1 Interpretation of simulation results

Our DGP is the following

$$y_t = \sum_{h=0}^{d} x_{tm-h} w_h + \varepsilon_t, \tag{1}$$

where ε_t is i.i.d white noise with variance σ_ε^2, and

$$x_\tau = x_{\tau-1} + v_\tau,$$

where v_τ is a stationary process with zero mean.

We propose 4 tests for testing the restriction

$$w_h = g(h, \gamma)$$

Our goal is to explore the properties of the proposed test statistics. In order to define tests rewrite the (1) model in the following way:

$$y_t = \sum_{h=0}^{d-1} v_{tm-h} \beta_h + \beta_d x_{tm-d} + \varepsilon_t, \tag{2}$$

where

$$\beta_h = \sum_{j=0}^{h} w_h.$$

We can also rewrite the model in the following way:

$$y_t = \sum_{h=0}^{d} v_{tm-h} \beta_h + \beta_d x_{tm-d-1} + \varepsilon_t \tag{3}$$

Note that coefficient β_d is used twice in this expression.

1.1 NLS tests

Estimate the (3) by OLS. Calculate the following residual:

$$\hat{u}_t = y_t - \hat{\beta}_d^{OLS} x_{tm-d-1}$$

and estimate $\hat{\gamma}$ by NLS from the following model:

$$\hat{u}_t = \sum_{h=0}^{d} v_{tm-h} f(h, \gamma) + \epsilon_t \tag{4}$$

where

$$f(h, \gamma) = \sum_{j=0}^{h} g(h, \gamma).$$

Define

$$\hat{D} = \frac{\partial \boldsymbol{f}}{\partial \gamma}(\hat{\gamma})$$

where $\boldsymbol{f}' = (f(1,\gamma),\dots,fd,\gamma)$. Denote $\hat{\beta}^{NLS} = \boldsymbol{f}(\hat{\gamma})$. Then the test for the restriction is

$$T_1 := h'\Sigma^{-1}h,$$

where

$$h = \frac{\sqrt{n_d}}{\hat{\sigma}_\varepsilon}(\hat{\beta}^{OLS} - \hat{\beta}^{NLS})$$
$$\Sigma = \left(\frac{1}{n_d}\boldsymbol{V}'\boldsymbol{V}\right)^{-1} - \hat{D}\left(\frac{1}{n_d}\hat{D}'\boldsymbol{V}'\boldsymbol{V}\hat{D}\right)\hat{D}'$$

Here n_d is the effective number of observations and $\boldsymbol{V}$ is the matrix of regressors from the vector form of (3).

The second test is the variation of this test. Estimate the following regression by OLS:

$$y_t = \beta_d x_{tm-d-1} + \eta_t$$

Calculate the following residual:

$$\hat{\mathfrak{u}}_t = y_t - \hat{\beta}_d x_{tm-d-1}.$$

Then proceed with the estimation of $\hat{\gamma}$ as in previous case, i.e. from the model (4), but with $\hat{\mathfrak{u}}_t$ instead of $\hat{u}_t$. The formula for the statistic T_2 is then identical to the one for statistic T_1, with the exception of $\hat{\beta}^{OLS}$ which is calculated via OLS from the following model:

$$\hat{\mathfrak{u}}_t = \sum_{h=0}^{d} v_{tm-h}\beta_h + e_t$$

For both the statistics the question is how to estimate the variance σ_ε^2 of the ε_t. For statistics T_1 and T_2 estimate it as the usual error variance from the OLS regression of (3). The variance can also be estimated from the NLS regression (4). Denote by T_1' and T_2', the variants of statistics T_1 and T_2 with σ_ε^2 estimated as normalized residual sum of squares of the NLS regression (4).

Under null hypothesis all the statistics have assymptotic χ_r^2 distribution, where r is the number of hyperparameters γ.

2 T_d tests

The idea behind the tests T_d is to compare the coefficient β_d estimated by OLS and NLS. Estimate by OLS the regression (2). Define the residual

$$\hat{u}_t = y_t - \beta_d^{OLS} x_{tm-d}$$

Note that compared to NLS test we estimate the same coefficient β_d, but the regressor now is x_{tm-d} instead of x_{tm-d-1}. Then proceed with estimation of $\hat{\gamma}$ by NLS from the regression

$$\hat{u}_t = \sum_{h=0}^{d-1} v_{tm-h} f(h,\gamma) + \epsilon_t \tag{5}$$

Note again, that we do not estimate restricted β_d in this model compared to model (4). On the other hand since we can calculate restrictedcoefficient β_d in the following way: $\beta_d^{NLS} = f(d,\hat{\gamma})$. Given $\hat{\gamma}$ we can calculate $\hat{D}$. Define $\boldsymbol{\hat{d}}$ the last row of this matrix, and $\hat{D}_{-d}$ the submatrix of $\hat{D}$ consisting of the first $d-1$ rows of $\hat{D}$. Then the test statistic T_3 is defined by

$$T_3 = \frac{h^2}{\Sigma},$$

where

$$h = \frac{\sqrt{n_d}}{\hat{\sigma}_\varepsilon}(\beta_d^{OLS} - \beta_d^{NLS}),$$

$$\Sigma = \hat{\boldsymbol{d}}\left(\frac{1}{n_d}\hat{D}'_{-d}\boldsymbol{V}'\boldsymbol{V}\hat{D}_{-d}\right)\hat{\boldsymbol{d}}'$$

The corresponding variant of T_3 is defined similar to T_2. First estimate β_d^{OLS} from the following regression:

$$y_t = \beta_d x_{tm-d} + \eta_t$$

Calculate the following residual:

$$\hat{\mathrm{u}}_t = y_t - \hat{\beta}_d^{OLS} x_{tm-d}.$$

and use it to estimate $\hat{\gamma}$ from the model (5) with $\hat{u}_t$ changed to $\hat{\mathrm{u}}_t$. The statistic T_4 is defined using the same formula as T_3.

The variants T_3' and T_4' are defined analogously, by using estimate $\hat{\sigma}_\varepsilon^2$ from the NLS regression (5) instead of OLS regression (2).

The statistics T_3, T_3', T_4, T_4' all have assymptotic distribution of χ_1^2.

3 Monte Carlo simulation

For calculating empirical size, simulate y_t as (1) with weights

$$g(h, \gamma) = \gamma_1 \frac{\exp(\gamma_2 h + \gamma_3 h^2)}{\sum_{j=0}^{d} \exp(\gamma_2 j + \gamma_3 j^2)} \tag{6}$$

with $\gamma = (10, 2, -10)$. The white noise ε_t is taken to be i.i.d random variables with normal distribution with zero mean and $\sigma_\varepsilon = 7$.

The innovations v_τ are taken to be AR(1) process with zero mean and $\rho = 0.5$ and $\rho = 0.9$. The innovations for v_τ are taken to be ni.i.d normal with unit variance. The burn-in for AR(1) process was the default setting used by `R` function `arima.sim`.

The first $10m$ values (m being the frequency ratio) of x_τ values were used as burn-in, i.e. were discarded, when calculating y_t.

To estimate the empirical size 2000 replications for each combination of sample size, frequency ratio, number of lags and AR(1) parameter were simulated. We used sample sizes 75, 125, 200, 500 and 1000 and frequency ratios 12, 24. The number of lags were chosen to be 11, 23 for frequency ratio 12 and 11, 23 and 47 for frequency ratio 24.

The optimisation for NLS problem was done using function `optim` with settings `method="BFGS"` and `control=list(maxit=1000,reltol=sqrt(.Machine$double.eps)/10)`. The starting values were chosen to be the $(10, 2, -10)$ with random noise added.

To guard against the divergence we discard the replications for which `optim` algorithm failed to converge. We note the effective number of replications.

3.1 Results for empirical sizes

The empirical sizes for statistics $T_1, ..., T_4$ and $T_1', ..., T_4'$ are shown in tables 1 and 2. We can immediately note that empirical sizes are off for statistics T_2 and T_4. They improve when NLS standard errors are used for statistics T_2' and T_4', but they do not converge to nominal sizes. This indicates that OLS standard error estimate is wrong for statistics T_2 and T_4. It is interesting to note that T_4' performs better when d is larger than m, which is opposite to behaviour of T_2'. This is especialy evident for $\rho = 0.9$. But in both cases the statistics over-estimate or under-estimate the nominal size.

If we look at statistics T_1 and T_3, we see that their performance is similar, only T_3 performs better on small sample sizes. The results for T_1' and T_3' are similar.

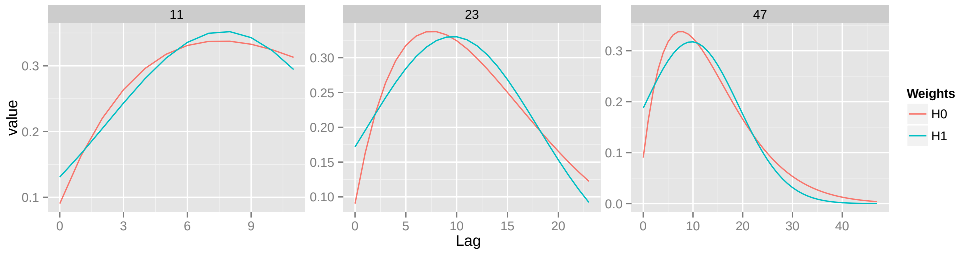

Figure 1: Weights for different lags

3.2 Analysis of empirical power

We also calculated the empirical power of the tests. For testing power we simulated the model (1) with weights:

$$w_h = g(h, \gamma) = \gamma_1 h \exp(\gamma_2 h + \gamma_3 h^2) \tag{7}$$

and $\gamma_0 = (10, -10, -10)$. We then estimated the model using the weights (6). The starting values for optimisation procedure were chosen to be

$$\operatorname{argmin}_\gamma \sum_{h=0}^{d} (g_0(h, \gamma_0) - g_1(h, \gamma))^2$$

where g_0 are weights (7) and g_1 are the weights (6). For each replication random noise was added to this value.

The simulation results are presented in tables 3 and 4

The weights for g_0 and g_1 are plotted in figure 1, the respective cumulative weights are plotted in figure 2. Looking at the graphs it is clear why the empirical power is low for $d = 11$. For this scenario we can see that T_3 and T_4 outperform T_1 and T_2. Using NLS standard errors decreases power for smaller sample sizes, but in the end the results are similar, i.e. T_3' is the best, with T_4' catching up for higher sample sizes.

3.3 Analysis of size-adjusted empirical power

We also calculated size-adjusted empirical power for all the statistics. We used the replications for empirical size calculations to compute the quantiles for empirical size 0.05. The results are presented in tables 5 and 6. The results are comparable to empirical power results, with expected drop in size-adjusted power for small sample sizes.

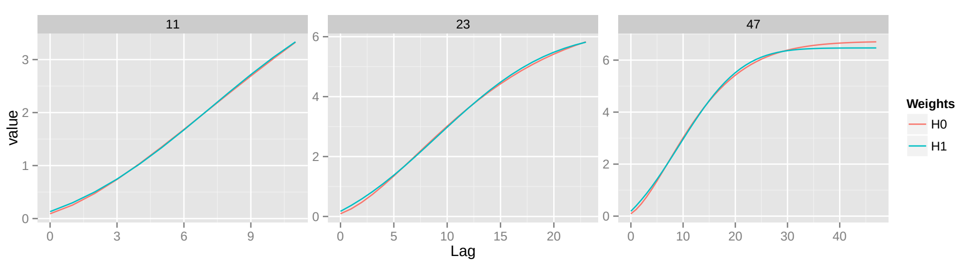

Figure 2: Cumulative weights for different lags

4 Appendix

Table 1: Empirical sizes for OLS standard errors

	n	d	m	ρ	T_1	R_{T_1}	T_2	R_{T_2}	T_3	R_{T_3}	T_4	R_{T_4}
1	75	11	12	0.5	0.080	2000	0.818	2000	0.048	2000	0.447	2000
2	125	11	12	0.5	0.067	2000	0.782	2000	0.058	2000	0.396	2000
3	200	11	12	0.5	0.061	2000	0.711	2000	0.055	2000	0.328	2000
4	500	11	12	0.5	0.044	2000	0.566	2000	0.045	2000	0.236	2000
5	1000	11	12	0.5	0.054	2000	0.414	2000	0.057	2000	0.145	2000
6	75	23	12	0.5	0.116	2000	0.948	2000	0.064	2000	0.595	2000
7	125	23	12	0.5	0.085	2000	0.914	2000	0.057	2000	0.538	2000
8	200	23	12	0.5	0.058	2000	0.903	2000	0.048	2000	0.497	2000
9	500	23	12	0.5	0.060	2000	0.829	2000	0.054	2000	0.367	2000
10	1000	23	12	0.5	0.055	2000	0.782	2000	0.056	2000	0.285	2000
11	75	11	24	0.5	0.075	2000	0.799	2000	0.059	2000	0.434	2000
12	125	11	24	0.5	0.066	2000	0.769	2000	0.046	2000	0.405	2000
13	200	11	24	0.5	0.053	2000	0.718	2000	0.048	2000	0.342	2000
14	500	11	24	0.5	0.056	2000	0.568	2000	0.050	2000	0.216	2000
15	1000	11	24	0.5	0.044	2000	0.428	2000	0.048	2000	0.150	2000
16	75	23	24	0.5	0.115	2000	0.927	2000	0.051	2000	0.596	2000
17	125	23	24	0.5	0.073	2000	0.895	2000	0.049	2000	0.519	2000
18	200	23	24	0.5	0.063	2000	0.858	2000	0.059	2000	0.468	2000
19	500	23	24	0.5	0.051	2000	0.769	2000	0.044	2000	0.325	2000
20	1000	23	24	0.5	0.049	2000	0.678	2000	0.059	2000	0.246	2000
21	75	47	24	0.5	0.251	2000	0.969	2000	0.065	2000	0.750	2000
22	125	47	24	0.5	0.120	2000	0.971	2000	0.059	2000	0.713	2000
23	200	47	24	0.5	0.085	2000	0.949	2000	0.046	2000	0.649	2000
24	500	47	24	0.5	0.056	2000	0.917	2000	0.048	2000	0.557	2000
25	1000	47	24	0.5	0.052	2000	0.904	1999	0.058	2000	0.472	2000
26	75	11	12	0.9	0.070	2000	0.949	1993	0.058	2000	0.747	1986
27	125	11	12	0.9	0.059	2000	0.925	2000	0.055	2000	0.664	1998
28	200	11	12	0.9	0.059	2000	0.908	2000	0.049	2000	0.617	2000
29	500	11	12	0.9	0.050	2000	0.854	1995	0.048	2000	0.527	1999
30	1000	11	12	0.9	0.046	2000	0.787	2000	0.053	1999	0.425	2000
31	75	23	12	0.9	0.112	2000	0.986	1996	0.063	2000	0.826	1999
32	125	23	12	0.9	0.088	2000	0.978	1992	0.058	2000	0.794	1997
33	200	23	12	0.9	0.059	2000	0.968	1996	0.048	2000	0.783	1997
34	500	23	12	0.9	0.059	2000	0.961	1998	0.052	2000	0.684	1998
35	1000	23	12	0.9	0.051	2000	0.939	1984	0.051	2000	0.622	1984
36	75	11	24	0.9	0.080	2000	0.933	2000	0.058	2000	0.689	1996
37	125	11	24	0.9	0.060	2000	0.898	1999	0.054	2000	0.625	1999
38	200	11	24	0.9	0.052	2000	0.891	2000	0.056	2000	0.565	2000
39	500	11	24	0.9	0.066	2000	0.809	1997	0.057	2000	0.464	2000
40	1000	11	24	0.9	0.048	2000	0.738	2000	0.047	2000	0.387	2000
41	75	23	24	0.9	0.121	2000	0.979	1999	0.059	2000	0.834	1998
42	125	23	24	0.9	0.081	2000	0.966	1997	0.047	2000	0.789	1999
43	200	23	24	0.9	0.074	2000	0.958	1998	0.049	2000	0.759	1999
44	500	23	24	0.9	0.056	2000	0.935	2000	0.050	2000	0.685	2000
45	1000	23	24	0.9	0.053	2000	0.908	1995	0.052	2000	0.587	1988
46	75	47	24	0.9	0.252	2000	0.994	1996	0.062	2000	0.923	1997
47	125	47	24	0.9	0.125	2000	0.992	1985	0.065	2000	0.911	1980
48	200	47	24	0.9	0.082	1999	0.989	1885	0.051	1999	0.897	1882
49	500	47	24	0.9	0.057	2000	0.981	1963	0.051	2000	0.847	1983
50	1000	47	24	0.9	0.065	1996	0.964	1677	0.045	1992	0.778	1721

Table 2: Empirical sizes for NLS standard errors

	n	d	m	ρ	T_1'	$R_{T_1'}$	T_2'	$R_{T_2'}$	T_3'	$R_{T_3'}$	T_4'	$R_{T_4'}$
1	75	11	12	0.5	0.041	2000	0.035	2000	0.047	2000	0.067	2000
2	125	11	12	0.5	0.051	2000	0.041	2000	0.058	2000	0.068	2000
3	200	11	12	0.5	0.050	2000	0.041	2000	0.054	2000	0.070	2000
4	500	11	12	0.5	0.040	2000	0.044	2000	0.045	2000	0.059	2000
5	1000	11	12	0.5	0.051	2000	0.049	2000	0.058	2000	0.051	2000
6	75	23	12	0.5	0.033	2000	0.051	2000	0.060	2000	0.035	2000
7	125	23	12	0.5	0.046	2000	0.072	2000	0.057	2000	0.029	2000
8	200	23	12	0.5	0.039	2000	0.082	2000	0.048	2000	0.034	2000
9	500	23	12	0.5	0.051	2000	0.090	2000	0.055	2000	0.028	2000
10	1000	23	12	0.5	0.051	2000	0.089	2000	0.056	2000	0.036	2000
11	75	11	24	0.5	0.043	2000	0.038	2000	0.062	2000	0.054	2000
12	125	11	24	0.5	0.047	2000	0.036	2000	0.045	2000	0.052	2000
13	200	11	24	0.5	0.041	2000	0.044	2000	0.050	2000	0.062	2000
14	500	11	24	0.5	0.052	2000	0.051	2000	0.052	2000	0.064	2000
15	1000	11	24	0.5	0.042	2000	0.051	2000	0.048	2000	0.049	2000
16	75	23	24	0.5	0.030	2000	0.022	2000	0.052	2000	0.059	2000
17	125	23	24	0.5	0.035	2000	0.032	2000	0.051	2000	0.062	2000
18	200	23	24	0.5	0.043	2000	0.043	2000	0.057	2000	0.060	2000
19	500	23	24	0.5	0.041	2000	0.051	2000	0.044	2000	0.056	2000
20	1000	23	24	0.5	0.041	2000	0.043	2000	0.060	2000	0.058	2000
21	75	47	24	0.5	0.001	2000	0.000	2000	0.058	2000	0.027	2000
22	125	47	24	0.5	0.020	2000	0.047	2000	0.060	2000	0.037	2000
23	200	47	24	0.5	0.035	2000	0.081	2000	0.045	2000	0.027	2000
24	500	47	24	0.5	0.041	2000	0.101	2000	0.048	2000	0.029	2000
25	1000	47	24	0.5	0.045	2000	0.092	1999	0.058	2000	0.041	2000
26	75	11	12	0.9	0.041	2000	0.034	1993	0.058	2000	0.148	1986
27	125	11	12	0.9	0.049	2000	0.048	2000	0.054	2000	0.113	1998
28	200	11	12	0.9	0.048	2000	0.056	2000	0.049	2000	0.108	2000
29	500	11	12	0.9	0.044	2000	0.047	1995	0.048	2000	0.094	1999
30	1000	11	12	0.9	0.043	2000	0.052	2000	0.053	1999	0.085	2000
31	75	23	12	0.9	0.030	2000	0.055	1996	0.068	2000	0.067	1999
32	125	23	12	0.9	0.044	2000	0.076	1992	0.057	2000	0.044	1997
33	200	23	12	0.9	0.037	2000	0.095	1996	0.046	2000	0.043	1997
34	500	23	12	0.9	0.051	2000	0.107	1998	0.053	2000	0.027	1998
35	1000	23	12	0.9	0.046	2000	0.108	1984	0.052	2000	0.030	1984
36	75	11	24	0.9	0.051	2000	0.034	2000	0.060	2000	0.100	1996
37	125	11	24	0.9	0.043	2000	0.041	1999	0.054	2000	0.080	1999
38	200	11	24	0.9	0.046	2000	0.042	2000	0.058	2000	0.061	2000
39	500	11	24	0.9	0.063	2000	0.055	1997	0.057	2000	0.062	2000
40	1000	11	24	0.9	0.046	2000	0.045	2000	0.047	2000	0.063	2000
41	75	23	24	0.9	0.032	2000	0.024	1999	0.057	2000	0.100	1998
42	125	23	24	0.9	0.041	2000	0.035	1997	0.045	2000	0.106	1999
43	200	23	24	0.9	0.049	2000	0.038	1998	0.050	2000	0.103	1999
44	500	23	24	0.9	0.047	2000	0.044	2000	0.050	2000	0.089	2000
45	1000	23	24	0.9	0.050	2000	0.046	1995	0.053	2000	0.082	1988
46	75	47	24	0.9	0.000	2000	0.001	1996	0.053	2000	0.052	1997
47	125	47	24	0.9	0.019	2000	0.042	1985	0.061	2000	0.030	1980
48	200	47	24	0.9	0.030	1999	0.081	1885	0.054	1999	0.024	1882
49	500	47	24	0.9	0.040	2000	0.108	1963	0.053	2000	0.021	1983
50	1000	47	24	0.9	0.056	1996	0.110	1677	0.046	1992	0.023	1721

Table 3: Empirical power for OLS standard errors

	n	d	m	ρ	T_1	R_{T_1}	T_2	R_{T_2}	T_3	R_{T_3}	T_4	R_{T_4}
1	75	11	12	0.5	0.108	2000	0.509	2000	0.141	2000	0.248	2000
2	125	11	12	0.5	0.111	2000	0.431	2000	0.171	2000	0.243	2000
3	200	11	12	0.5	0.123	2000	0.362	2000	0.213	2000	0.272	2000
4	500	11	12	0.5	0.243	2000	0.369	2000	0.425	2000	0.442	2000
5	1000	11	12	0.5	0.479	2000	0.533	2000	0.674	2000	0.675	2000
6	75	23	12	0.5	0.543	2000	0.958	2000	0.734	2000	0.718	2000
7	125	23	12	0.5	0.784	2000	0.976	2000	0.882	2000	0.784	2000
8	200	23	12	0.5	0.968	2000	0.995	2000	0.979	2000	0.868	2000
9	500	23	12	0.5	1.000	2000	1.000	2000	1.000	2000	0.988	2000
10	1000	23	12	0.5	1.000	2000	1.000	2000	1.000	2000	1.000	2000
11	75	11	24	0.5	0.093	2000	0.484	2000	0.132	2000	0.249	2000
12	125	11	24	0.5	0.103	2000	0.419	2000	0.154	2000	0.244	2000
13	200	11	24	0.5	0.118	2000	0.372	2000	0.234	2000	0.302	2000
14	500	11	24	0.5	0.250	2000	0.364	2000	0.464	2000	0.477	2000
15	1000	11	24	0.5	0.466	2000	0.526	2000	0.676	2000	0.684	2000
16	75	23	24	0.5	0.538	2000	0.936	2000	0.708	2000	0.699	2000
17	125	23	24	0.5	0.793	2000	0.972	2000	0.895	2000	0.778	2000
18	200	23	24	0.5	0.958	2000	0.994	2000	0.978	2000	0.873	2000
19	500	23	24	0.5	1.000	2000	1.000	2000	1.000	2000	0.988	2000
20	1000	23	24	0.5	1.000	2000	1.000	2000	1.000	2000	1.000	2000
21	75	47	24	0.5	0.964	2000	0.999	2000	0.968	2000	0.829	2000
22	125	47	24	0.5	1.000	2000	1.000	1999	0.999	2000	0.839	2000
23	200	47	24	0.5	1.000	2000	1.000	2000	1.000	2000	0.872	2000
24	500	47	24	0.5	1.000	2000	1.000	2000	1.000	2000	0.962	2000
25	1000	47	24	0.5	1.000	2000	1.000	2000	1.000	2000	0.996	2000
26	75	11	12	0.9	0.101	2000	0.821	1969	0.197	2000	0.532	1970
27	125	11	12	0.9	0.117	2000	0.778	1999	0.260	1988	0.478	1972
28	200	11	12	0.9	0.150	2000	0.733	2000	0.384	2000	0.503	2000
29	500	11	12	0.9	0.320	2000	0.736	2000	0.708	2000	0.699	2000
30	1000	11	12	0.9	0.622	2000	0.799	2000	0.918	2000	0.888	2000
31	75	23	12	0.9	0.919	2000	0.999	1996	0.998	2000	0.906	1999
32	125	23	12	0.9	0.997	2000	1.000	1993	1.000	2000	0.914	1995
33	200	23	12	0.9	1.000	2000	1.000	2000	1.000	2000	0.962	2000
34	500	23	12	0.9	1.000	2000	1.000	1998	1.000	2000	0.996	2000
35	1000	23	12	0.9	1.000	2000	1.000	1998	1.000	2000	1.000	1996
36	75	11	24	0.9	0.106	2000	0.770	1988	0.204	2000	0.460	1996
37	125	11	24	0.9	0.114	2000	0.738	2000	0.274	1982	0.445	1979
38	200	11	24	0.9	0.140	2000	0.686	2000	0.383	2000	0.480	1999
39	500	11	24	0.9	0.310	2000	0.660	2000	0.691	2000	0.658	2000
40	1000	11	24	0.9	0.625	2000	0.761	2000	0.928	2000	0.873	2000
41	75	23	24	0.9	0.932	2000	0.999	1999	0.998	2000	0.895	2000
42	125	23	24	0.9	0.999	2000	1.000	1997	1.000	2000	0.928	2000
43	200	23	24	0.9	1.000	2000	1.000	2000	1.000	2000	0.938	1999
44	500	23	24	0.9	1.000	2000	1.000	2000	1.000	2000	0.991	2000
45	1000	23	24	0.9	1.000	2000	1.000	2000	1.000	2000	0.998	1999
46	75	47	24	0.9	1.000	2000	1.000	1997	1.000	2000	0.958	1994
47	125	47	24	0.9	1.000	2000	1.000	1975	1.000	2000	0.961	1972
48	200	47	24	0.9	1.000	2000	1.000	1933	1.000	1999	0.966	1962
49	500	47	24	0.9	1.000	2000	1.000	1941	1.000	2000	0.988	1973
50	1000	47	24	0.9	1.000	1997	1.000	1855	1.000	2000	0.998	1867

Table 4: Empirical power for NLS standard errors

	n	d	m	ρ	T_1'	$R_{T_1'}$	T_2'	$R_{T_2'}$	T_3'	$R_{T_3'}$	T_4'	$R_{T_4'}$
1	75	11	12	0.5	0.061	2000	0.049	2000	0.142	2000	0.129	2000
2	125	11	12	0.5	0.081	2000	0.067	2000	0.172	2000	0.153	2000
3	200	11	12	0.5	0.109	2000	0.089	2000	0.214	2000	0.195	2000
4	500	11	12	0.5	0.230	2000	0.203	2000	0.423	2000	0.388	2000
5	1000	11	12	0.5	0.474	2000	0.439	2000	0.673	2000	0.648	2000
6	75	23	12	0.5	0.292	2000	0.083	2000	0.693	2000	0.231	2000
7	125	23	12	0.5	0.658	2000	0.197	2000	0.860	2000	0.374	2000
8	200	23	12	0.5	0.955	2000	0.367	2000	0.973	2000	0.544	2000
9	500	23	12	0.5	1.000	2000	0.785	2000	1.000	2000	0.880	2000
10	1000	23	12	0.5	1.000	2000	0.980	2000	1.000	2000	0.989	2000
11	75	11	24	0.5	0.057	2000	0.044	2000	0.134	2000	0.121	2000
12	125	11	24	0.5	0.078	2000	0.059	2000	0.154	2000	0.147	2000
13	200	11	24	0.5	0.104	2000	0.078	2000	0.232	2000	0.217	2000
14	500	11	24	0.5	0.237	2000	0.200	2000	0.463	2000	0.426	2000
15	1000	11	24	0.5	0.461	2000	0.417	2000	0.675	2000	0.653	2000
16	75	23	24	0.5	0.278	2000	0.055	2000	0.668	2000	0.282	2000
17	125	23	24	0.5	0.681	2000	0.197	2000	0.871	2000	0.439	2000
18	200	23	24	0.5	0.942	2000	0.414	2000	0.969	2000	0.619	2000
19	500	23	24	0.5	1.000	2000	0.874	2000	1.000	2000	0.936	2000
20	1000	23	24	0.5	1.000	2000	0.993	2000	1.000	2000	0.999	2000
21	75	47	24	0.5	0.115	2000	0.003	2000	0.913	2000	0.277	2000
22	125	47	24	0.5	0.996	2000	0.180	1999	0.987	2000	0.387	2000
23	200	47	24	0.5	1.000	2000	0.353	2000	1.000	2000	0.478	2000
24	500	47	24	0.5	1.000	2000	0.730	2000	1.000	2000	0.717	2000
25	1000	47	24	0.5	1.000	2000	0.951	2000	1.000	2000	0.934	2000
26	75	11	12	0.9	0.058	2000	0.044	1969	0.198	2000	0.254	1970
27	125	11	12	0.9	0.091	2000	0.064	1999	0.260	1988	0.217	1972
28	200	11	12	0.9	0.130	2000	0.093	2000	0.384	2000	0.276	2000
29	500	11	12	0.9	0.311	2000	0.195	2000	0.708	2000	0.517	2000
30	1000	11	12	0.9	0.615	2000	0.425	2000	0.918	2000	0.804	2000
31	75	23	12	0.9	0.747	2000	0.085	1996	0.998	2000	0.251	1999
32	125	23	12	0.9	0.992	2000	0.161	1993	1.000	2000	0.365	1995
33	200	23	12	0.9	1.000	2000	0.240	2000	1.000	2000	0.502	2000
34	500	23	12	0.9	1.000	2000	0.538	1998	1.000	2000	0.877	2000
35	1000	23	12	0.9	1.000	2000	0.788	1998	1.000	2000	0.989	1996
36	75	11	24	0.9	0.070	2000	0.041	1988	0.203	2000	0.190	1996
37	125	11	24	0.9	0.088	2000	0.050	2000	0.276	1982	0.186	1979
38	200	11	24	0.9	0.119	2000	0.073	2000	0.384	2000	0.231	1999
39	500	11	24	0.9	0.302	2000	0.186	2000	0.689	2000	0.509	2000
40	1000	11	24	0.9	0.618	2000	0.397	2000	0.928	2000	0.784	2000
41	75	23	24	0.9	0.774	2000	0.067	1999	0.996	2000	0.322	2000
42	125	23	24	0.9	0.994	2000	0.150	1997	1.000	2000	0.451	2000
43	200	23	24	0.9	1.000	2000	0.272	2000	1.000	2000	0.572	1999
44	500	23	24	0.9	1.000	2000	0.625	2000	1.000	2000	0.871	2000
45	1000	23	24	0.9	1.000	2000	0.897	2000	1.000	2000	0.989	1999
46	75	47	24	0.9	0.989	2000	0.014	1997	1.000	2000	0.289	1994
47	125	47	24	0.9	1.000	2000	0.125	1975	1.000	2000	0.376	1972
48	200	47	24	0.9	1.000	2000	0.246	1933	1.000	1999	0.501	1962
49	500	47	24	0.9	1.000	2000	0.535	1941	1.000	2000	0.700	1973
50	1000	47	24	0.9	1.000	1997	0.818	1855	1.000	2000	0.919	1867

Table 5: Adjusted empirical power for OLS standard errors

	n	d	m	ρ	T_1	R_{T_1}	T_2	R_{T_2}	T_3	R_{T_3}	T_4	R_{T_4}
1	75	11	12	0.5	0.070	2000	0.000	2000	0.143	2000	0.003	2000
2	125	11	12	0.5	0.081	2000	0.000	2000	0.162	2000	0.002	2000
3	200	11	12	0.5	0.107	2000	0.000	2000	0.203	2000	0.004	2000
4	500	11	12	0.5	0.265	2000	0.000	2000	0.441	2000	0.038	2000
5	1000	11	12	0.5	0.472	2000	0.001	2000	0.664	2000	0.418	2000
6	75	23	12	0.5	0.385	2000	0.000	2000	0.704	2000	0.035	2000
7	125	23	12	0.5	0.670	2000	0.003	2000	0.874	2000	0.045	2000
8	200	23	12	0.5	0.965	2000	0.002	2000	0.979	2000	0.082	2000
9	500	23	12	0.5	1.000	2000	0.011	2000	1.000	2000	0.627	2000
10	1000	23	12	0.5	1.000	2000	0.024	2000	1.000	2000	0.995	2000
11	75	11	24	0.5	0.070	2000	0.000	2000	0.129	2000	0.004	2000
12	125	11	24	0.5	0.084	2000	0.000	2000	0.167	2000	0.003	2000
13	200	11	24	0.5	0.116	2000	0.000	2000	0.236	2000	0.007	2000
14	500	11	24	0.5	0.234	2000	0.000	2000	0.464	2000	0.067	2000
15	1000	11	24	0.5	0.469	2000	0.000	2000	0.682	2000	0.382	2000
16	75	23	24	0.5	0.368	2000	0.004	2000	0.707	2000	0.035	2000
17	125	23	24	0.5	0.741	2000	0.004	2000	0.897	2000	0.035	2000
18	200	23	24	0.5	0.952	2000	0.004	2000	0.973	2000	0.091	2000
19	500	23	24	0.5	1.000	2000	0.009	2000	1.000	2000	0.859	2000
20	1000	23	24	0.5	1.000	2000	0.103	2000	1.000	2000	0.999	2000
21	75	47	24	0.5	0.764	2000	0.020	2000	0.961	2000	0.102	2000
22	125	47	24	0.5	0.999	2000	0.013	1999	0.998	2000	0.080	2000
23	200	47	24	0.5	1.000	2000	0.022	2000	1.000	2000	0.158	2000
24	500	47	24	0.5	1.000	2000	0.031	2000	1.000	2000	0.516	2000
25	1000	47	24	0.5	1.000	2000	0.062	2000	1.000	2000	0.958	2000
26	75	11	12	0.9	0.074	2000	0.000	1969	0.181	2000	0.016	1970
27	125	11	12	0.9	0.092	2000	0.000	1999	0.253	1988	0.025	1972
28	200	11	12	0.9	0.135	2000	0.000	2000	0.389	2000	0.039	2000
29	500	11	12	0.9	0.312	2000	0.000	2000	0.712	2000	0.007	2000
30	1000	11	12	0.9	0.626	2000	0.000	2000	0.920	2000	0.048	2000
31	75	23	12	0.9	0.828	2000	0.006	1996	0.998	2000	0.063	1999
32	125	23	12	0.9	0.992	2000	0.007	1993	1.000	2000	0.061	1995
33	200	23	12	0.9	1.000	2000	0.005	2000	1.000	2000	0.059	2000
34	500	23	12	0.9	1.000	2000	0.004	1998	1.000	2000	0.078	2000
35	1000	23	12	0.9	1.000	2000	0.015	1998	1.000	2000	0.541	1996
36	75	11	24	0.9	0.070	2000	0.000	1988	0.195	2000	0.038	1996
37	125	11	24	0.9	0.098	2000	0.000	2000	0.264	1982	0.030	1979
38	200	11	24	0.9	0.124	2000	0.000	2000	0.359	2000	0.018	1999
39	500	11	24	0.9	0.260	2000	0.000	2000	0.673	2000	0.007	2000
40	1000	11	24	0.9	0.631	2000	0.001	2000	0.935	2000	0.086	2000
41	75	23	24	0.9	0.820	2000	0.005	1999	0.998	2000	0.051	2000
42	125	23	24	0.9	0.997	2000	0.009	1997	1.000	2000	0.050	2000
43	200	23	24	0.9	1.000	2000	0.003	2000	1.000	2000	0.050	1999
44	500	23	24	0.9	1.000	2000	0.009	2000	1.000	2000	0.111	2000
45	1000	23	24	0.9	1.000	2000	0.015	2000	1.000	2000	0.954	1999
46	75	47	24	0.9	1.000	2000	0.021	1997	1.000	2000	0.070	1994
47	125	47	24	0.9	1.000	2000	0.014	1975	1.000	2000	0.059	1972
48	200	47	24	0.9	1.000	2000	0.011	1933	1.000	1999	0.099	1962
49	500	47	24	0.9	1.000	2000	0.060	1941	1.000	2000	0.511	1973
50	1000	47	24	0.9	1.000	1997	0.035	1855	1.000	2000	0.859	1867

Table 6: Adjusted empirical power for NLS standard errors

	n	d	m	ρ	T_1'	$R_{T_1'}$	T_2'	$R_{T_2'}$	T_3'	$R_{T_3'}$	T_4'	$R_{T_4'}$
1	75	11	12	0.5	0.070	2000	0.067	2000	0.146	2000	0.106	2000
2	125	11	12	0.5	0.081	2000	0.083	2000	0.161	2000	0.125	2000
3	200	11	12	0.5	0.107	2000	0.097	2000	0.203	2000	0.170	2000
4	500	11	12	0.5	0.265	2000	0.210	2000	0.441	2000	0.356	2000
5	1000	11	12	0.5	0.472	2000	0.440	2000	0.663	2000	0.650	2000
6	75	23	12	0.5	0.385	2000	0.079	2000	0.657	2000	0.286	2000
7	125	23	12	0.5	0.670	2000	0.159	2000	0.843	2000	0.427	2000
8	200	23	12	0.5	0.965	2000	0.276	2000	0.975	2000	0.597	2000
9	500	23	12	0.5	1.000	2000	0.725	2000	1.000	2000	0.905	2000
10	1000	23	12	0.5	1.000	2000	0.970	2000	1.000	2000	0.992	2000
11	75	11	24	0.5	0.070	2000	0.052	2000	0.126	2000	0.113	2000
12	125	11	24	0.5	0.084	2000	0.079	2000	0.171	2000	0.144	2000
13	200	11	24	0.5	0.116	2000	0.088	2000	0.233	2000	0.189	2000
14	500	11	24	0.5	0.234	2000	0.195	2000	0.460	2000	0.394	2000
15	1000	11	24	0.5	0.469	2000	0.411	2000	0.681	2000	0.655	2000
16	75	23	24	0.5	0.368	2000	0.106	2000	0.665	2000	0.273	2000
17	125	23	24	0.5	0.741	2000	0.245	2000	0.867	2000	0.413	2000
18	200	23	24	0.5	0.952	2000	0.426	2000	0.964	2000	0.605	2000
19	500	23	24	0.5	1.000	2000	0.871	2000	1.000	2000	0.936	2000
20	1000	23	24	0.5	1.000	2000	0.993	2000	1.000	2000	0.998	2000
21	75	47	24	0.5	0.764	2000	0.113	2000	0.900	2000	0.325	2000
22	125	47	24	0.5	0.999	2000	0.184	1999	0.985	2000	0.409	2000
23	200	47	24	0.5	1.000	2000	0.323	2000	1.000	2000	0.508	2000
24	500	47	24	0.5	1.000	2000	0.676	2000	1.000	2000	0.741	2000
25	1000	47	24	0.5	1.000	2000	0.935	2000	1.000	2000	0.941	2000
26	75	11	12	0.9	0.074	2000	0.057	1969	0.173	2000	0.173	1970
27	125	11	12	0.9	0.092	2000	0.070	1999	0.249	1988	0.125	1972
28	200	11	12	0.9	0.135	2000	0.084	2000	0.386	2000	0.158	2000
29	500	11	12	0.9	0.312	2000	0.219	2000	0.713	2000	0.426	2000
30	1000	11	12	0.9	0.626	2000	0.413	2000	0.919	2000	0.799	2000
31	75	23	12	0.9	0.828	2000	0.076	1996	0.997	2000	0.292	1999
32	125	23	12	0.9	0.992	2000	0.129	1993	1.000	2000	0.445	1995
33	200	23	12	0.9	1.000	2000	0.176	2000	1.000	2000	0.539	2000
34	500	23	12	0.9	1.000	2000	0.452	1998	1.000	2000	0.904	2000
35	1000	23	12	0.9	1.000	2000	0.703	1998	1.000	2000	0.991	1996
36	75	11	24	0.9	0.070	2000	0.055	1988	0.186	2000	0.148	1996
37	125	11	24	0.9	0.097	2000	0.054	2000	0.262	1982	0.136	1979
38	200	11	24	0.9	0.124	2000	0.091	2000	0.364	2000	0.204	1999
39	500	11	24	0.9	0.260	2000	0.165	2000	0.671	2000	0.473	2000
40	1000	11	24	0.9	0.631	2000	0.414	2000	0.934	2000	0.781	2000
41	75	23	24	0.9	0.820	2000	0.106	1999	0.996	2000	0.248	2000
42	125	23	24	0.9	0.997	2000	0.175	1997	1.000	2000	0.376	2000
43	200	23	24	0.9	1.000	2000	0.292	2000	1.000	2000	0.482	1999
44	500	23	24	0.9	1.000	2000	0.645	2000	1.000	2000	0.835	2000
45	1000	23	24	0.9	1.000	2000	0.903	2000	1.000	2000	0.982	1999
46	75	47	24	0.9	1.000	2000	0.090	1997	1.000	2000	0.291	1994
47	125	47	24	0.9	1.000	2000	0.135	1975	1.000	2000	0.408	1972
48	200	47	24	0.9	1.000	2000	0.213	1933	1.000	1999	0.537	1962
49	500	47	24	0.9	1.000	2000	0.427	1941	1.000	2000	0.754	1973
50	1000	47	24	0.9	1.000	1997	0.737	1855	1.000	2000	0.934	1867