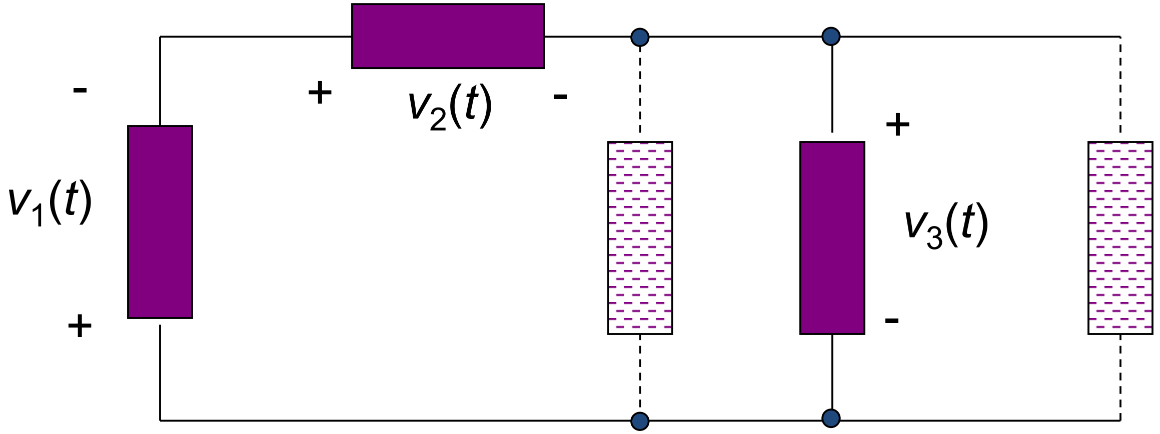
$$\boxed{\sum_{j=1}^{n} i_j(t) = 0}$$

KCL and KVL

KVL

The sum of voltages around any loop in a circuit is zero.

$$\sum_{j=1}^{n} v_j(t) = 0$$



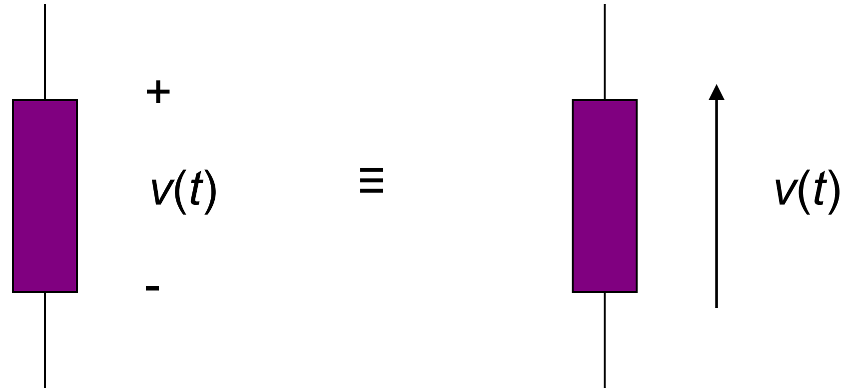
KCL and KVL

In KVL:

A voltage encountered + to - is positive.

A voltage encountered - to + is negative.

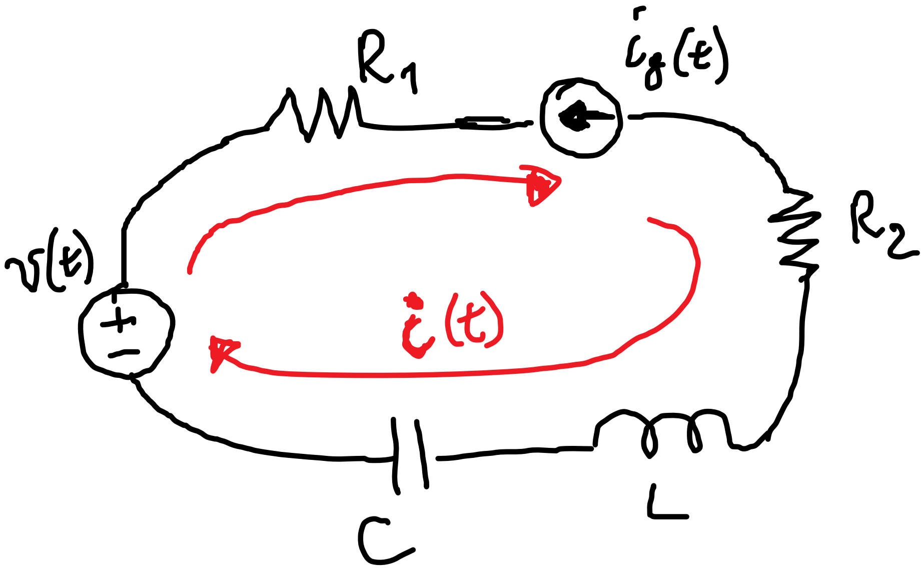
Arrows are sometimes used to represent voltage differences; they point from low to high voltage.



Resistors in Series

A single loop circuit is one which has only a single loop.

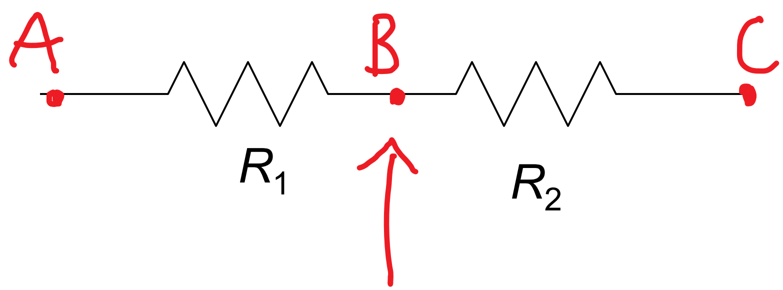
The same current flows through each element of the circuit - the elements are in ***series***.



$$i(t) = i_g(t) =$$
$$= i_v(t) = i_{R_1} = i_{R_2}$$
$$= i_C = i_L$$

Resistors in Series

Two elements are in series if the current that flows through one must also flow through the other.

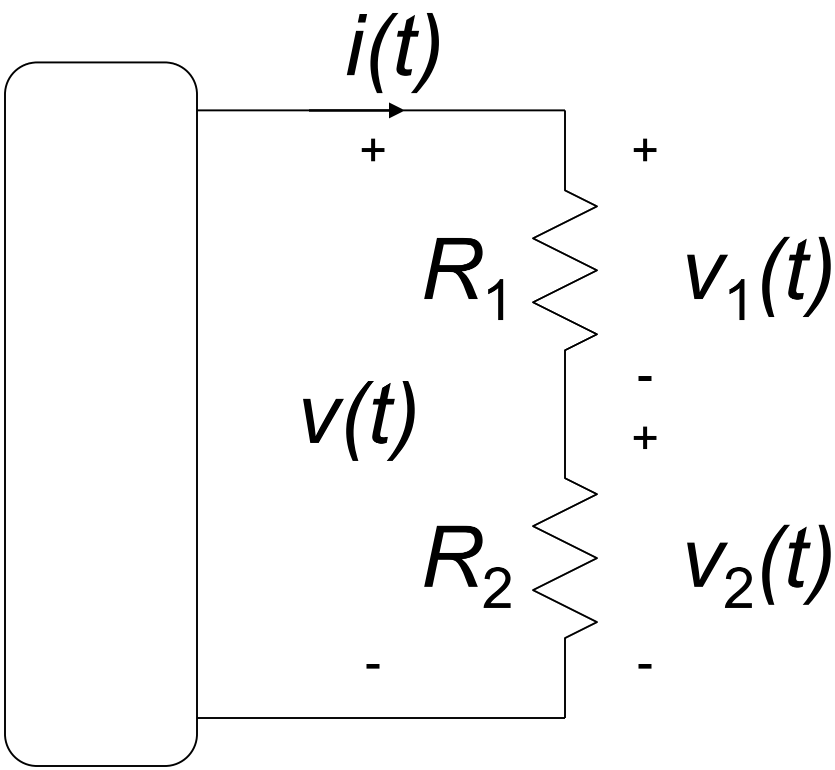


Series

The elements share a single node exclusively. No other branches can be connected to B

Resistors in Series

Consider two resistors *in series* with a voltage *v(t)* across them:



Voltage division:

$$v_1(t) = v(t)\frac{R_1}{R_1 + R_2}$$

$$v_2(t) = v(t)\frac{R_2}{R_1 + R_2}$$

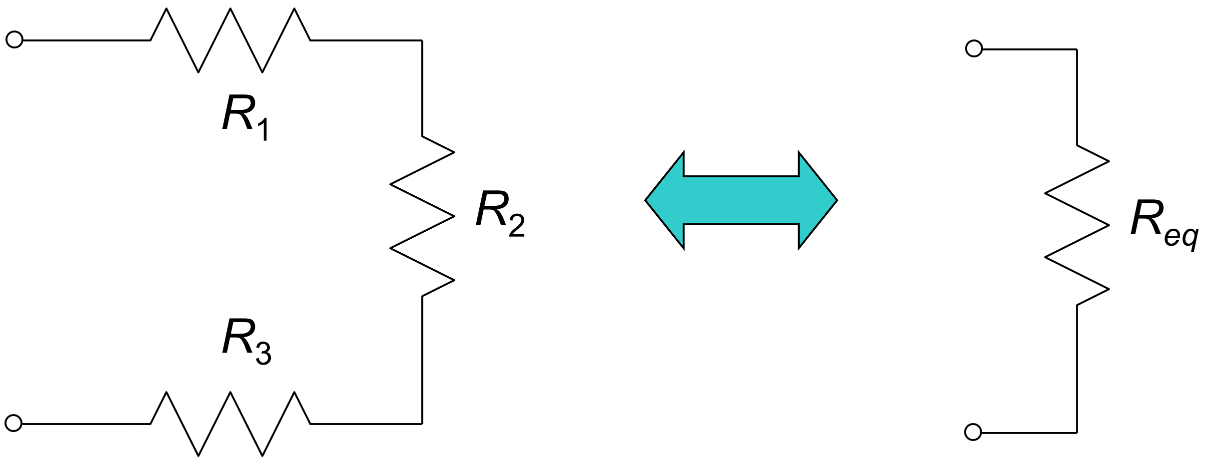
Resistors in Series

If we wish to replace the two series resistors with a single ***equivalent*** resistor whose voltage-current relationship is the same, the *equivalent* resistor has a value given by

$$R_{eq} = R_1 + R_2$$

Resistors in Series

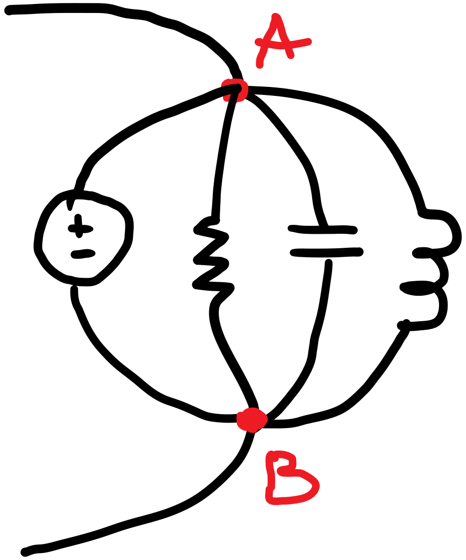
For N resistors in series, the *equivalent* resistor has a value given by



$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N$$

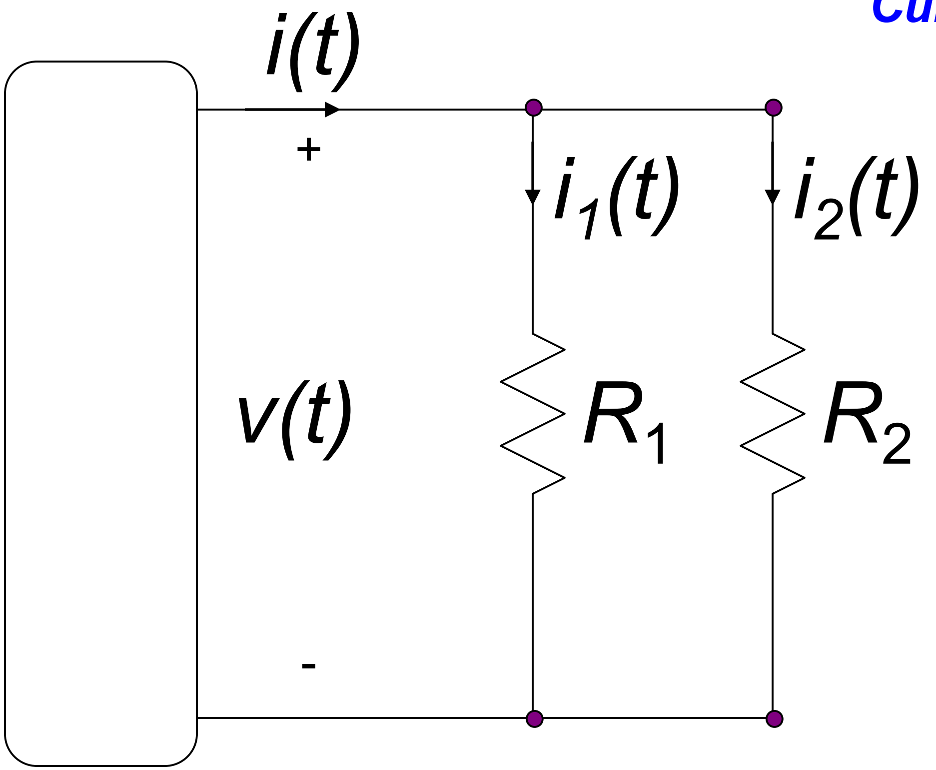
Resistors in Parallel

When the terminals of two or more circuit elements are connected to the same two nodes, the circuit elements are said to be in ***parallel***.



Resistors in Parallel

Consider two resistors *in parallel* with a voltage *v(t)* across them:



Current division:

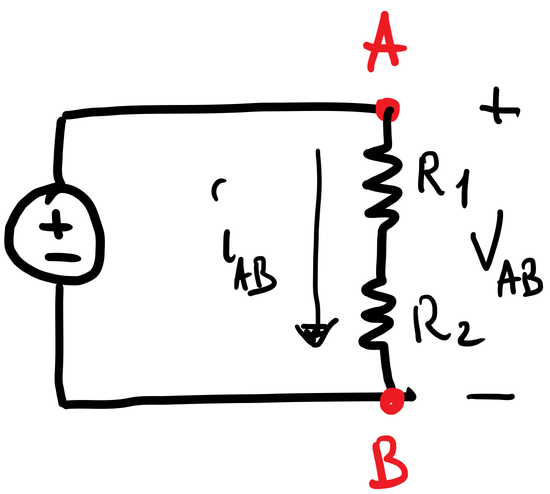
$$i_1(t) = i(t)\frac{R_2}{R_1 + R_2}$$

$$i_2(t) = i(t)\frac{R_1}{R_1 + R_2}$$

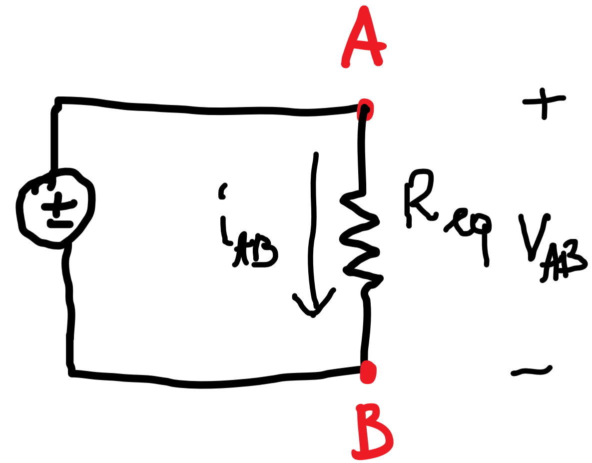
Resistors in Parallel

If we wish to replace the two parallel resistors with a single ***equivalent*** resistor whose voltage-current relationship is the same, the *equivalent* resistor has a value given by

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad \frac{\min(R_1, R_2)}{2} \le R_{eq} < \min(R_1, R_2)$$

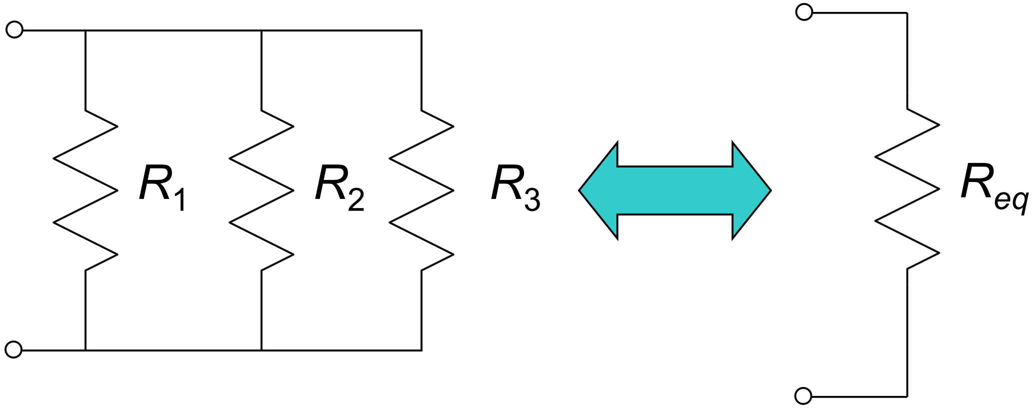


V_{AB}, i_{AB} DO NOT CHANGE IF WE REPLACE R_1 AND R_2 WITH R_{eq} AS IN THE DRAW



Resistors in Parallel

For N resistors in parallel, the *equivalent* resistor has a value given by



$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

Resistors in Parallel

If we use the conductance instead of the resistance we obtain

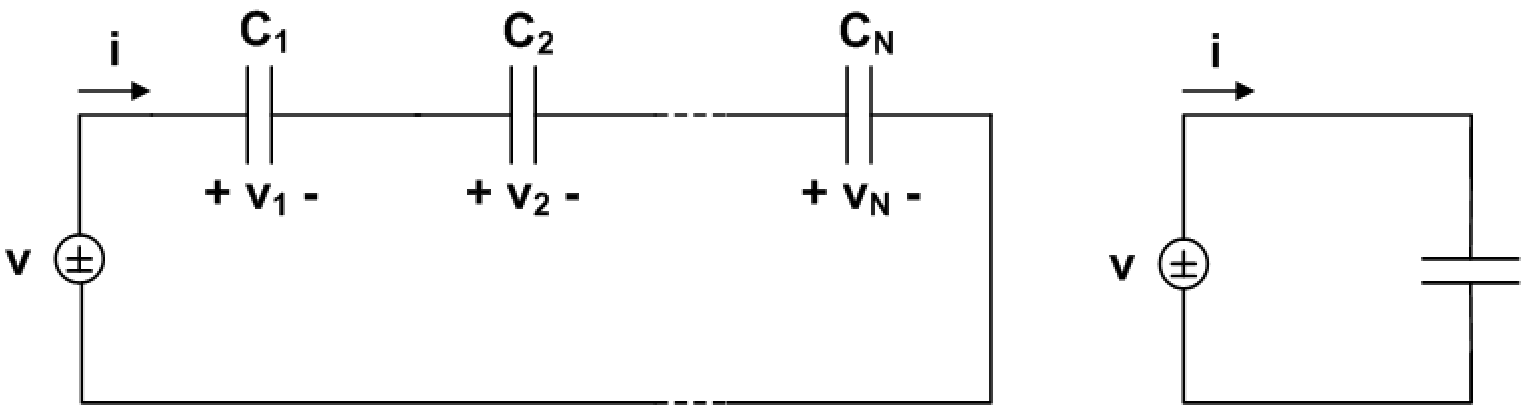
$$R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}$$

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

In series we add resistance values, in parallel we add conductance values, this another example of duality of electrical circuits

Condensatori in serie

La capacità equivalente di N condensatori collegati in serie è pari al reciproco della somma dei reciproci delle singole capacità.



$$v = v_1 + v_2 + \ldots + v_N$$

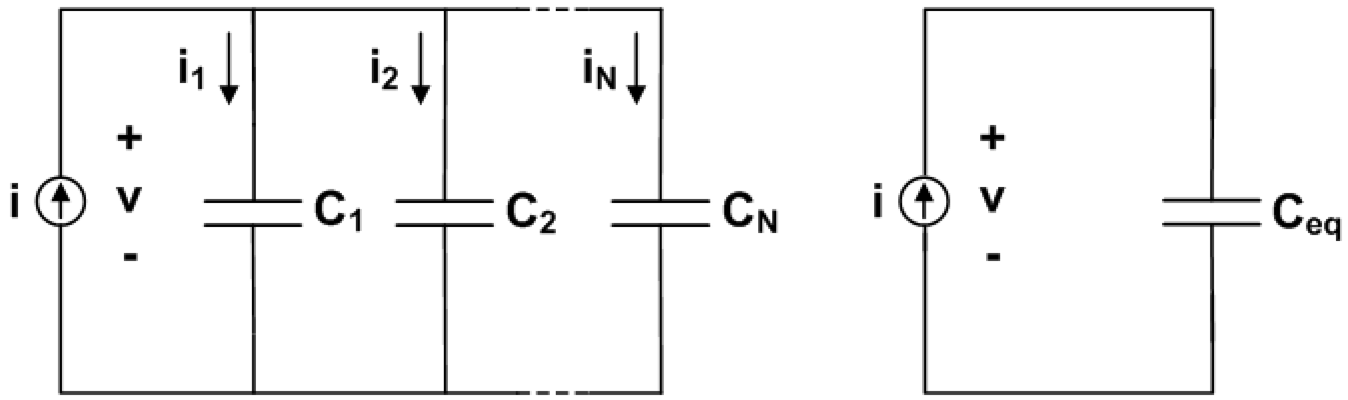
$$v = \frac{1}{C_1}\int_{t_0}^{t} i(t)dt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(t)dt + v_2(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(t)dt + v_N(t_0) =$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(t)dt + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0) = \frac{1}{C_{eq}}\int_{t_0}^{t} i(t)dt + v(t_0)$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}}$$

Condensatori in parallelo

La capacità equivalente di N condensatori collegati in parallelo è pari alla somma delle singole capacità.



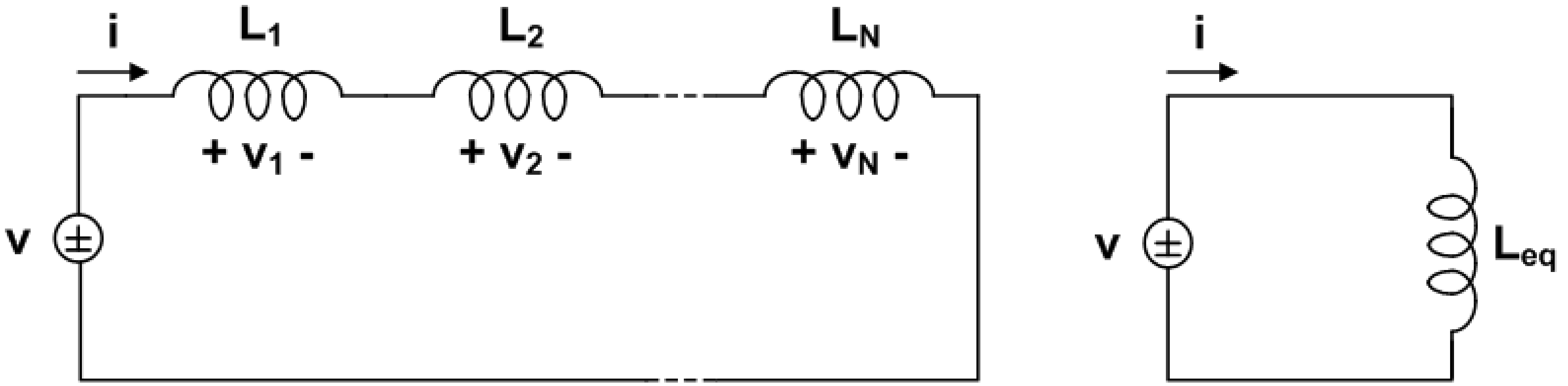
$$i = i_1 + i_2 + \dots + i_N$$

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

$$\boxed{C_{eq} = C_1 + C_2 + \dots + C_N}$$

Induttori in serie

L'induttanza equivalente di N induttori collegati in serie è pari alla somma delle singole induttanze.



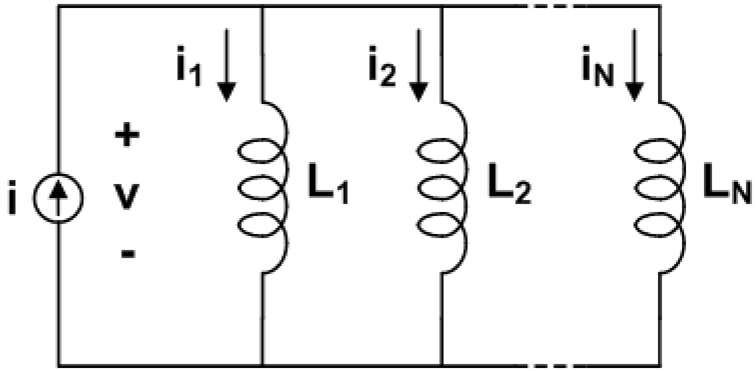
$$v = v_1 + v_2 + \dots v_N$$

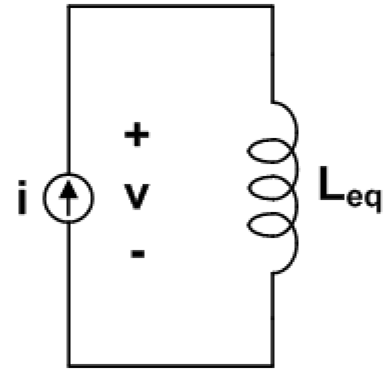
$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \dots + L_N \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + \dots + L_N}$$

Induttori in parallelo

L'induttanza equivalente di N induttori collegati in parallelo è pari al reciproco della somma dei reciproci delle singole induttanze.





$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$i = \frac{1}{L_1}\int_{t_0}^{t} v(t)dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v(t)dt + i_2(t_0) + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v(t)dt + i_N(t_0) =$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v(t)dt + i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v(t)dt + i(t_0)$$

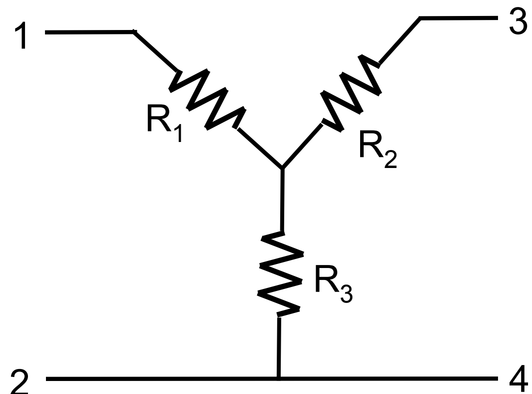
$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}}$$

Riepilogo connessioni serie e parallelo

	SERIE	PARALLELO
RESISTORI	$R_{eq} = R_1 + R_2 + \ldots + R_N$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$
CONDENSATORI	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$	$C_{eq} = C_1 + C_2 + \ldots + C_N$
INDUTTORI	$L_{eq} = L_1 + L_2 + \ldots + L_N$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$

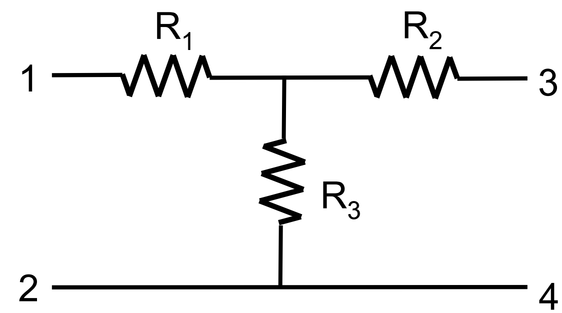
Trasformazioni stella-triangolo

STELLA



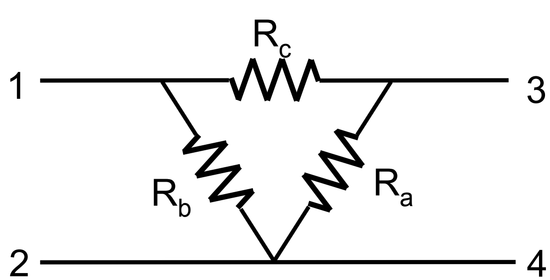
(a)

T



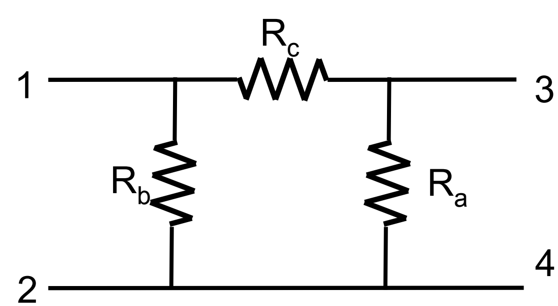
(b)

TRIANGOLO



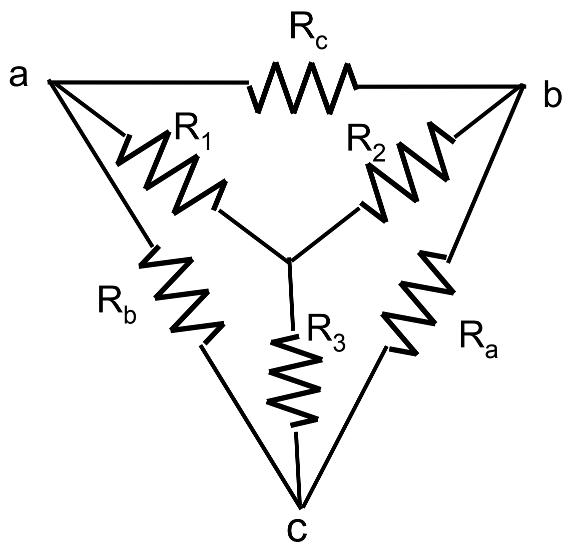
(a)

π



(b)

Trasformazioni stella-triangolo



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

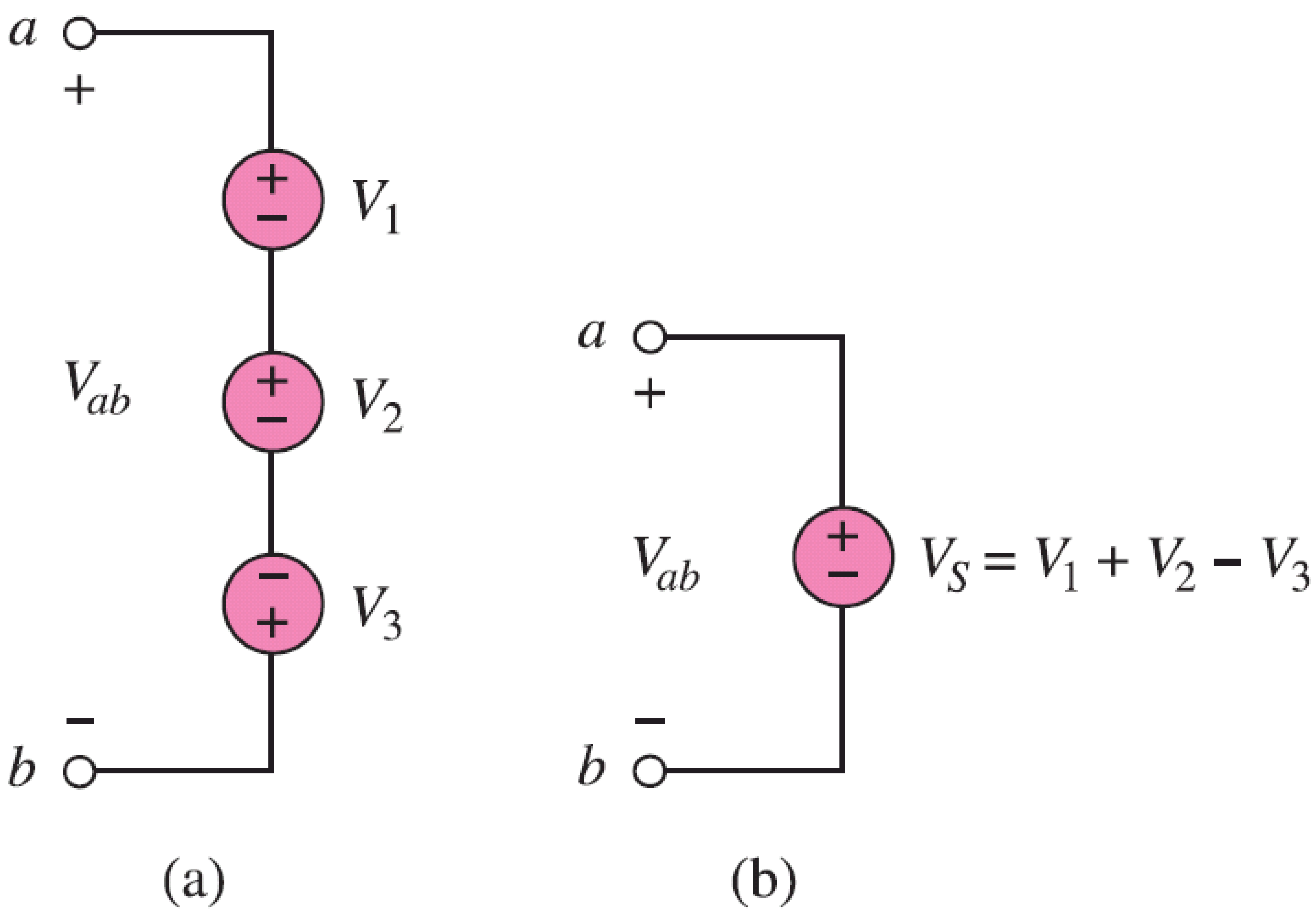
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Ciascun resistore della rete a **stella** è il prodotto dei resistori nei due rami adiacenti delle rete a triangolo, diviso per la somma dei tre resistori del triangolo.

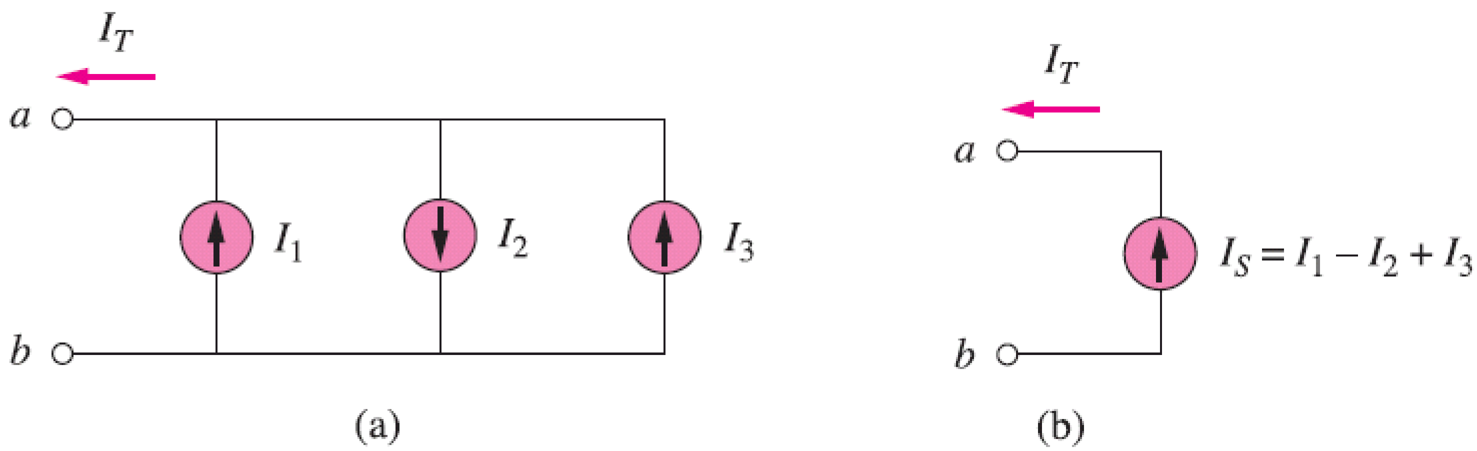
Ciascun resistore della rete a **triangolo** è pari alla somma di tutti i prodotti dei resistori della stella presi a due a due, divisa per il resistore ad esso opposto nella stella.

$$R_Y = \frac{R_\Delta}{3} \rightarrow R_\Delta = 3R_Y$$

Generatori di tensione in serie



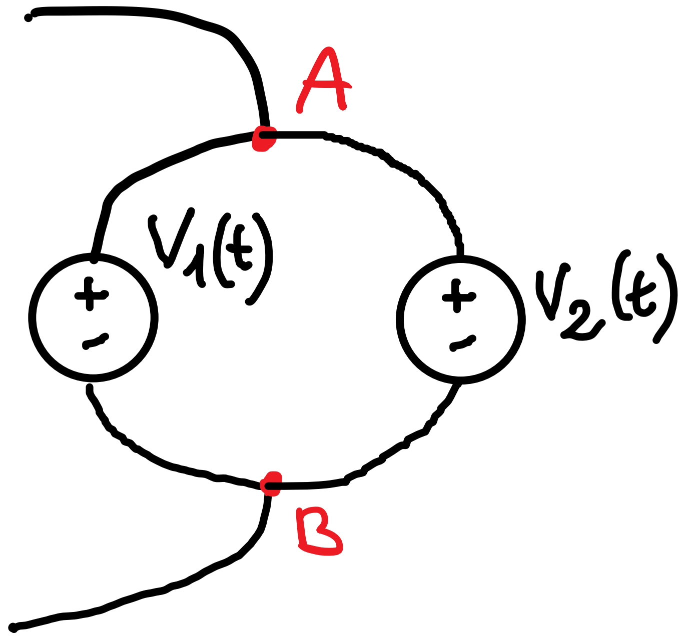
Generatori di corrente in parallelo



Generatori reali

INCONGRUENZE LEGATE AI MODELLI IDEALI

Se $v_1(t) \neq v_2(t)$
Pretenderei che agli stessi nodi si abbiano contemporaneamente due ddp differenti



Se $v_1(t) = v_2(t)$
L'equilibrio alla maglia

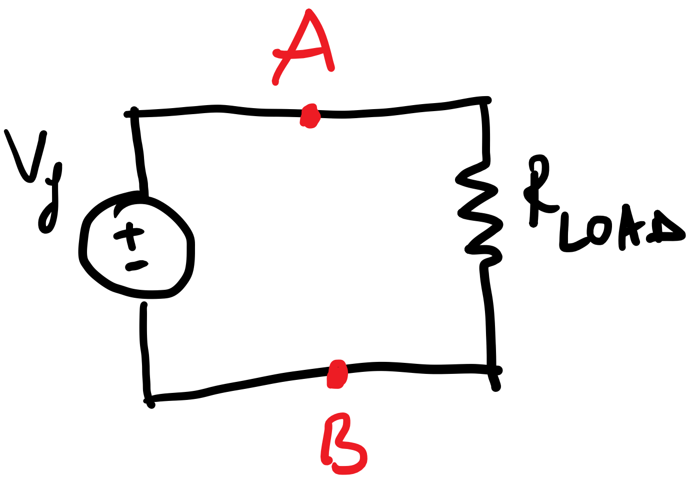
$$v_1(t) - v_2(t) = 0$$

non consente di calcolare la corrente che scorre (infinite soluzioni).

Generatori reali

INCONGRUENZE LEGATE AI MODELLI IDEALI

Un generatore ideale può erogare una potenza infinita



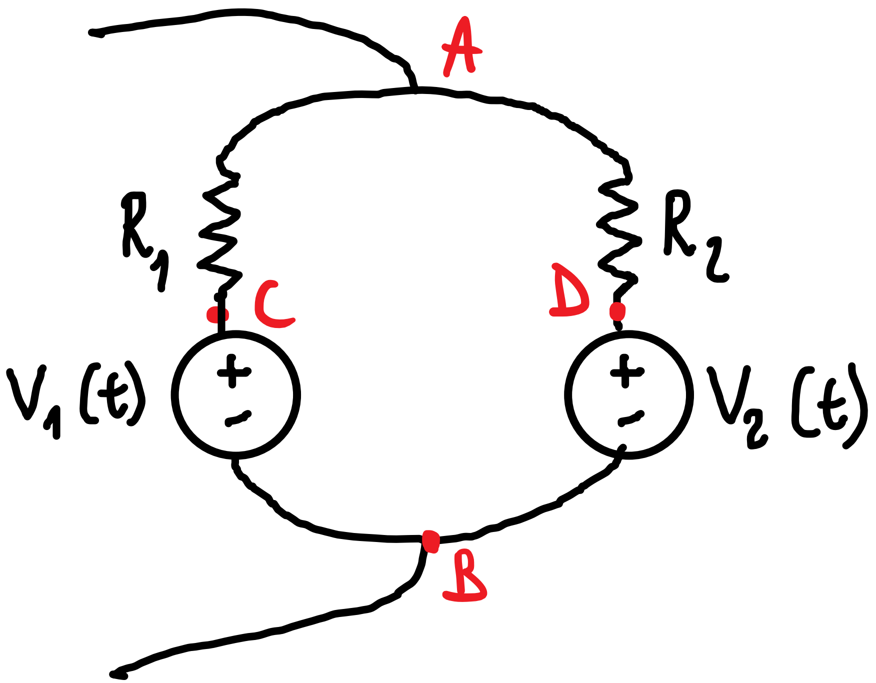
$$V_{R_{LOAD}} = V_g$$

$$P_{R_{LOAD}} = \frac{V_g^2}{R_{LOAD}}$$

SE $R_{LOAD} \rightarrow 0 \Rightarrow P_{R_{LOAD}} \rightarrow +\infty$

Generatori reali

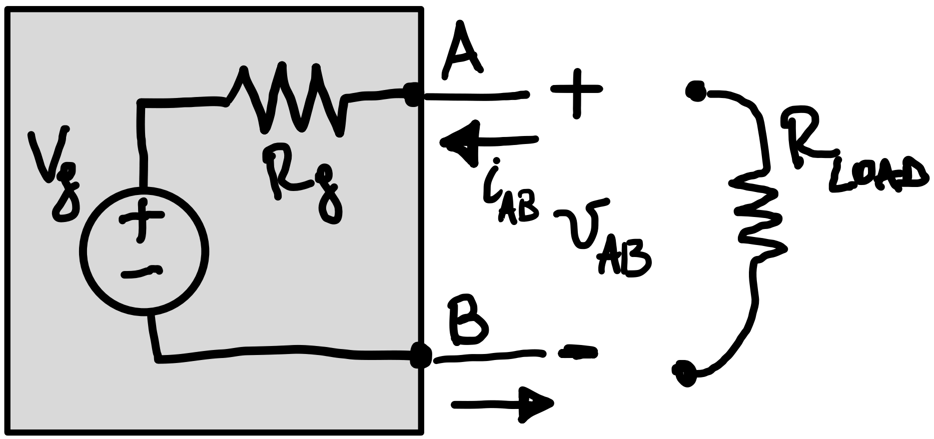
Occorre introdurre un modello più accurato, combinando più elementi ideali (serie di un generatore di tensione e di un resistore).



Questo modello più accurato consente di evitare le incongruenze e le limitazioni evidenziate.

Generatori reali

NOTA BENE: Un generatore reale di tensione non può mai erogare potenza infinita, un generatore reale ha sempre una resistenza interna che limita la corrente erogabile



Se chiudiamo un generatore reale di tensione su un carico resistivo otteniamo un partitore di tensione

Questo modello più accurato consente di evitare le incongruenze e le limitazioni evidenziate.

Generatori reali

INCONGRUENZE LEGATE AI MODELLI IDEALI

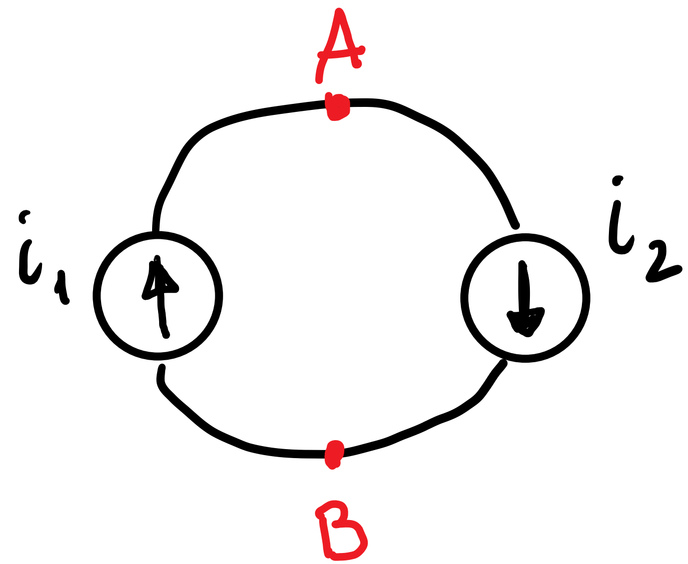
Se $\quad i_1(t) \neq i_2(t)$

Pretenderei che sulla stessa

Maglia si abbiano

contemporaneamente

due correnti differenti



Se $\quad i_1(t) = i_2(t)$

La KLC ai nodi A o B

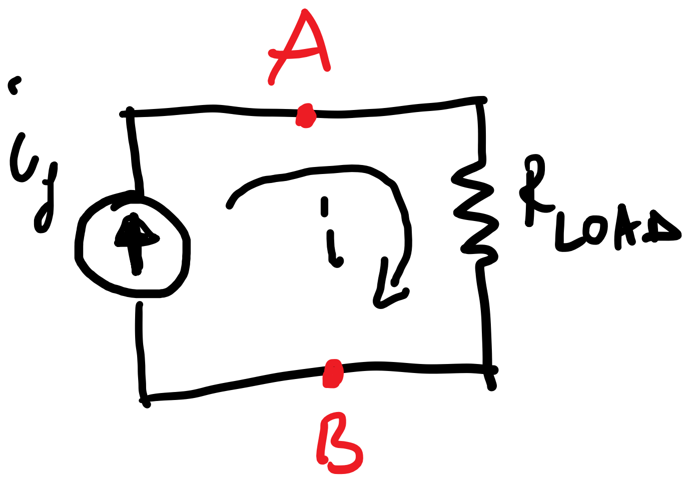
$$i_1(t) - i_2(t) = 0$$

non consente di calcolare la tensione ai capi dei generatori (infinite soluzioni).

Generatori reali

INCONGRUENZE LEGATE AI MODELLI IDEALI

Un generatore ideale può erogare una potenza infinita



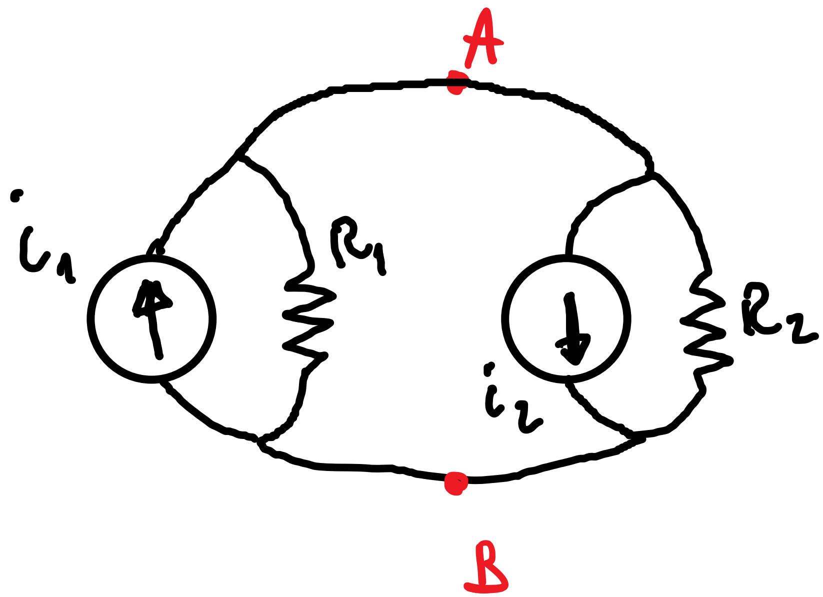
$$i_{R_{LOAD}} = i_g$$

$$P_{R_{LOAD}} = (i_g)^2 \cdot R_{LOAD}$$

se $R_{LOAD} \to +\infty \Rightarrow P_{R_{LOAD}} \to +\infty$

Generatori reali

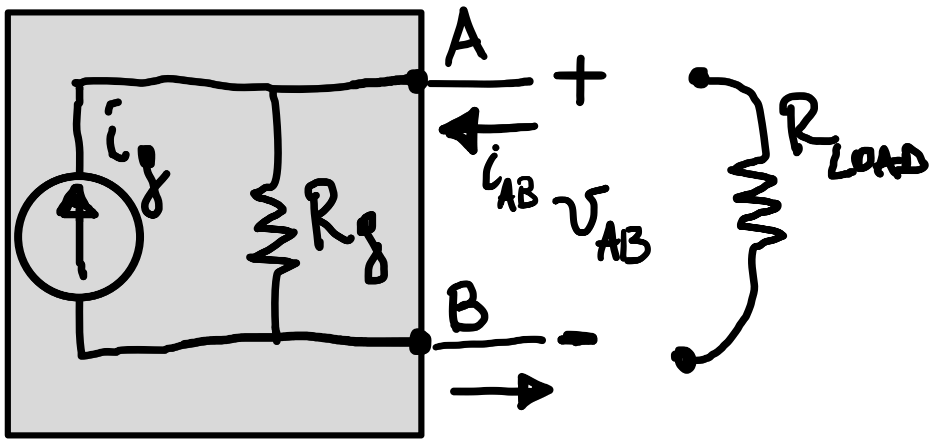
Occorre introdurre un modello più accurato, combinando più elementi ideali (parallelo di un generatore di corrente e di un resistore).



Questo modello più accurato consente di evitare le incongruenze e le limitazioni evidenziate.

Generatori reali

NOTA BENE: Un generatore reale di corrente non può mai erogare potenza infinita, un generatore reale ha sempre una resistenza interna che limita la tensione ai morsetti

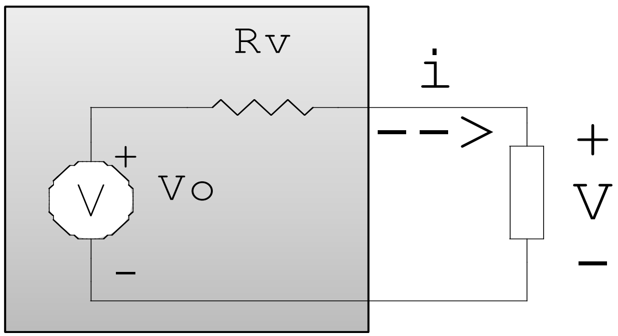


Se chiudiamo un generatore reale di corrente su un carico resistivo otteniamo un partitore di tensione

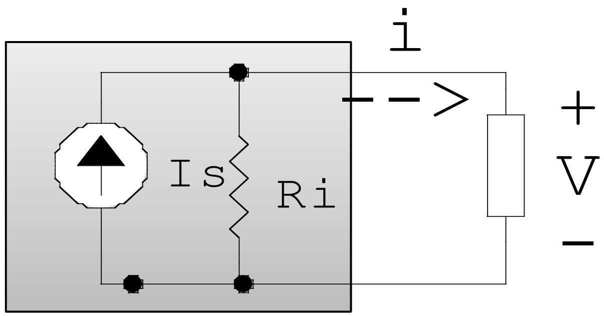
Questo modello più accurato consente di evitare le incongruenze e le limitazioni evidenziate.

Generatori reali

Equivalenza **esterna** dei generatori reali



$$\mathrm{V} = \mathrm{V}_0 - R_V i$$



$$i = i_0 - \frac{1}{R_i} v$$

I due generatori possono essere sostituiti tra loro in maniera equivalente.

Dalla equazione di equilibrio delle correnti al generatore di corrente ho:

$$\boxed{i + \frac{v}{R_i} - I_S = 0 \Rightarrow I_S - i = \frac{v}{R_i}}$$

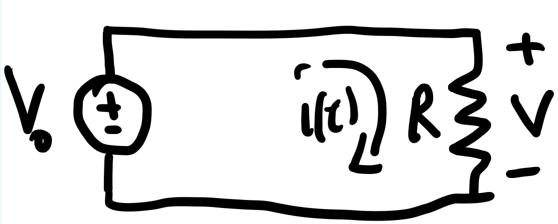
equivalente a quella del generatore di tensione se

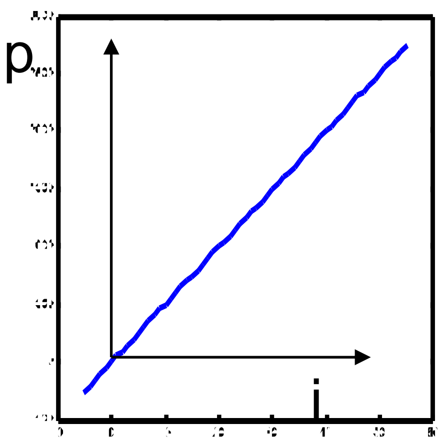
$V_0 = R_I I_S$ ed $R_v = R_i$

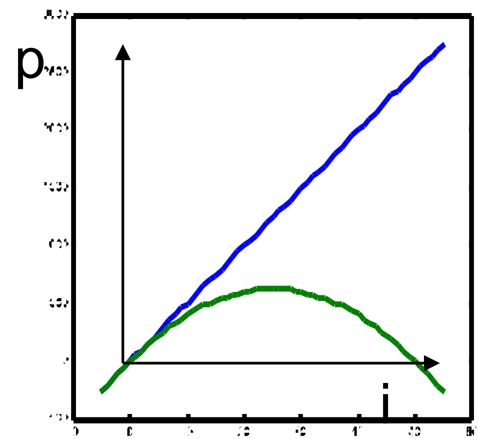
Generatori reali

Confronto tra modelli del generatore reale

Confrontiamo il generatore ideale di tensione con il modello del generatore reale di tensione tramite la potenza erogata sul carico esterno







$$P = v \cdot i = v_0 \cdot i$$

dove $i = V_0/R$ (carico resistivo)

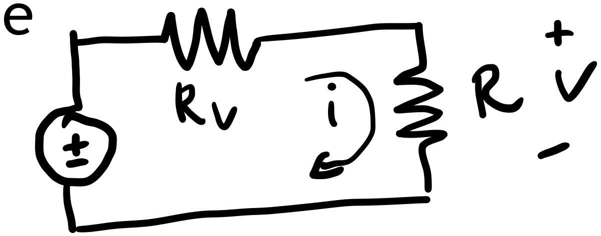
segue $\lim\limits_{\substack{i \to \infty \\ R=0}} (P) = \infty$

$$P = v \cdot i = (v_0 - R_v i) \cdot i = v_0 i - R_v i^2$$

dove $i = V_0 / (R_v + R)$ (carico resistivo)

Generatori reali

La massima corrente che il generatore reale può far circolare si ha per la chiusura in corto circuito



$$i_{MAX} = i_{CC} = \frac{V_0}{R_v}$$

a cui corrisponde una potenza erogata al carico R nulla ()

$$p_{icc} = v_0 \cdot \frac{v_0}{R_v} - R_v \left(\frac{v_0}{R_v} \right)^2 = 0$$

La potenza massima su R si ha per $i = \frac{i_{cc}}{2}$ e vale $P_m = \frac{v_0^2}{4R_v}$

Per R_v=0 la potenza cresce indefinitamente al crescere di i (generatore ideale).

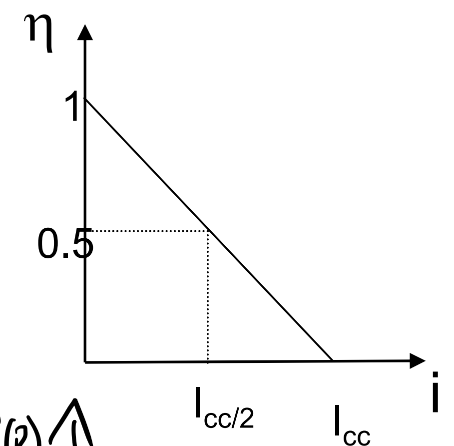
LA LIMITAZIONE NELLA MASSIMA POTENZA EROGABILE E' LEGATA ALLA PRESENZA DI Rv

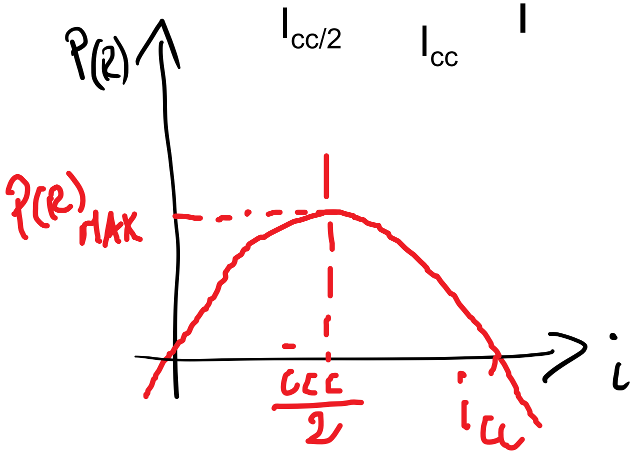
Generatori reali

Rendimento

$$\eta = \frac{\text{potenza assorbita dal carico}}{\text{Potenza erogata dal generatore ideale}}$$

$$\eta = \frac{p}{p_0} = \frac{V \cdot I}{V_0 \cdot I} = \frac{(V_0 - V_R) \cdot I}{V_0 \cdot I} = 1 - \frac{V_R \cdot I}{V_0 \cdot I} = 1 - \frac{R \cdot I}{V_0}$$





- Elevato rendimento (trasporto di energia)
- Massima potenza trasferita (trasmissione segnali)