

IMPEDENZA E AMMETTENZA

$$\frac{\bar{V}}{\bar{I}} = Z \in \mathbb{C} \quad \frac{\bar{I}}{\bar{V}} = Y \in \mathbb{C}$$

$$\frac{v(t)}{i(t)} = R \in \mathbb{R} = \frac{i(t)}{v(t)} = G \in \mathbb{R}$$

CONDENSATORE

$$Z_C = \frac{1}{j\omega C}$$

$$Y_C = j\omega C$$

$$\lim_{\omega/f \rightarrow 0} |Z_C| \rightarrow +\infty \quad |Y_C| \rightarrow 0 \quad \text{circuito aperto}$$

$$\frac{|\bar{I}|}{|\bar{V}|} = |Y_C|$$

$$\lim_{\omega/f \rightarrow +\infty} |Z_C| \rightarrow 0 \quad |Y_C| \rightarrow +\infty$$

$$|V| = |Z_C| |I| \Rightarrow \text{corto circuito}$$

INDUTTORE

$$Z_L = j\omega L$$

$$Y_L = \frac{1}{j\omega L}$$

$$\lim_{\omega/f \rightarrow 0} |Z_L| \rightarrow 0 \Rightarrow \text{corto circuito}$$

$$\lim_{\omega/f \rightarrow \infty} |Y_L| \rightarrow 0 \Rightarrow \text{circuito aperto}$$

LEGGI DI KIRCHHOFF PER I FASORI

$$\sum_{\alpha} \pm V_{\alpha}(t) = 0 \quad \sum_{\beta} \pm i_{\beta}(t) = 0$$

$$\begin{aligned} V_{\alpha}(t) &= \operatorname{Re} \left\{ \bar{V}_{\alpha} e^{j\omega_0 t} \right\} \\ &= \frac{\bar{V}_{\alpha} e^{j\omega_0 t} + \bar{V}_{\alpha}^* e^{-j\omega_0 t}}{2} \end{aligned} \quad \left. \vphantom{\begin{aligned} V_{\alpha}(t) &= \operatorname{Re} \left\{ \bar{V}_{\alpha} e^{j\omega_0 t} \right\} \\ &= \frac{\bar{V}_{\alpha} e^{j\omega_0 t} + \bar{V}_{\alpha}^* e^{-j\omega_0 t}}{2} \end{aligned}} \right]$$

$$\cos(\varphi) = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$$

$$\sum_{\alpha} \pm V_{\alpha}(t) = \sum_{\alpha} \pm \operatorname{Re} \left\{ \bar{V}_{\alpha} e^{j\omega_0 t} \right\} = 0$$

$$\forall t \quad = \sum_{\alpha} \pm \operatorname{Re} \left\{ \bar{V}_{\alpha} \right\} = 0$$

$$\sum_{\alpha} \pm \bar{V}_{\alpha} = 0$$

$$A_1 \cos(\omega_0 t + \varphi_1) + A_2 \cos(\omega_0 t + \varphi_2) =$$

$$A_3 \cos(\omega_0 t + \varphi_3) \doteq$$

$$\underbrace{A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2}}_{\text{SOMMA DEI FASORI}} = \underbrace{A_3 e^{j\varphi_3}}_{\substack{\text{FASONE} \\ \text{DELLA} \\ \text{SOMMA}}}$$

$$\sum \pm V_{\alpha}(t) = 0$$

$$\sum \pm \bar{V}_{\alpha} = 0 \quad \nwarrow \substack{\text{FASONE} \\ \text{DELLA} \\ \text{SOMMA}}$$

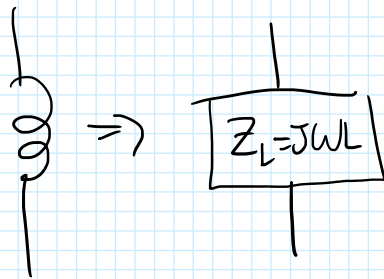
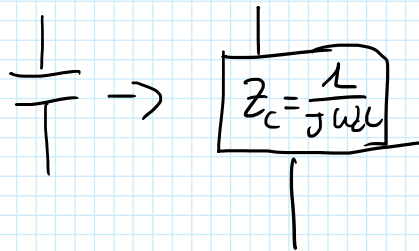
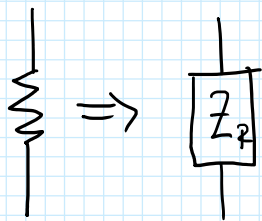
CIRCUITO SIMBOICO NEL DOMINIO DEI FASORI

- 1) sostituire i generatori sinusoidali con generatori costanti a valori complessi pari al fasore dei generatori

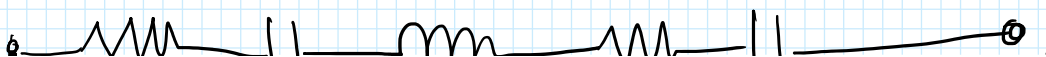
$$V_g \quad \text{~} \quad V_g = A \cos(\omega t + \varphi_{V_g}) \Leftrightarrow \frac{+}{-} \bar{V}_g$$

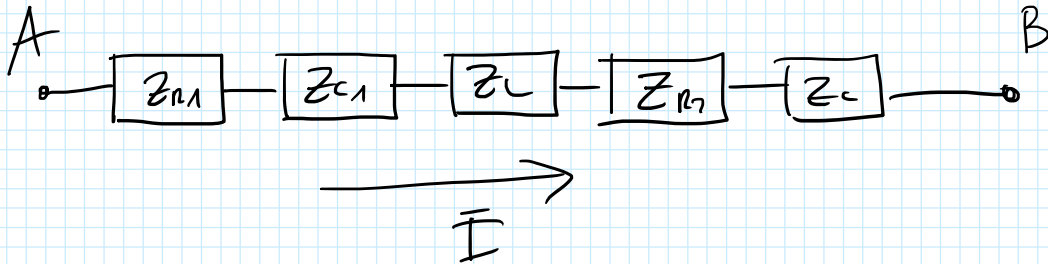
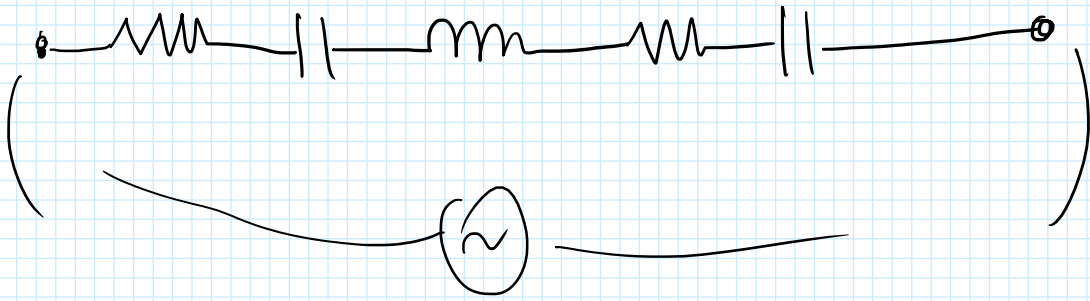
$$I_g \quad \text{~} \quad I_g = B \cos(\omega t + \varphi_{I_g}) \Rightarrow \text{~} \bar{I}$$

2) Sostituiamo i bipoli con le
rispettive impedenze/admittenze

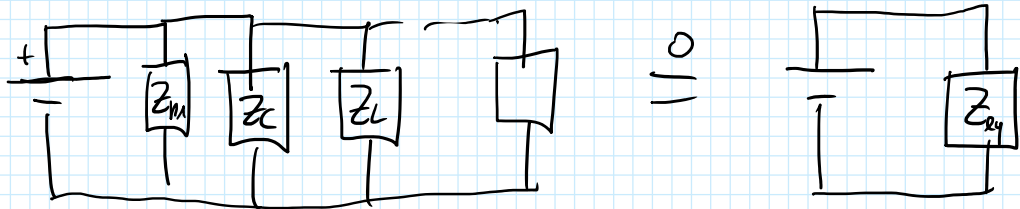
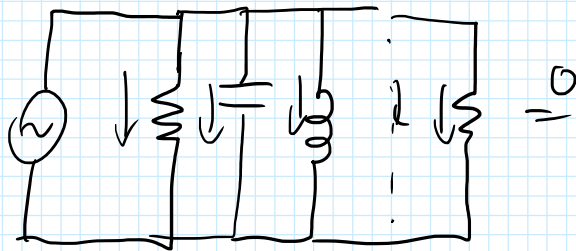


PER IL CIRCUITO SIMBOLICO VALGONO
TUTTI I METODI DI ANALISI E TUTTI
I TEOREMI (THEVENIN, NORTON, MILLMAN, SOSTITUZIONE
SOV, PSE, EQUIVALENZA DEI
GEN. REALI)





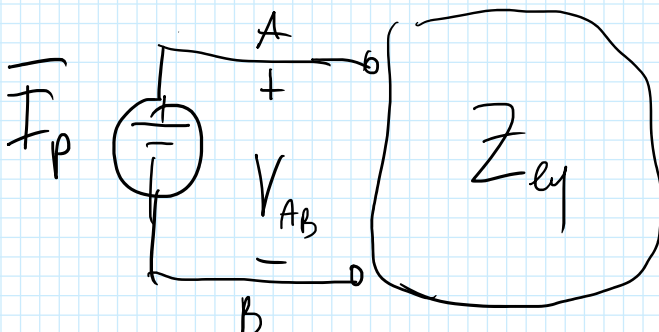
$$\bar{V} = \bar{I} \left(\sum_k Z_k \right)$$



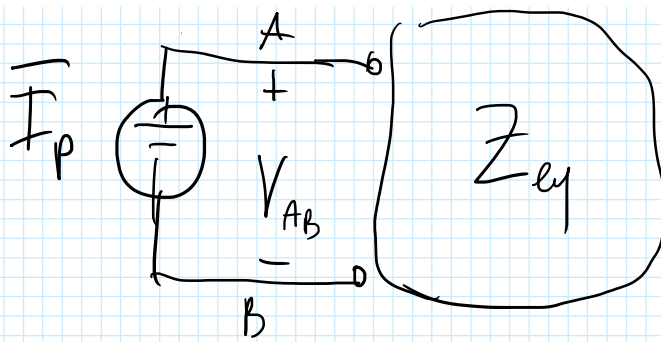
$$Y_{eq} = \sum_{\alpha} Y_{\alpha} \Rightarrow Z_{eq} = \left(\sum_{\alpha} \frac{1}{Z_{\alpha}} \right)^{-1}$$

$$Z = [\Omega]$$

$$Y = [\Omega^{-1}]$$

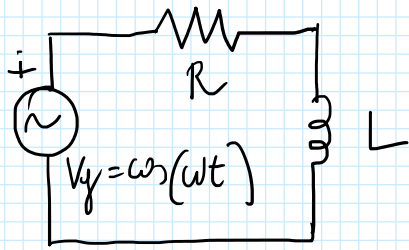


$$Z_{eq} = \frac{\bar{V}_{AB}}{\bar{I}_p}$$



$$Z_{Ly} = \frac{\overline{V_{AB}}}{\overline{I_p}}$$

$$p(t) = v(t) \dot{u}(t)$$



DISEGNARE IL CIRCUITO SIMBOLICO

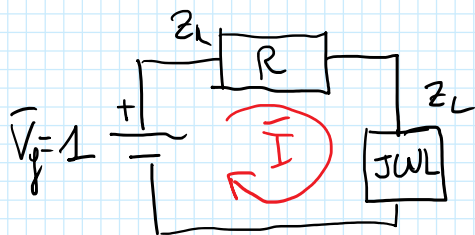
→ SOSTITUIRE I GEN. CON I FASORI

→ SOSTITUIRE I BIPOLI CON LE RISPETTIVE IMPEDENZE

$$V_g = \cos(\omega t) \Rightarrow \bar{V}_g = 1$$

$$R \rightarrow Z_R = R$$

$$L \rightarrow Z_L = j\omega L$$



$$\bar{I} = \frac{\bar{V}_g}{Z_R + Z_L} = \frac{\bar{V}_g}{R + j\omega L}$$

$$\bar{I} = \left(\frac{1}{R + j\omega L} \right) \xrightarrow{\text{TEMPO}} i(t) = I_0 \cos(\omega t + \varphi_I)$$

$$I_0 = |\bar{I}| \quad \varphi_I = \text{Arg}(\bar{I})$$

$$I_0 = \frac{1}{\sqrt{R^2 + \omega^2 L^2}} \quad \frac{a + jb}{c + jd} = \bar{X} = \frac{A_1 e^{j\varphi_1}}{A_2 e^{j\varphi_2}}$$

$$|\bar{X}| = \frac{A_1}{A_2} = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$$

$$\varphi_I = \text{Arg}(1) - \text{Arg}(R + j\omega L) \quad \text{Arg}(\bar{v}) = \varphi_v - \theta$$

$$\varphi_I = \text{Arg}(1) - \text{Arg}(R + j\omega L) \quad \text{Arg}(\bar{x}) = \varphi_1 - \varphi_2$$

$$\varphi_I = 0 - \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\hat{i}(t) = \frac{1}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) [A]$$

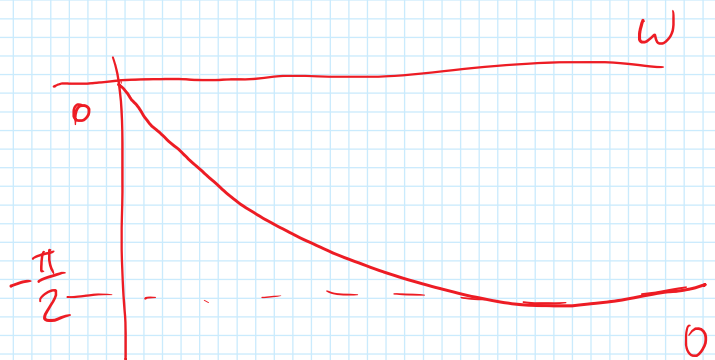
$$\hat{i}(t) = \underbrace{\frac{1}{\sqrt{R^2 + \omega^2 L^2}}}_A \cos\left(\underbrace{\omega_0 t}_{\omega_0} + \underbrace{\arctan\left(-\frac{\omega L}{R}\right)}_{\varphi}\right)$$

$-\tan^{-1}\left(\frac{\omega L}{R}\right)$

$$|\hat{i}(t)| = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\omega \rightarrow \infty \rightarrow |\hat{i}(t)| \rightarrow 0$$

$$\varphi_I = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$



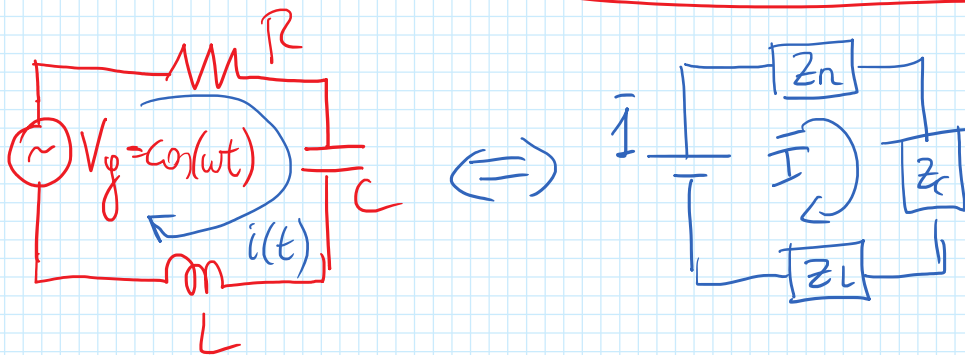
$$\omega \rightarrow 0 \quad |\hat{i}(t)| = \frac{1}{R}$$

$$|Z_n| = R = |Z_L| \quad \omega?$$

$$R = \omega L \quad \omega^* = \left(\frac{R}{L}\right) = \frac{1}{\tau}$$

$\sim R$

$|Z_n|$



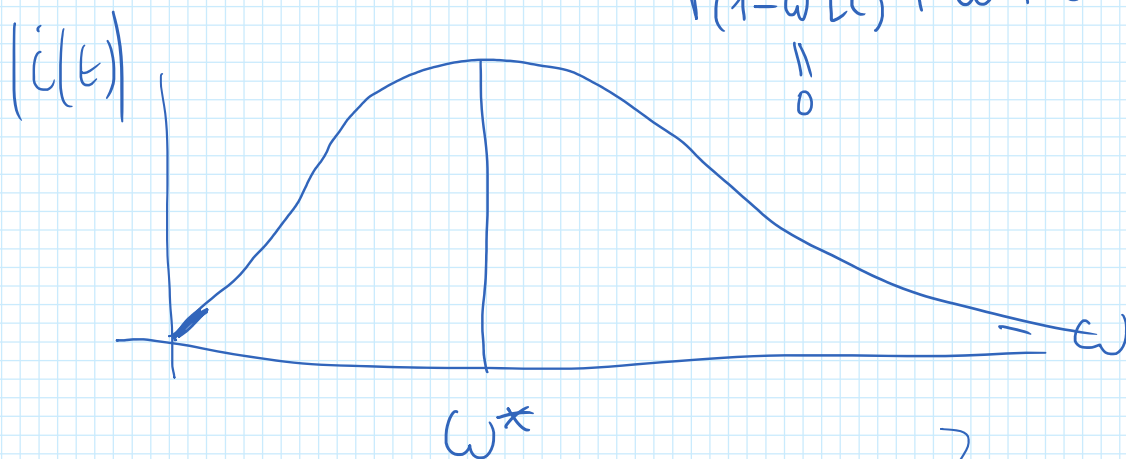
$$\bar{I} = \frac{\bar{V}_g}{Z_n + Z_L + Z_C} = \frac{\bar{V}_g}{R + j\omega L - \frac{j}{\omega C}}$$

$$\frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$\bar{I} = \frac{\omega C \bar{V}_g}{\omega R C + j\omega^2 L C - j} \cdot \frac{j}{j} = \frac{j\omega C}{1 + j\omega R C - \omega^2 L C} \bar{V}_g$$

$$\bar{I} = \bar{V}_g \cdot \frac{j\omega C}{(1 - \omega^2 L C) + j\omega R C}$$

$$|\dot{i}(t)| = |\bar{I}| = |\bar{V}_g| \cdot \frac{\omega C}{\sqrt{(1 - \omega^2 L C)^2 + \omega^2 R^2 C^2}}$$



$$\left. \begin{aligned} 1 - \omega^{*2} L C &= 0 \\ \omega^{*2} L C &= 1 \end{aligned} \right\} \omega^* = \frac{1}{\sqrt{LC}}$$

$$\lim_{\omega \rightarrow \omega^*} |\bar{I}| = |\bar{V}_g| \cdot \frac{\omega C}{1} = |\bar{V}_g|$$

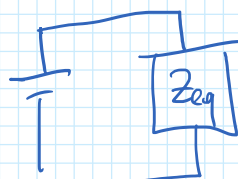
$$|\bar{i}(t)| = |\bar{I}| = |V_g| \cdot \frac{\omega C}{\omega R C} = \frac{|V_g|}{R}$$

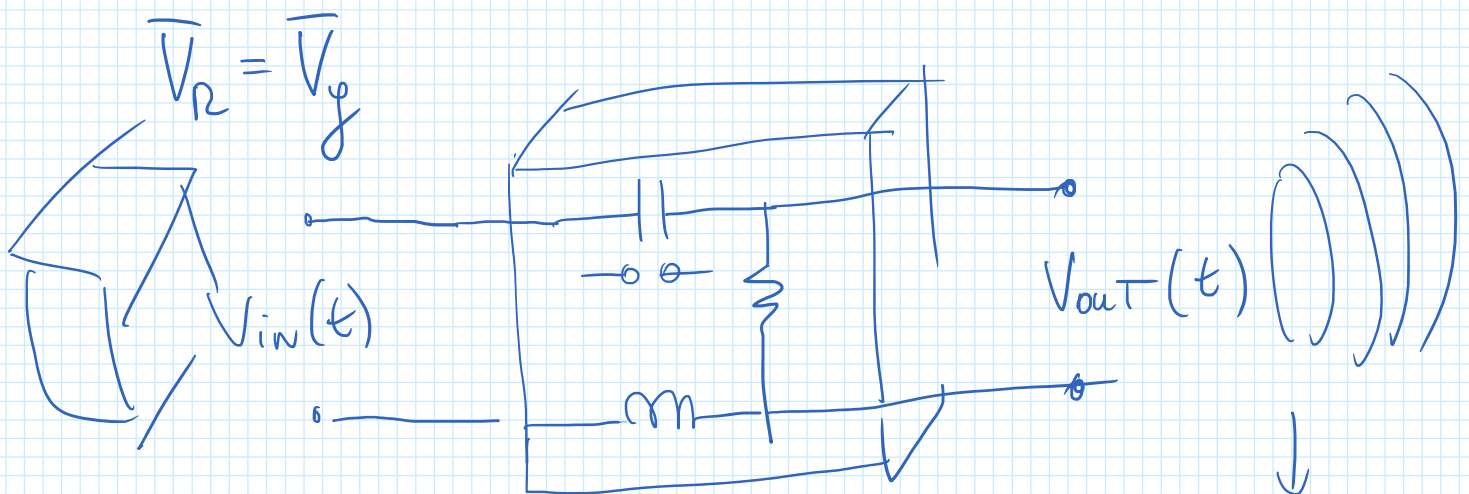
per $\omega = \omega^* = \frac{1}{\sqrt{LC}}$ $Z_L = -Z_C$

$$j\omega^* L = \frac{j}{\omega^* C}$$

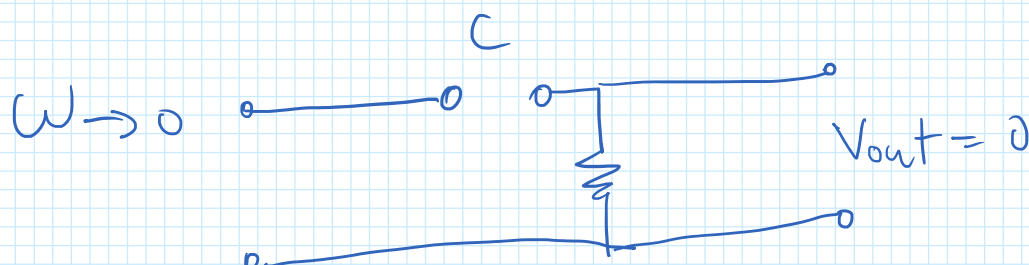
$$\frac{L}{\sqrt{LC}} = \frac{\sqrt{LC}}{C}$$

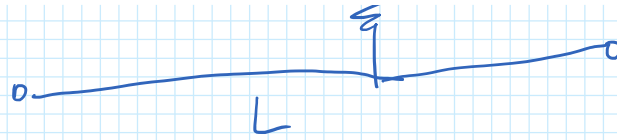
$$\sqrt{\frac{L}{C}} = \sqrt{\frac{L}{C}}$$

 $Z_{eq} = Z_R + Z_L + Z_C = Z_R$ $\omega = (\sqrt{LC})^{-1}$

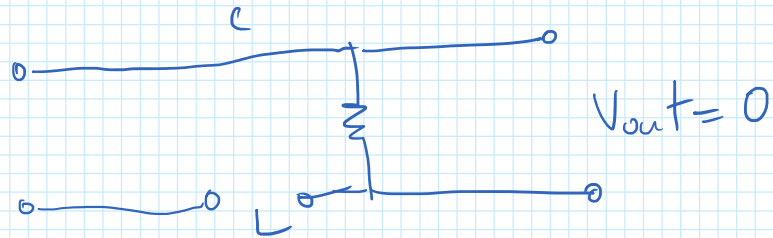


$$V_R(\omega) = I(t, \omega) \cdot R$$

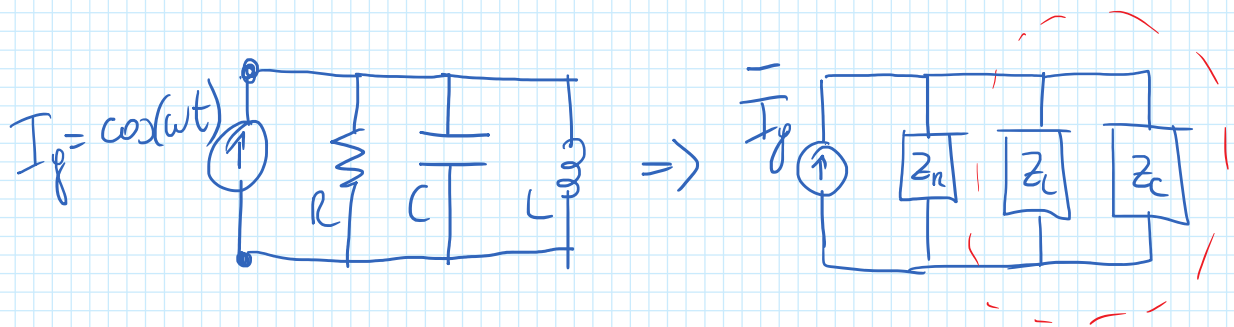
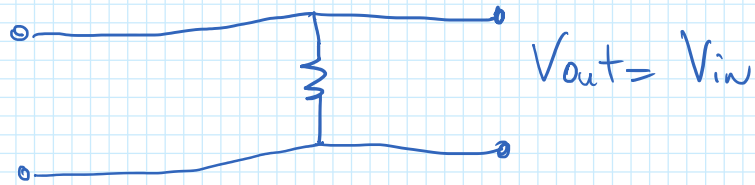




$\omega \rightarrow \infty$



$\omega = \omega^* = \frac{1}{\sqrt{LC}}$



$$Z_{LC} = \frac{Z_L Z_C}{Z_L + Z_C}$$

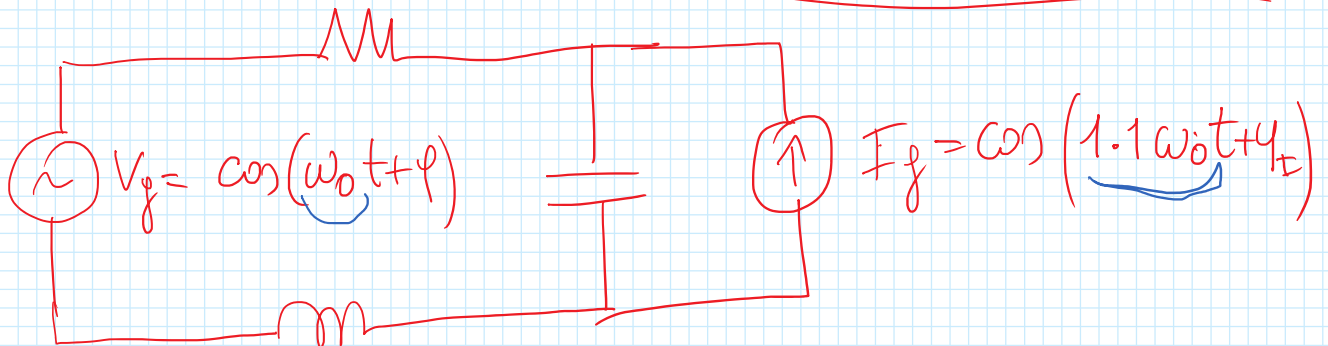
$$\omega = \omega^* = \frac{1}{\sqrt{LC}}$$

$$Z_L + Z_C = 0$$

$$Z_{LC} \rightarrow +\infty$$

$$Y_C + Y_L = 0$$

$Y_{eq} = Y_{in} + Y_L + Y_C$



IN PRESENZA DI ECCITAZIONI SINUSOIDALI
A FREQ DIFFERENTI APPLICO IL PSE

→ SPENGO TUTTI I GEN. TRANNÈ QUELLI
AD UNA STESSA FREQ f_0

→ APPLICO IL METODO DEI FASORI
ALLA FREQ f_0

→ CALCOLO LE GRANDEZZE TEMPORALI
ALLA FREQ f_0

→ RIPETO I PASSAGGI SO PRA $\forall f_k$

→ SOMMO LE GRANDEZZE TEMPORALI
ALLE VARI FREQUENZE