ANALISI IN REGIME PERMANENTE SINUSOIDALE

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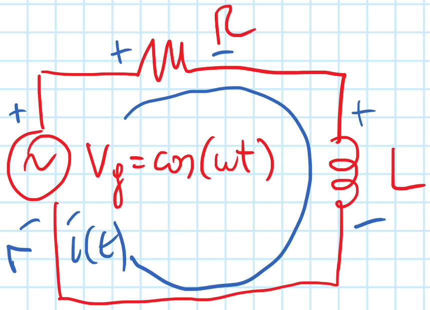
$$\left.\begin{aligned} V_g &= V_0 \cos(\omega_0 t + \varphi_V) \\ I_g &= I_0 \cos(\omega_0 t + \varphi_I) \end{aligned}\right\}$$

TUTTI I GENERATORI HANNO LA STESSA PULSAZIONE / FREQUENZA

$$\omega = 2\pi f$$

$$\frac{1}{f} = T \quad \text{periodo}$$



C, L

$$i_C(t) = C\,\frac{dV_C(t)}{dt}$$

$$V_L(t) = L\,\frac{di_L(t)}{dt}$$

$$R\,i(t) + L\,\dot{i}(t) = V_g(t) = \cos(\omega t)$$

$$\dot{i}(t) + \frac{R}{L}\,i(t) = \frac{\cos(\omega t)}{L}$$

$i(t) =$ risposta naturale + risposta forzata

risposta transitoria + risposta a regime

$$i^{OMO}(t) = t/c \qquad \dot{i}(t) + \frac{R}{L}\,i(t) = 0 \;\Rightarrow\; i^{omo}(t) = i_0\,e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R}$$

$$\boxed{\dot{i}(t) + \frac{R}{L}\,i(t) = \frac{\cos(\omega t)}{L}}$$

t

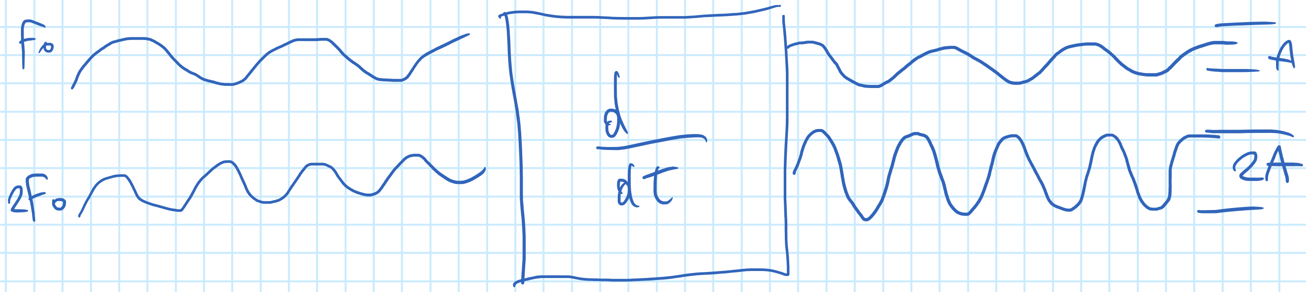
$$\frac{d}{dt} A\cos(\omega t + \varphi) = \omega A\,(-\sin(\omega t + \varphi)) =$$

$\omega A \cos(\omega t + \varphi + \pi)$

$\omega A \operatorname{sen}(\omega t+\varphi) =$

$$\int_{-\infty}^{t} A\cos(\omega t')\,dt' = \frac{A}{\omega}\operatorname{sen}(\omega t) \qquad \underset{\text{cost multiplicat.}}{\omega A} \cos\big(\omega t+\varphi+\underset{\text{sfasamento}}{\tfrac{\pi}{2}}\big)$$

$$= \frac{A}{\omega}\cos\left(\omega t - \frac{\pi}{2}\right)$$



$$A_0, \varphi_0, \omega_0 \left\{ \frac{d}{dt}, \int_{-\infty}^{t}, \mathcal{L}\{\} \right\} \quad A_1, \varphi_1, \omega_0$$

$$\dot{i}(t) + \frac{R}{L} i(t) = \frac{\cos(\omega_0 t)}{L} \qquad i(t) = A\cos(\omega_0 t+\varphi)$$

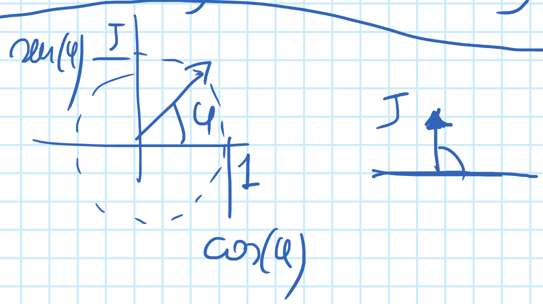
$$i(t) = \mathcal{L}\{\cos(\omega_0 t)\} \qquad \dot{i}(t) = \omega A\cos\left(\omega_0 t+\varphi+\frac{\pi}{2}\right)$$

$$\omega A\cos\left(\omega_0 t+\varphi+\frac{\pi}{2}\right) + \frac{R}{L} A\cos(\omega_0 t+\varphi) = \frac{\cos(\omega_0 t)}{L}$$

$$\|$$

$$\boxed{\operatorname{Re}\left\{\omega A\, e^{J(\omega_0 t+\varphi+\frac{\pi}{2})} + \frac{R}{L} A\, e^{J(\omega_0 t+\varphi)}\right\} = \operatorname{Re}\left\{\frac{e^{J\omega_0 t}}{L}\right\}}$$

$$e^{J\varphi} = \cos(\varphi) + J\operatorname{sen}(\varphi)$$



$$\cos(\varphi) = \operatorname{Re}\{e^{J\varphi}\}$$

$$\operatorname{sen}(\varphi) = \operatorname{Im}\{e^{J\varphi}\}$$

$$\mathrm{Re}\left\{e^{j\omega_0 t}\left[\omega A e^{j\varphi} \underbrace{e^{j\frac{\pi}{2}}}_{=\,j} + \frac{R}{L} A e^{j\varphi}\right]\right\} = \mathrm{Re}\left\{\frac{e^{j\omega_0 t}}{L}\right\}$$

$$j\omega A e^{j\varphi} + \frac{R}{L} A e^{j\varphi} = \frac{1}{L}$$

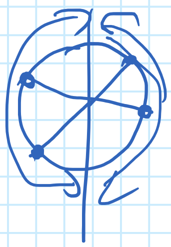
$$A e^{j\varphi}\left(\frac{R + j\omega L}{L}\right) = \frac{1}{L}$$

$$A e^{j\varphi} = \frac{1}{R + j\omega L}$$

$i(t) = A\cos(\omega t + \varphi) \Rightarrow$ è soluzione se

$$A e^{j\varphi} = \underbrace{\frac{1}{R + j\omega L}}_{a + jb}$$

modulo fase



$$A e^{j\varphi} = \frac{1}{R + j\omega L} \cdot \frac{R - j\omega L}{R - j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

rapp polare

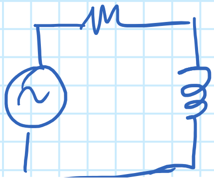
rapp cartesiana: $a + jb$

$$a = \frac{R}{R^2 + \omega^2 L^2}$$

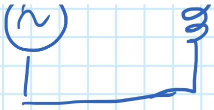
$$A = \sqrt{a^2 + b^2} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$$

$$b = \frac{-\omega L}{R^2 + \omega^2 L^2}$$

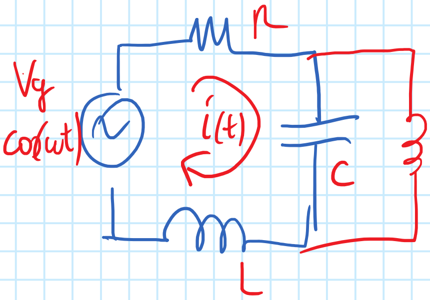
$$\varphi = \operatorname{arctg}\left(\frac{b}{a}\right) + \begin{cases} 0 & \text{se } a > 0 \\ \pi & \text{se } a < 0 \end{cases}$$



$$i(t) = \frac{1}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega_0 t + \operatorname{arctg}\left(-\frac{\omega L}{R}\right)\right)$$



$$i(t) = \underbrace{\frac{1}{\sqrt{R^2+\omega^2L^2}}}_{A} \cos(\underbrace{\omega_0}_{\omega_0} t + \underbrace{\mathrm{arctg}\left(-\frac{\omega L}{R}\right)}_{\varphi})$$



$$\frac{R}{L} i(t) + \frac{1}{CL}\int_{-\infty}^{t} i(t')\,dt' + \frac{\cancel{L}}{\cancel{L}}\frac{d\,i(t)}{dt} = \frac{\cos(\omega_0 t)}{L}$$

$$\Downarrow$$

$$\boxed{\ddot{i}(t) + \frac{R}{L}\dot{i}(t) + \frac{1}{CL} i(t)} = \frac{\omega\cos(\omega_0 t + \frac{\pi}{2})}{L}$$

$i(t) = A\cos(\omega_0 t + \varphi) \Rightarrow$

tutte le grandezze elettriche sono del tipo

$i_\alpha(t) = A_\alpha \cos(\omega_0 t + \varphi_\alpha)$

$V_\beta(t) = A_\beta \cos(\omega_0 t + \varphi_\beta)$

$$\{A_\alpha, A_\beta, \varphi_\alpha, \varphi_\beta\}$$

ad ogni grandezza elettrica si associa un numero complesso chiamato FASORE

$V_\beta(t) = A_\beta\cos(\omega_0 t + \varphi_\beta) \Rightarrow \overline{V}_\beta$ = FASORE DI $V_\beta(t)$ =

$= A_\beta\cos(\varphi_\beta) + j A_\beta \operatorname{sen}(\varphi_\beta) = A_\beta e^{j\varphi_\beta} =$

$\mathbb{C} \leftrightarrow \mathbb{R}\times\mathbb{R}$

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come trovo A e φ ?

$$\mathcal{L}(f(t)) = F(s)$$

$$\mathcal{L}\frac{d}{dt}(f(t)) = sF(s)$$

$$\mathcal{L}\left\{\int_{-\infty}^{t} f(t)\right\} = \frac{1}{s}F(s) - F(0)$$

$$\mathcal{F}\{f(t)\} = F(\omega)$$

$$\mathcal{F}\left\{\frac{dF(t)}{dt}\right\} = j\omega F(\omega)$$

$$\mathcal{F}\left\{\int_{-\infty}^{t} f(t')dt'\right\} = \frac{1}{j\omega}F(\omega)$$

R

$$V_R(t) = V_R^0 \cos(\omega_0 t + \varphi_{V_R})$$
$$i_R(t) = I_R^0 \cos(\omega_0 t + \varphi_{I_R})$$

C

$$V_C(t) = V_C^0 \cos(\omega_0 t + \varphi_{V_C})$$
$$i_C(t) = i_C^0 \cos(\omega_0 t + \varphi_{I_C})$$

L

$$V_L(t) = V_L^0 \cos(\omega_0 t + \varphi_{V_L})$$
$$i_L(t) = I_L^0 \cos(\omega_0 t + \varphi_{I_L})$$

$$V_R(t) = R\, i_R(t) \qquad i_C(t) = C\frac{dV_C(t)}{dt} \qquad V_L(t) = L\frac{di_L(t)}{dt}$$

Rel costitutive mettono in relazione ampiezza e fase di tensione e corrente

$$V_R(t) \Rightarrow V_R^0 e^{j\varphi_{V_R}} = \bar{V}_R$$

$$V_R(t) = V_R^0 \cos(\omega_0 t + \varphi_{V_R}) = Re\{\bar{V}_R e^{j\omega_0 t}\} = \frac{\bar{V}_R e^{j\omega_0 t} + \bar{V}_R^* e^{-j\omega_0 t}}{2}$$

$$i_R(t) = \frac{V_R^0}{R} \cos(\omega_0 t + \varphi_{V_R})$$

$$\bar{V}_R = V_R^0 e^{j\varphi_{V_R}} \qquad \bar{I}_R = \frac{V_R^0}{R} e^{j\varphi_{V_R}}$$

$\bar{V}_R = R\,\bar{I}_R$ legge di Ohm simbolica

$$\frac{\bar{V}_R}{\bar{I}_R} = R$$

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induttore $V_L(t) = L \frac{d\,i_L(t)}{dt} \rightarrow i_L(t) = i_L^0 \cos(\omega_0 t + \varphi_{i_L})$

$$\bar{I}_L = i_L^0 e^{j\varphi_{I_L}}$$

$$V_L(t) = \underbrace{L\,\omega_0\, i_L^0}\cos\left(\omega_0 t + \varphi_{I_L} + \frac{\pi}{2}\right)$$

$$\bar{V}_L = \omega_0 L\, i_L^0\, e^{j\left(\varphi_{I_L} + \frac{\pi}{2}\right)} = \omega_0 L\, \underbrace{i_L^0\, e^{j\varphi_{I_L}}}_{=\bar{I}_L} \cdot e^{j\frac{\pi}{2}}$$

$$\bar{V}_L = \omega_0 L e^{j\frac{\pi}{2}}\, \bar{I}_L$$

$$\bar{V}_L = \underbrace{j\omega_0}_{=\frac{d}{dt}} L\, \bar{I}_L$$

$$\frac{\bar{V}_L}{\bar{I}_L} = j\omega L \qquad \frac{\bar{V}_R}{\bar{I}_R} = R$$

$$\frac{\bar{V}_{Bip}}{\bar{I}_{Bip}} = Z$$ IMPEDENZA $\in \mathbb{C}$

$$\bar{V} = Z\bar{I} \rightarrow Z \in \mathbb{C}$$

$$i_C(t) = C\frac{dV(t)}{dt} \qquad \bar{I} = \boxed{JWC}\,\bar{V}$$

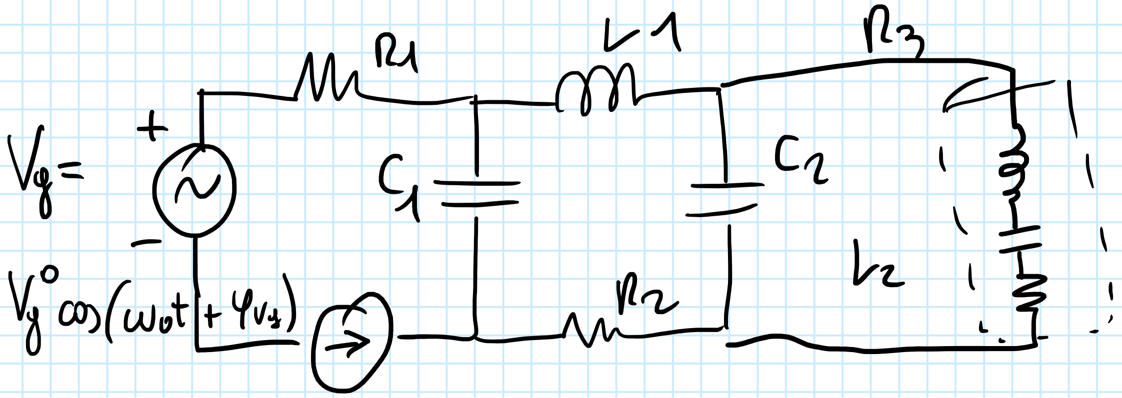
$$i(t) = G\,V(t)$$

CONDENSATORE

$$\frac{\bar{V}}{\bar{I}} = \frac{1}{JWC}$$ → integrazione

IMPEDENZA DEL condensatore

$$V(t) = \frac{1}{C}\int_{-\infty}^{t} i(t')\,dt'$$



$$I_g = I_g^o \cos(\omega_0 t + \varphi_I)$$

$$\bar{V}_{R_1} = R_1 \bar{I}_{R1} \qquad \bar{V}_{L_1} = JWL\,\bar{I}_{L1}$$