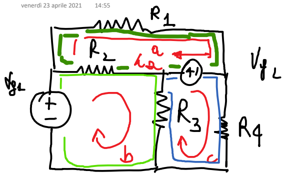
Esercitazione V - 23 Aprile
venerdì 23 aprile 2021 14:55



$V_{g1} = 1\,[V]$

$V_{g2} = 2\,[V]$

$R_1 = R_2 = 1\,[\Omega]$

$R_3 = R_4 = 2\,[\Omega]$

BILANCIO POTENZE E POTENZE DISSIPATE DAI RESISTORI

(a) $i_a R_1 + (i_a - i_b)\cdot R_2 - V_{g2} = 0 \Rightarrow i_a(R_1+R_2) + i_b(-R_2) = V_{g2}$

(b) $(i_b - i_a)R_2 + (i_b - i_c)\cdot R_3 - V_{g1} = 0 \Rightarrow i_a(-R_2) + i_b(R_2+R_3) + i_c(-R_3) = V_{g1}$

(c) $i_c \cdot R_4 + (i_c - i_b)\cdot R_3 + V_{g2} = 0 \Rightarrow i_b(-R_3) + i_c(R_3+R_4) = -V_{g2}$

$$\underbrace{\begin{bmatrix} R_1+R_2 & -R_2 & 0 \\ -R_2 & R_2+R_3 & -R_3 \\ 0 & -R_3 & R_3+R_4 \end{bmatrix}}_{\hat{R}} \underbrace{\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}}_{\vec{i}} = \underbrace{\begin{bmatrix} V_{g2} \\ V_{g1} \\ -V_{g2} \end{bmatrix}}_{\vec{V}} \Rightarrow \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$\Delta_{\hat{R}} = 12$

$$i_a = \frac{\begin{vmatrix} 2 & -1 & 0 \\ 1 & 3 & -2 \\ -2 & -2 & 4 \end{vmatrix}}{\Delta_{\hat{R}}} = \frac{24-8}{12} = \frac{\cancel{16}\,4}{\cancel{12}\,3} = 4/3\ \boxed{A}$$

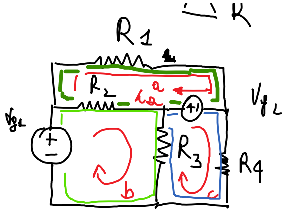
$$i_b = \frac{\begin{vmatrix} 2 & 2 & 0 \\ -1 & 1 & -2 \\ 0 & -2 & 4 \end{vmatrix}}{\Delta_{\hat{R}}} = \frac{8}{12} = \frac{2}{3}\ [A]$$

$$i_c = \frac{\begin{vmatrix} 2 & -1 & 2 \\ -1 & 3 & 1 \\ 0 & -2 & -2 \end{vmatrix}}{\Delta_{\hat{R}}} = \frac{-2}{12} = -\frac{1}{6}\,A$$

$V_{g1} = 1\,[V]$

$i_a = 4/3\ [A]$

$P \quad V^2 \quad R I^2$



$V_{g1} = 1 [V]$
$V_{g2} = 2 [V]$
$R_1 = R_2 = 1 [\Omega]$
$R_3 = R_4 = 2 [\Omega]$

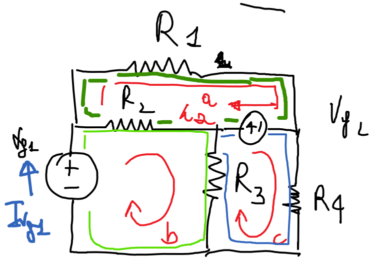
$i_a = 4/3 [A]$
$i_b = \frac{2}{3} [A]$
$i_c = -\frac{1}{6} [A]$

$$P = \frac{V^2}{R} = R \cdot I^2$$

$$P_{R_1} = i_a^2 \cdot R_1 = \frac{16}{9} \cdot 1 = \frac{16}{9} [W]$$
$$P_{R_2} = (i_b - i_a)^2 \cdot R_2 = (i_a - i_b)^2 \cdot R_2 = \left(\frac{4}{3} - \frac{2}{3}\right)^2 \cdot 1 = \frac{4}{9} [W]$$
$$P_{R_3} = (i_c - i_b)^2 R_3 = (i_b - i_c)^2 \cdot R_3 = \left(\frac{2}{3} + \frac{1}{6}\right)^2 \cdot 2 = \frac{25}{36} \cdot 2 = \frac{50}{36} [W] = \frac{25}{18} [W]$$
$$P_{R_4} = i_c^2 R_4 = \frac{1}{36} \cdot 2 = \frac{1}{18} [W]$$

$$\sum_{i=1}^{4} P_{R_i}^{ass} = \left(\frac{16}{9} + \frac{4}{9} + \frac{25}{18} + \frac{1}{18}\right) [W] = \frac{20}{9} + \frac{\cancel{26}}{\cancel{18}} \frac{13}{9} = \frac{11}{3} \frac{\cancel{33}}{\cancel{9}} [W]$$

$$\sum P^{GEN}_{GENERATE}$$

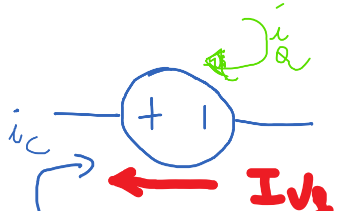


$V_{g1} = 1 [V]$
$V_{g2} = 2 [V]$
$R_1 = R_2 = 1 [\Omega]$
$R_3 = R_4 = 2 [\Omega]$

$$P_{GENERATORI} = V_g \cdot I_{V_g}$$

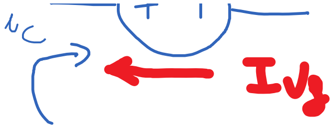
$$P^{GEN}_{V_{g1}} = V_{g1} \cdot I_{V_{g1}} = 1 [V] (i_b) = \frac{2}{3} [W]$$
$$P^{GEN}_{V_{g2}} = V_{g2} (I_{V_{g2}}) = \frac{3}{\cancel{2}} [A] \cancel{2} [V] = 3 [W]$$

$$\sum P^{gen}_{g} = \frac{11}{3} [W] =$$

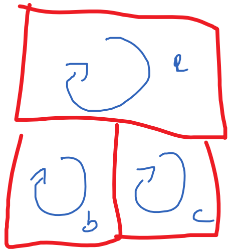


$$I_{V_{g2}} = i_a - i_c = \frac{4}{3} + \frac{1}{6} = \frac{3}{2} \cancel{\frac{9}{6}} [A]$$

$$\sum P_{ASS}^{R_r}$$



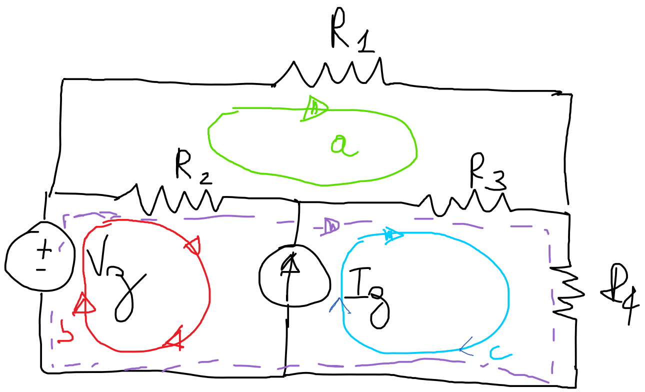
$$I_{V_{g2}} = i_a - i_c = \frac{4}{3} + \frac{1}{6} = \boxed{\frac{\cancel{9}\,3}{\cancel{6}\,2}\ [A]}$$



$$\begin{bmatrix} R_1+R_2 & -R_3 & \\ -R_3 & R_2+R_3 & \\ & & R_2+R_3+R_4 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} V_{g1} \\ V_{g2} \\ 0 \\ V_{g3} + V_{g4} \end{bmatrix}$$

$$\hat{R} \qquad \vec{i} \qquad \vec{V}$$

ESEMPIO



$I_g = 1\ [A]$

$V_g = 2\ [V]$

$R_1 = R_2 = 1\ [\Omega]$

$R_3 = R_4 = 2\ [\Omega]$

VERIFICARE BILANCIO

$$I_g = i_c - i_b \Rightarrow i_c = I_g + i_b \Rightarrow i_b = i_c - I_g$$

$$\text{a)} \quad i_a(R_1) + (i_a - i_c)\cdot R_3 + (i_a - i_b)R_2 = 0 \Rightarrow i_a(R_1 + R_3 + R_2) + i_c(-R_3 - R_2) + I_g \cdot R_2 = 0$$

$$\text{b+c)} \quad (i_b - i_a)R_2 + (i_c - i_a)\cdot R_3 + i_c R_4 - V_g = 0 \qquad i_c(R_2 + R_3 + R_4) - I_g R_2 + i_a(-R_2 - R_3) + \quad - V_g = 0$$

$$i_a(R_1 + R_2 + R_3) + i_c(-R_3 - R_2) = -I_g R_2$$

$$i_a(-R_2 - R_3) + i_c(R_2 + R_3 + R_4) = V_g + I_g R_2$$

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 - R_2 \\ -R_3 - R_2 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_a \\ i_c \end{bmatrix} = \begin{bmatrix} -I_g R_2 \\ V_g + I_g R_2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} i_a \\ i_c \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\begin{vmatrix} 4 & -3 \\ -3 & 5 \end{vmatrix} = \Delta \hat{R} = 11$$

$$i_a = \frac{\begin{vmatrix} -1 & -3 \\ 3 & 5 \end{vmatrix}}{\Delta \hat{R}} = \frac{-5+9}{11} = \frac{4}{11} \, [A] \simeq 0.36$$

$$i_c = \frac{\begin{vmatrix} 4 & -1 \\ -3 & 3 \end{vmatrix}}{11} = \frac{12-3}{11} = \frac{9}{11} \simeq 0.81 \, [A]$$

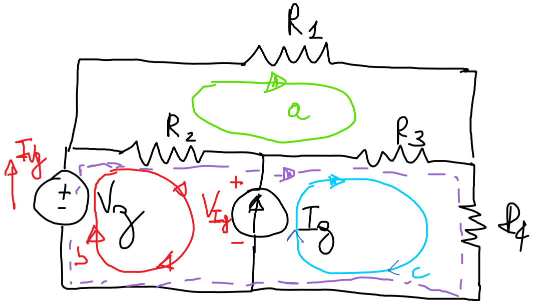
$6[\bar{A}]$

$$\begin{vmatrix} 4 & -3 \\ -3 & 5 \end{vmatrix} = \Delta \hat{R} = 11$$

$$i_c = i_b + I_g$$

$i_a = 0.36\,[A]$ $i_b = -0.19\,[A]$ $i_c = +0.81\,[A]$

$$i_b = i_c - I_g = 0.81\,[A] - 1\,[A] = -0.$$



$I_g = 1\,[A]$

$V_g = 2\,[V]$

$R_1 = R_2 = 1\,[\Omega]$

$R_3 = R_4 = 2\,[\Omega]$

$P_{R_1} = i_a^2 \cdot R_1 \approx 0.13\,[W]$

$P_{R_2} = R_2 \cdot (i_a - i_b)^2 \simeq 0.3\,[W]$

$P_{R_3} = R_3\,(i_a - i_c)^2 \simeq 0.4\,[W]$

$P_{R_4} = R_4\, i_c^2 \simeq 1.31\,[W]$

$$\sum 0.3 + 0.4 + 0.13 +$$

$P^g_{V_g} = V_g \cdot I_{V_g} = 2\,[V] \cdot (-0.19)\,[A] \simeq -0.38\,[W]$

$P^g_{I_g} = I_g \cdot V_{I_g} = 1\,[A] \cdot 2.55\,[V] = 2.55\,[W]$

b $\quad -V_g + (i_b - i_a) R_2 + V_{I_g} = 0 \Rightarrow$

$V_{I_g} = (i_a - i_b) \cdot R_2 + V_g$

$V_{I_g} = 0.55 + 2 = 2.55\,[V]$

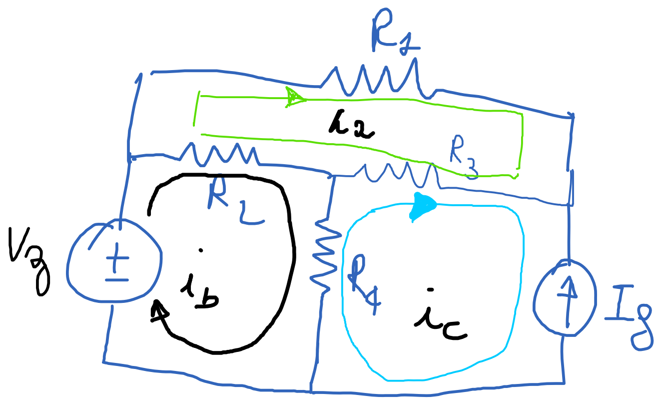
$$\sum 2.55 - 0.38 = 2.14\,? \Rightarrow 2.17 \simeq 2.14$$

$[W]$

19 [A]

$1.31 \simeq 2.14$ [W]

$[V]$



$I_g = 1\,[A] \quad V_g = 2\,[V]$

$R_1, R_2 = 1\,[\Omega]$

$R_3, R_4 = 2\,\Omega$

$i_c = -I_g = -1\,[A] \quad i_a = ? \quad i_b = ?$

(a) $i_a R_1 + (i_a + I_g) R_3 + (i_a - i_b) R_2 = 0$

(b) $(i_b - i_a) R_2 + (i_b + I_g) \cdot R_4 - V_g = 0$

$$i_a (R_1 + R_2 + R_3) + i_b (-R_2) = -I_g R_3$$

$$i_a (-R_2) + i_b (R_2 + R_4) = V_g - I_g R_4$$

$$\underbrace{\begin{bmatrix} R_1+R_2+R_3 & -R_2 \\ -R_2 & R_2+R_4 \end{bmatrix}}_{\hat{R}} \underbrace{\begin{bmatrix} i_a \\ i_b \end{bmatrix}}_{\vec{i}} = \underbrace{\begin{bmatrix} -I_g R_3 \\ V_g - I_g R_4 \end{bmatrix}}_{\vec{V}} \qquad \begin{bmatrix} 4 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

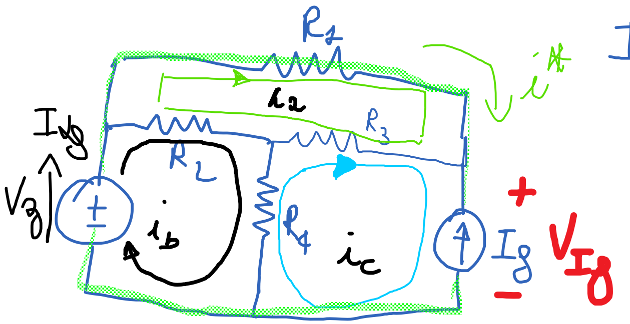
$\Downarrow$

$\Delta_{\hat{R}} = 12 - 1 = 11$

$$i_a = \frac{\begin{vmatrix} -2 & -1 \\ 0 & 3 \end{vmatrix}}{11} = \frac{-6}{11} = \frac{-6}{11} \; [A]$$

$$i_b = \frac{\begin{vmatrix} 4 & 4 \\ -1 & 0 \end{vmatrix}}{11} = \frac{0-2}{11} = -\frac{2}{11} \; [A]$$

$i_c = -1 \; [A]$



$I_g = 1 \, [A]$ $\quad V_g = 2 \, [V]$

$R_1, R_2 = 1 \, [\Omega]$

$R_3, R_4 = 2 \, \Omega$

$P_{R_1} = i_a^2 \cdot R_1 \simeq 0.297 \, [W]$

$P_{R_2} = (i_a - i_b)^2 \cdot R_2 \simeq 0.132 \, [W]$

$P_{R_3} = (i_a - i_c)^2 \cdot R_3 \simeq 0.413 \, [W]$

$P_{R_4} = (i_c - i_b)^2 \cdot R_4 \simeq 1.34 \, [W]$

$\sum \simeq 2.1$

$$P_{V_g} = V_g \cdot I_{V_g} = 2[V] \cdot \left(-\frac{2}{11}\right) A = -0.36 \, [W]$$

$P_{I_g} = I_g (V_{I_g}) \Rightarrow$ LKT $\quad -V_g + i_a R_1 + V_{I_g} = 0 \Rightarrow$

$$V_{I_g} = V_g - i_a R_1 = 2[V] + \frac{6}{11}[V] = \frac{28}{11}[V] \simeq 2.54 \, [V]$$

$P_{I_g} = 1A \; 2.54V = 2.54 \, [W]$

-3 [W]

$$\sum P^{gen} = \underbrace{2.54 - 0.36}_{2.18\ [W]}\ [W] = \sum P_{ass} = 2.13\ [W]$$

BILANCIO VERIFICATO