1. (10 points) Multiple-Choice Questions. Select a single answer choice that best fits the question.

   (a) (1 point) Consider two bank accounts, one earning **simple interest** and one earning **monthly compound interest.** Suppose both start with the same initial deposit (and you make no other deposits or withdrawals) and earn the same APR.
   After two years, the balance of the account paying **simple interest** will be _____ the balance of the account paying **monthly compound interest.**

   A. greater than
   B. less than
   C. greater than or equal to
   D. less than or equal to

   B

   (b) (1 point) With the same initial deposit, APR, and length of time, which of the following accounts would yield the greatest interest?

   A. continuous compounding
   B. quarterly compounding
   C. daily compounding
   D. monthly compounding

   A

   (c) (1 point) Is it possible that APR and APY are the same?

   A. Yes. They are the same if continuous compounding is applied.
   B. Yes. They are the same if there is only one compounding during the year.
   C. No. It's impossible that APR and APY are the same. APR is always greater than APY.
   D. No. It's impossible that APR and APY are the same. APR is always less than APY.

   B

   (d) (1 point) Suppose you deposited $50 per month into a savings plan for 4 years and at the end of that period your balance was $3000. What was the amount you earned in interest?

   A. $200
   B. $400
   C. $600
   D. Impossible to compute without knowing the APR

   C

(e) (1 point) Government support for education is falling 1.75% per year. What is the most appropriate pattern for this situation?

D

A. Linear Growth
B. Linear Decay
C. Exponential Growth
D. Exponential Decay

(f) (1 point) Which of the following is an example of linear growth?

A

A. Money in a bank account earning simple interest at an APR of 3%.
B. Money in a bank account earning compound interest at an APR of 3%.
C. Money in a bank account earning compound interest at an APR of 30%.
D. You have a magic bank account in which your balance doubled each day.

(g) (2 points) At 9:30 you place a single bacterium in a bottle, and at 9:31 it divides into 2 bacteria, which at 9:32 divide into 4 bacteria, and so on. How many bacteria will be in the bottle at 10:00?

B

A. 2×30
B. 2^{30}
C. 30^2
D. 2×10^{30}

(h) (2 points) Radioactive tritium (hydrogen-3) has a half-life about 12 years, which means that if you start with 200 kilograms of tritium, 100 kilograms will decay during the first 12 years. How much will decay during the next 12 years?

C

A. 24 kilograms
B. 25 kilograms
C. 50 kilograms
D. 100 kilograms

2. (5 points) The half-life of methadone in the blood stream is 36 hours.

   (a) (1 point) What portion of the original amount 45 hours after ingesting methadone? $\left(\frac{1}{2}\right)^{\frac{45h}{36h}} \approx 42\%$

   (b) (2 points) What portion of the original amount 20 hours after ingesting methadone? *Express your answer as a percentage.*

$\left(\frac{1}{2}\right)^{\frac{20h}{36h}} \approx 68\%$

   (c) (2 points) If the current level in an individual's bloodstream is 750 mg/mL, calculate the level of methadone we expect to find in them 20 hours later.

$$750 \text{ mg/ mL} \times \left(\frac{1}{2}\right)^{\frac{20}{36}} = 510.30 \text{ mg/ mL}$$

3. (5 points) You invest \$8000 in a retirement account that has a doubling time of six years.

   (a) (2 points) What is the value of the account after 18 years?

$\$8000 \times 2^{\frac{18y}{6y}} = \$64{,}000$

   (b) (3 points) Calculate the value of the account after 11 years. *Hint: think about how you would express the answer from (a) in the same format with the powers of 2 ... and don't be afraid of fractions.*

$\$8000 \times 2^{\frac{11y}{6y}} = \$28{,}508.80$

4. (2 points) It took me ninety minutes to write the first fifth of this exam. *Assuming I continue at this pace,* how much time in total should I expect to spend writing this exam? *Fill in the blanks. This is standard short conversion problem. Show fractions and ALL units to receive full credit.*

90 mins $= \frac{1}{5}$ exam

450 mins = 1 exam

(7.5 hrs)

5. (15 points) For each problem, select a single answer choice that (i) expresses the correct type of growth, (ii) the correct formula to use, **and** (iii) **if asked**, use this formula to calculate the answer to a related question, including the appropriate units (**no units = no credit, even if numerical answer is correct**), but you don't need to show your work.

(a) (2 points) The property values in Denver increase 21.7% each year. The average home price in 2016 is \$350,000. Calculate the average price in 2025.

C

A. Linear Growth
B. Linear Decay
C. Exponential Growth
D. Exponential Decay

A

A. $\$350,000 \times (1 + 0.217)^9$
B. $\$350,000 \times (1 - 0.217)^9$
C. $\$350,000 + (9 \times 0.217)$
D. $\$350,000 - (9 \times 0.217)$

(b) (2 points) The balance of your savings account increases by \$40 each month. Your balance at the beginning of February is \$750. Calculate the balance at the beginning of November that same year.

A

A. Linear Growth
B. Linear Decay
C. Exponential Growth
D. Exponential Decay

C

A. $\$750 - 40 \times 9$
B. $\$750 \times (1 + .40)^9$
C. $\$750 + 40 \times 9$
D. $\$750 \times (1 - .40)^9$

(c) (2 points) The amount of radioactive material in a sample is cut in half every 15,000 years. How much of a 10 gram sample will remain after 30,000 years?

D

A. Linear Growth
B. Linear Decay
C. Exponential Growth
D. Exponential Decay

B

A. $10\text{g} \times (1 - \frac{1}{2})^{15,000}$
B. $10\text{g} \times (1 - \frac{1}{2})^2$
C. $10\text{g} - \frac{30,000}{15,000}$
D. $10\text{g} + \frac{30,000}{15,000}$

(d) (4 points) How much remains after 45,000 years?

$\left(\frac{1}{2}\right)^{\frac{45000}{15000}} = \frac{1}{2}^3 = \frac{1}{8}$ $10\text{g} \times \frac{1}{8} = \frac{5}{4}\text{g}$ $= 1.25\text{g}$

(e) (2 points) The length of time required to complete MyMathLab homework decreases by 3 minutes each week. If you spent 50 minutes the first week, how much time was spent eight weeks later?

B

A. Linear Growth
B. Linear Decay
C. Exponential Growth
D. Exponential Decay

A

A. 50 mins $- 8 \times 3$ mins
B. 50 mins $+ 8 \times 3$ mins
C. 50 mins $\times (1 - \frac{3}{8})$
D. 50 mins $\times (1 - \frac{3}{8})$

(f) (3 points) How much was spent in the third week?

Wk 1 50

50 mins $- 3 \times 3$ mins

41 mins

Rephrase for clarity.

6. (5 points) Use the appropriate formula to compute **the balance in the following accounts after the stated period of time.**

   (a) (2 points) $7,000 is invested in an account that pays **simple interest** of 4% per year for 15 years.

   $\$7000(1.04)^{15} = \$11{,}200$

   (b) (3 points) $10,000 is invested for 5 years with an APR of 3% and **daily compounding.**

   $\$10{,}000\left(1+\frac{.03}{365}\right)^{365\times 5} = \$11{,}618.27$

7. (7 points) Suppose you place a **single bacterium** in a bottle at 11:00. It grows, and at 11:01 divides into **two bacteria.** These two bacteria each grow, and at 11:02, each divides into two bacteria, resulting in a total of **four bacteria** in the bottle.
   Now, suppose the bacteria continue to double in this way **every minute,** and the bottle is completely full at 12:00.

   (a) (2 points) At what time was the bottle a quarter full?

   11:58

   $\frac{1}{4} = (2)^{58}/2^{60}$ 11:58

   12 - full
   11:59 - 1/2
   11:58 - 1/4

   (b) (5 points) What is the total number of bacterium in the bottle at 12:00? Express your answer both as a power of two and evaluate it with your calculator. *Don't forget to include units as part of your answer. Evaluating your answer is only 2 of these points.*

   1.15×10^{18}

   $1\,\text{bac}\times 2^{60} = 2^{60}\,\text{bac} \approx 1.15\times10^{18}$

8. (8 points) Radioactive carbon-14 has a half-life of about 5,700 years. It collects in organisms while they are alive. Once they are dead, it only decays.

   (a) (1 point) What *fraction* of the carbon-14 in an animal's bones remains 5,700 years after the animal dies?

   $\frac{1}{2}$

   $\left(\frac{1}{2}\right)^{\frac{5700}{5700}} = \left(\frac{1}{2}\right)^{1}$ 1 opportunity

   (b) (2 points) What fraction of the carbon-14 in an animal's bones remains 11,400 years (*i.e. another 5,700 after part (a)*) after the animal dies?

   $\frac{1}{4}$

   $\left(\frac{1}{2}\right)^{\frac{11400}{5700}} = \left(\frac{1}{2}\right)^{2} = \frac{1}{4}$ 2 opportunities

   (c) (5 points) What fraction of the carbon-14 in an animal's bones remains 10,000 years after the animal dies? *Hint: Look at the setup in the first part of the methadone problem. Also, evaluating your answer only counts for 2 of these points.*

   30%

   $\left(\frac{1}{2}\right)^{\frac{10{,}000}{5700}} \approx 30\%$

9. (10 points) Kathleen deposits $100 a month into an account with an APR of 10% compounded monthly starting in 2010.

   (a) (5 points) How much will she have in 2020? *Evaluating your answer only counts for 2 of these points.*

$$A = \$100 \frac{\left(1 + \frac{.10}{12}\right)^{10 \cdot 12} - 1}{\left(\frac{.10}{12}\right)} = \$20,484.50$$

   (b) (5 points) Suppose after 2020, she *stops* making monthly deposits and lets the balance of the account grow *only* through the power of the compounding interest rate (compounded monthly). How much will she have in another ten years (i.e. in 2030)? *Again, evaluating your answer only counts for 2 of these points.*

$$A \times \left(1 + \frac{0.10}{12}\right)^{10 \cdot 12} = (\$20,484.50) \times \left(1 + \frac{.1}{12}\right)^{10 \times 12}$$

$$= \$155452.39$$

10. (10 points) The half-life of a particular drug in the bloodstream is 18 hours. Suppose that your current amount of drug in the bloodstream is 2 grams. Find the amount of drug in the bloodstream two days from now.

$$2 \text{ days} \times \frac{24 \text{ hrs}}{\text{day}} = 48 \text{ hr}$$

$$2g \times \left(\frac{1}{2}\right)^{\frac{48 \text{ hrs}}{18 \text{ hr}}} = .315g$$

11. (5 points) The population of a town is increasing by 550 people per year. State whether this growth is linear or exponential. If the population is 3720 today, what will the population be in three years?

(Circle one) The population growth is LINEAR or EXPONENTIAL

Show your work below: *Hint: refer to earlier problems*

$3720 \text{ ppl} + \frac{550 \text{ ppl}}{\text{yr}} \times 3 \text{ yr}$

$= 5370 \text{ ppl}$

$\frac{550 \text{ people}}{\text{year}}$

12. (3 points) Humans radiate an average of 360,000 Joules of energy each hour as heat. A gummy bear contains around 36 kiloJoules worth of energy. *Hint: "kilo" means "thousand".*

$$\frac{1 \text{ gummy bear}}{36 \text{ kJ}} \times \frac{1 \text{ kJ}}{1000 \text{ J}} \times \frac{360{,}000 \text{ J of heat}}{1 \text{ hour}}$$

$= 10$ gummy bear of heat per hour

13. (20 points) Suppose you have student loans totaling \$40,000 when you graduate from college. The interest rate (APR) is 5.5%, and the loan term is 30 years.

(a) (5 pts) What are your monthly payments? *Evaluating your answer is only worth 2 of these points.*

$$PMT = \$40{,}000 \times \frac{\left(\frac{0.055}{12}\right)}{1-\left(1+\frac{0.055}{12}\right)^{-12\cdot 30}} \approx \$227.16$$

(b) (3 pts) How much will you pay over the lifetime of the loan?

$(a) \times 30 \times 12 \approx \$81{,}761.62$

(c) (2 pts) How much will you pay **in interest** over the lifetime of the loan?

Difference $= \$41{,}761.62$

(d) (5 pts) **For the first month,** calculate the: (1) amount of your payment that goes towards interest, (2) the amount of your payment that goes towards principal, and (3) the remaining principal after the payment. *Hint: APR/12 is the "monthly interest rate." (This is the portion of the principle that represents the amount you owe in interest for that first month). You know the monthly payment from part (a). These are not the same (one is bigger than the other). Proceed accordingly.*

$\frac{0.055}{12} \times \$40{,}000 = \$40{,}183.33$

$-\$40{,}000.00$

$\$183.33$ ← Interest

PMT = \$227.16

$-\$183.33$ ← Interest

$\$43.78$ ← Towards Principal

New: \$39956.22

(e) (5 pts) For the **second month,** calculate the: (1) amount of your payment that goes towards interest, (2) the amount of your payment that goes towards principal, and (3) the remaining principal after the payment.

$\left(\frac{0.055}{12}\right)\$39956.22 = 40134.35$

$-40{,}000.00$

$\$134.35$ ← Interest

227.16

-134.35

43.58 ← Principal

New: 39956.22

-43.58

\$39912.23 ← NEW BAL