

```

/** Boxed newtype for F[G[A]]. */
case class Nested[F[_], G[_], A](value: F[G[A]])

/** Nested covariant functors yield a covariant functor. */
implicit def `+`[+] = +`[F[_]: Functor, G[_]: Functor]: Functor[({type l[a] = Nested[F, G, a]})#l] =
  new Functor[({type l[a] = Nested[F, G, a]})#l] {
    def map[A, B](nested: Nested[F, G, A])(f: A => B): Nested[F, G, B] = {
      val fga = nested.value
      val fgb = fga.map((ga: G[A]) => ga.map(f))
      Nested(fgb)
    }
  }

/** Contravariant functor in a covariant functor yields a contravariant functor. */
implicit def `+`[-] = -`[F[_]: Functor, G[_]: Contravariant]: Contravariant[({type l[a] = Nested[F, G, a]})#l] =
  new Contravariant[({type l[a] = Nested[F, G, a]})#l] {
    def contramap[A, B](nested: Nested[F, G, A])(f: B => A): Nested[F, G, B] = {
      val fga = nested.value
      val fgb = fga.map((ga: G[A]) => ga.contramap(f))
      Nested(fgb)
    }
  }

/** Covariant functor in a contravariant functor yields a contravariant functor. */
implicit def `-`[+] = -`[F[_]: Contravariant, G[_]: Functor]: Contravariant[({type l[a] = Nested[F, G, a]})#l] =
  new Contravariant[({type l[a] = Nested[F, G, a]})#l] {
    def contramap[A, B](nested: Nested[F, G, A])(f: B => A): Nested[F, G, B] = {
      val fga = nested.value
      val fgb = fga.contramap((gb: G[B]) => gb.map(f))
      Nested(fgb)
    }
  }

/** Nested contravariant functors yield a covariant functor. */
implicit def `-`[-] = +`[F[_]: Contravariant, G[_]: Contravariant]: Functor[({type l[a] = Nested[F, G, a]})#l] =
  new Functor[({type l[a] = Nested[F, G, a]})#l] {
    def map[A, B](nested: Nested[F, G, A])(f: A => B): Nested[F, G, B] = {
      val fga = nested.value
      val fgb = fga.contramap((gb: G[B]) => gb.contramap(f))
      Nested(fgb)
    }
  }

/** Covariant functor in an invariant functor yields an invariant functor. */
implicit def `i`[+] = i`[F[_]: InvariantFunctor, G[_]: Functor]: InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] =
  new InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] {
    def xmap[A, B](nested: Nested[F, G, A], f: A => B, g: B => A): Nested[F, G, B] = {
      val fga = nested.value
      val fgb: F[G[B]] = fga.xmap((ga: G[A]) => ga.map(f), (gb: G[B]) => gb.map(g))
      Nested(fgb)
    }
  }

/** Contravariant functor in an invariant functor yields an invariant functor. */
implicit def `i`[-] = i`[F[_]: InvariantFunctor, G[_]: Contravariant]: InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] =
  new InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] {
    def xmap[A, B](nested: Nested[F, G, A], f: A => B, g: B => A): Nested[F, G, B] = {
      val fga = nested.value
      val fgb: F[G[B]] = fga.xmap((ga: G[A]) => ga.contramap(g), (gb: G[B]) => gb.contramap(f))
      Nested(fgb)
    }
  }

/** Invariant functor in a covariant functor yields an invariant functor. */
implicit def `+`[i] = i`[F[_]: Functor, G[_]: InvariantFunctor]: InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] =
  new InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] {
    def xmap[A, B](nested: Nested[F, G, A], f: A => B, g: B => A): Nested[F, G, B] = {
      val fga = nested.value
      val fgb: F[G[B]] = fga.map((ga: G[A]) => ga.xmap(f, g))
      Nested(fgb)
    }
  }

/** Invariant functor in a contravariant functor yields an invariant functor. */
implicit def `-`[i] = i`[F[_]: Contravariant, G[_]: InvariantFunctor]: InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] =
  new InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] {
    def xmap[A, B](nested: Nested[F, G, A], f: A => B, g: B => A): Nested[F, G, B] = {
      val fga = nested.value
      val fgb: F[G[B]] = fga.contramap((gb: G[B]) => gb.xmap(g, f))
      Nested(fgb)
    }
  }

/** Invariant functor in an invariant functor yields an invariant functor. */
implicit def `i`[i] = i`[F[_]: InvariantFunctor, G[_]: InvariantFunctor]: InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] =
  new InvariantFunctor[({type l[a] = Nested[F, G, a]})#l] {
    def xmap[A, B](nested: Nested[F, G, A], f: A => B, g: B => A): Nested[F, G, B] = {
      val fga = nested.value
      val fgb: F[G[B]] = fga.xmap((ga: G[A]) => ga.xmap(f, g), (gb: G[B]) => gb.xmap(g, f))
      Nested(fgb)
    }
  }

```

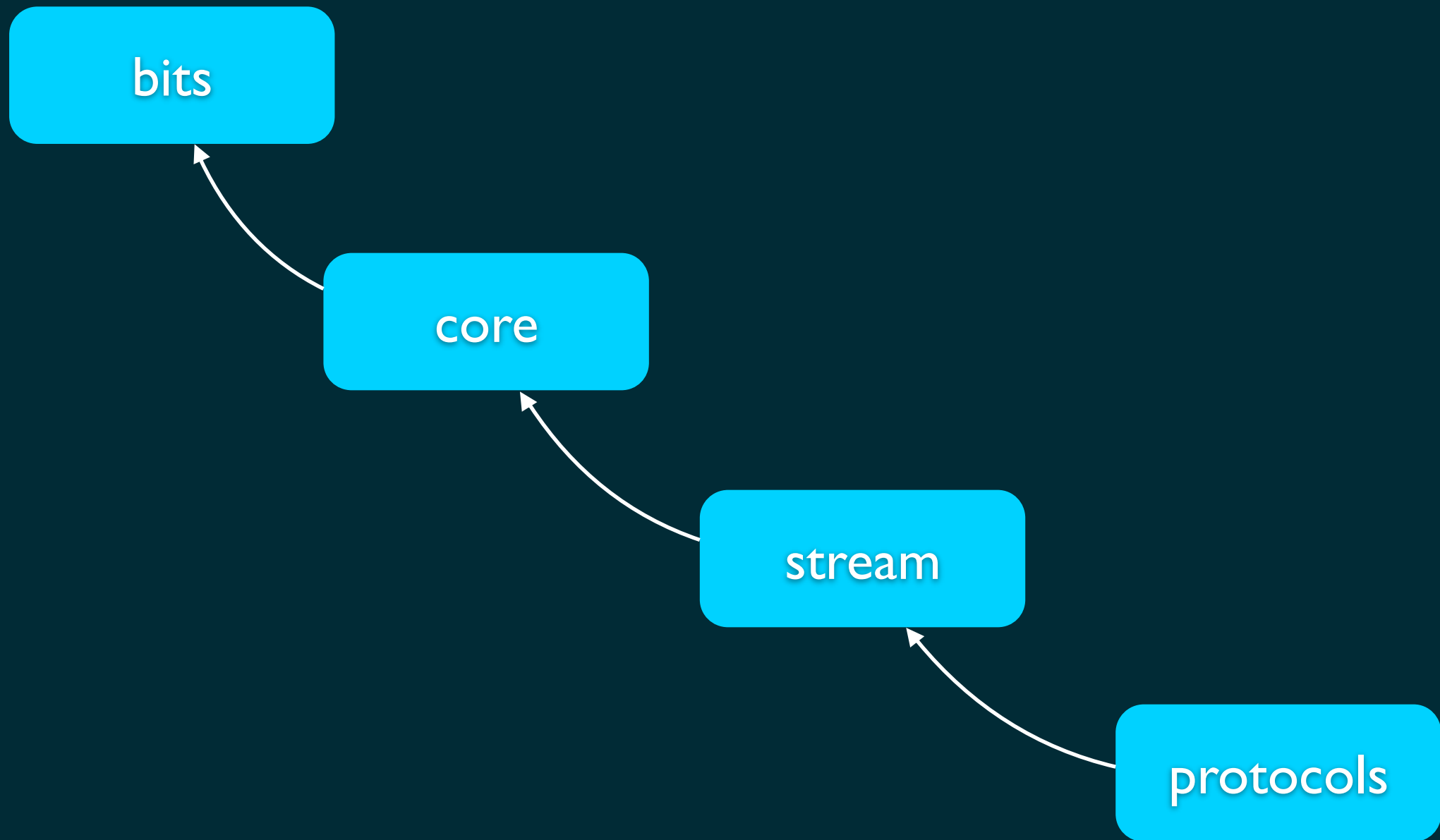
Explorations in Variance

What is scodec?

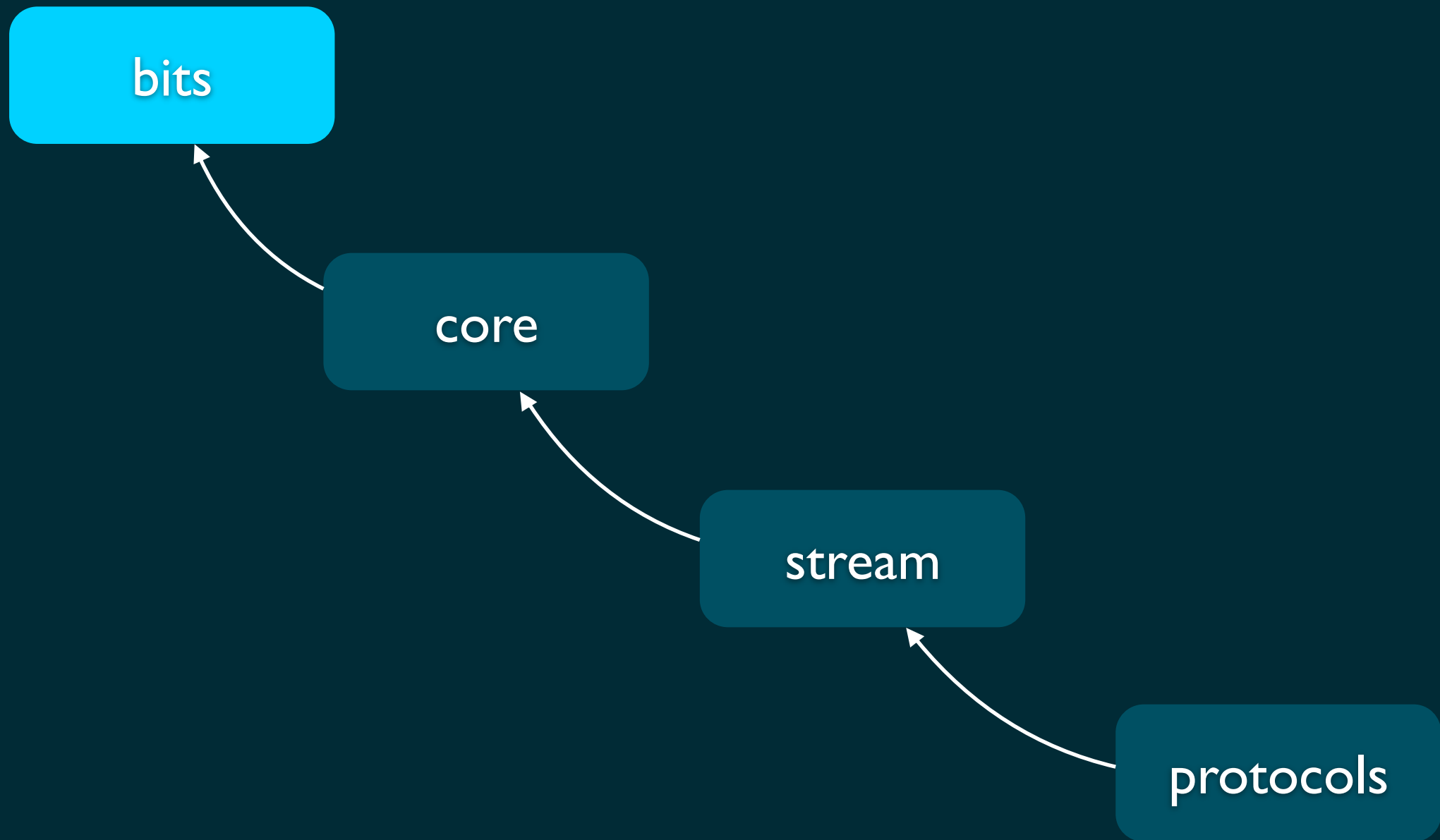
Scala combinator library that supports contract-first and pure functional encoding and decoding of binary data

<http://typelevel.org/projects/scodec>

Modules



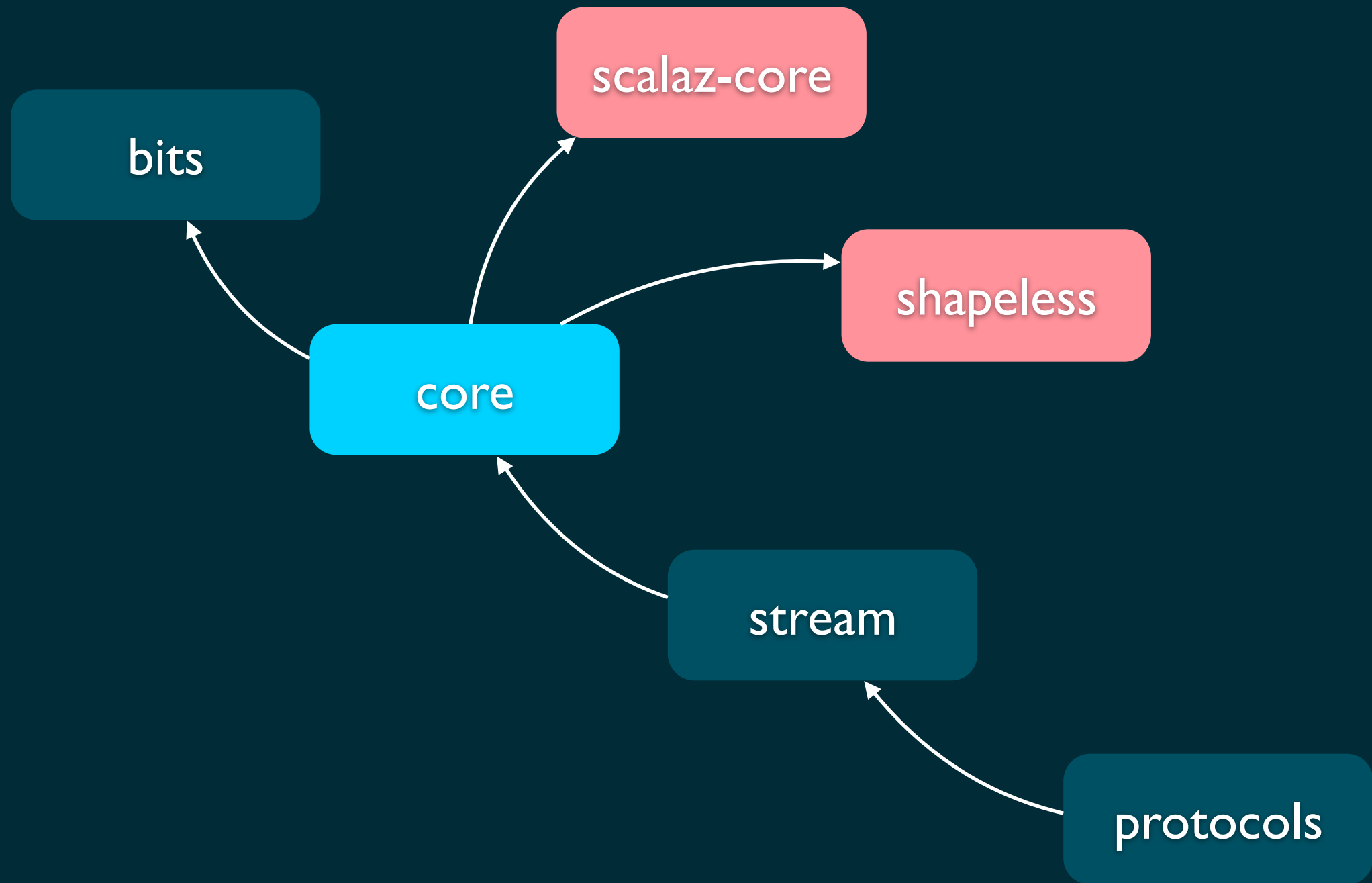
Modules



scodec-bits

- Zero dependencies
- Performant immutable data structures for working with bits (`BitVector`) and bytes (`ByteVector`)
- Base conversions (bin, hex, base64)
- CRCs (arbitrary! want a 93-bit CRC with a reflected output register?)
- Lots and lots of methods (>90 on `BitVector`)

Modules



Design Constraints

- Binary structure should mirror protocol definitions and be self-evident under casual reading
- Mapping of binary structures to types should be statically verified
- Encoding and decoding should be purely functional
- Failures in encoding and decoding should provide descriptive errors
- Compiler plugin should not be used

Example Usage

```
case class EthernetFrameHeader(  
  destination: MacAddress,  
  source: MacAddress,  
  ethertypeOrLength: Int  
) {  
  def length: Option[Int] =  
    (ethertypeOrLength <= 1500).option(ethertypeOrLength)  
  def ethertype: Option[Int] =  
    (ethertypeOrLength > 1500).option(ethertypeOrLength)  
}
```


Example Usage

```
case class EthernetFrameHeader(  
  destination: MacAddress,  
  source: MacAddress,  
  ethertypeOrLength: Int  
) {  
  def length: Option[Int] =  
    (ethertypeOrLength <= 1500).option(ethertypeOrLength)  
  def ethertype: Option[Int] =  
    (ethertypeOrLength > 1500).option(ethertypeOrLength)  
}
```

```
object EthernetFrameHeader {  
  implicit val codec: Codec[EthernetFrameHeader] = {  
    val macAddress = Codec[MacAddress]  
    ("destination" | macAddress) ::  
    ("source" | macAddress) ::  
    ("ethertype" | uint16)  
  }.as[EthernetFrameHeader]  
}
```

```
trait Decoder[+A] {  
  def decode(bits: BitVector): String \/ (BitVector, A)  
}
```

```
trait Decoder[+A] {  
  def decode(bits: BitVector): String \/ (BitVector, A)  
}
```

Input

Error

Remainder

Decoded Value

```
trait Decoder[+A] {  
  def decode(bits: BitVector): String \/ (BitVector, A)  
}
```

- Decoder is a *type constructor*
- applying a *proper type* to Decoder yields a *proper type*
- e.g., applying Int to Decoder yields Decoder[Int]

```
trait Decoder[+A] {  
  def decode(bits: BitVector): String \/ (BitVector, A)  
}
```

- Decoder is *covariant* in its only type parameter
⇒ Decoder[X] a subtype of Decoder[Y] if X is a subtype of Y
- <: < shorthand for “is a subtype of”
- e.g., Decoder[Int] <: <
Decoder[AnyVal] <: < Decoder[Any]

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
}
```

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
}
```

Input

Error

Encoded Value

```
trait Encoder[-A] {  
  def encode(value: A): String \/ BitVector  
}
```

- Encoder is *contravariant* in its only type parameter
- $\text{Encoder}[X] < : < \text{Encoder}[Y]$ if $Y < : < X$
- e.g., $\text{Encoder}[\text{Animal}] < : < \text{Encoder}[\text{Cat}]$

Higher-Order Subtyping Variance

Subtyping Transitivity

```
trait Coll[+A]
```

```
trait List[+A] extends Coll[A]
```

```
List[Int] <::< Coll[AnyVal] ?
```

Subtyping Transitivity

```
trait Coll[+A]
```

```
List[Int] <:< Coll[AnyVal] ?
```

```
trait List[+A] extends Coll[A]
```

Given type constructors F and G and proper types X and Y:

$$F <: < G \wedge X <: < Y \Rightarrow F[X] <: < G[Y]$$

Variance Annotations

- Type constructor parameter variance is specified via *variance annotations*
 - + for covariance, - for contravariance
- If not annotated, parameter is *invariant*, meaning there is no subtyping relationship between $F[X]$ and $F[Y]$ for given distinct proper types X and Y
- +/- has origins in literature - see papers on Higher Order Subtyping Polarization by Cardelli, Steffen, Pierce, et al

Bivariance

- Bivariance defines a subtyping relationship where $F[X] <: F[Y]$ for all X, Y
- Scala does not support bivariance

n-ary Type Constrs

- Subtype relationship for type constructors with multiple arguments follows from single arg
- Intuition: consider each parameter in isolation and logical-and the result
- Example:
 - given $F[+A, +B]$ and proper types X_1, Y_1, X_2, Y_2
 $X_1 < : < X_2 \wedge Y_1 < : < Y_2 \Rightarrow F[X_1, Y_1] < : < F[X_2, Y_2]$

n-ary Type Constrs

Given: $\text{Dog} <::< \text{Pet}$
 $\text{Person} <::< \text{Owner}$

Which are true?

$\text{Dog} \Rightarrow \text{Owner} <::< \text{Pet} \Rightarrow \text{Owner}$

$\text{Pet} \Rightarrow \text{Owner} <::< \text{Dog} \Rightarrow \text{Owner}$

$\text{Dog} \Rightarrow \text{Person} <::< \text{Dog} \Rightarrow \text{Owner}$

$\text{Dog} \Rightarrow \text{Owner} <::< \text{Dog} \Rightarrow \text{Person}$

n-ary Type Constrs

Given: $\text{Dog} <::< \text{Pet}$
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Which are true?

 $\text{Dog} \Rightarrow \text{Owner} <::< \text{Pet} \Rightarrow \text{Owner}$

$\text{Pet} \Rightarrow \text{Owner} <::< \text{Dog} \Rightarrow \text{Owner}$

$\text{Dog} \Rightarrow \text{Person} <::< \text{Dog} \Rightarrow \text{Owner}$

$\text{Dog} \Rightarrow \text{Owner} <::< \text{Dog} \Rightarrow \text{Person}$

$f(\text{cat})$

n-ary Type Constrs

Given: $\text{Dog} <::< \text{Pet}$
 $\text{Person} <::< \text{Owner}$

Which are true?

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$f(\text{cat})$

 $\text{Pet} \Rightarrow \text{Owner} <::< \text{Dog} \Rightarrow \text{Owner}$

$\text{Dog} \Rightarrow \text{Person} <::< \text{Dog} \Rightarrow \text{Owner}$

$\text{Dog} \Rightarrow \text{Owner} <::< \text{Dog} \Rightarrow \text{Person}$

n-ary Type Constrs

Given: $\text{Dog} <::< \text{Pet}$
 $\text{Person} <::< \text{Owner}$

Which are true?

 $\text{Dog} \Rightarrow \text{Owner} <::< \text{Pet} \Rightarrow \text{Owner}$

$f(\text{cat})$

 $\text{Pet} \Rightarrow \text{Owner} <::< \text{Dog} \Rightarrow \text{Owner}$

 $\text{Dog} \Rightarrow \text{Person} <::< \text{Dog} \Rightarrow \text{Owner}$

$\text{Dog} \Rightarrow \text{Owner} <::< \text{Dog} \Rightarrow \text{Person}$

n-ary Type Constrs

Given: `Dog <::< Pet`
`Person <::< Owner`

Which are true?

 `Dog => Owner <::< Pet => Owner`

`f(cat)`

 `Pet => Owner <::< Dog => Owner`

 `Dog => Person <::< Dog => Owner`

 `Dog => Owner <::< Dog => Person`

`f(dog).height`

Function Types

Scala models a single argument function as a binary type constructor

```
trait Function1[-A, +B] {  
  def apply(a: A): B  
}
```

Variance Positions

- Covariant params cannot appear in argument lists of methods

```
scala> trait Foo[+A] { def f(a: A): Unit }
```

```
<console>:7: error: covariant type A occurs in contravariant position  
in type A of value a
```

```
trait Foo[+A] { def f(a: A): Unit }  
                   ^
```

- Contravariant params cannot appear as a return type of methods

```
scala> trait Bar[-A] { def g(): A }
```

```
<console>:7: error: contravariant type A occurs in covariant position  
in type ()A of method g
```

```
trait Bar[-A] { def g(): A }  
                   ^
```

Declaration Site

- Scala supports *declaration-site* (or *definition-site*) variance annotations
 - Parameters of type constructors are annotated
- C# also has declaration-site variance and uses *out* for covariant and *in* for contravariant
- Contrast with *use-site* annotations, where the type is not annotated
 - e.g., Java generics + wildcards

Use Site

```
public interface List<A> {  
    void add(A a);  
    A head();  
    A set(int idx, A a);  
    void clear();  
}
```

Use Site

```
public interface List<A> {  
    void add(A a);  
    A head();  
    A set(int idx, A a);  
    void clear();  
}
```

```
public <A> void covariant(List<? extends A> list) {  
    // can call head or clear  
    // cannot call add or set  
}
```

App.java:4: add(capture#772 of ? extends A) in List<capture#772 of ? extends A> cannot be applied to (A)

```
    list.add(list.head());
```

^

Use Site

```
public interface List<A> {  
    void add(A a);  
    A head();  
    A set(int idx, A a);  
    void clear();  
}
```

```
public <A> void contravariant(List<? super A> list) {  
    // can call add or clear  
    // cannot call head or set  
}
```

Use Site

```
public interface List<A> {  
    void add(A a);  
    A head();  
    A set(int idx, A a);  
    void clear();  
}
```

```
public <A> void bivariant(List<?> list) {  
    // can only call clear  
}
```

Use Site

```
public interface List<A> {  
    void add(A a);  
    A head();  
    A set(int idx, A a);  
    void clear();  
}
```

```
public <A> void invariant(List<A> list) {  
    // can call all methods  
}
```

Use Site

- Use site variance is generally harder to work with but is more flexible in some circumstances
- Recent research by John Altidor et al incorporates declaration site variance in to Java, without removing use site variance
- See “Taming the Wildcards: Combining Definition- and Use-Site Variance”
- May end up as JEP (<http://mail.openjdk.java.net/pipermail/compiler-dev/2014-April/008745.html>)

Cheating Decl Variance

Consider the beginnings of an immutable list:

```
trait List[+A] {  
  def head: A  
}
```

Challenge: add a cons method

Cheating Decl Variance

Consider the beginnings of an immutable list:

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trait List[+A] {  
  def head: A  
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```

Challenge: add a cons method

```
trait List[+A] {  
  def head: A  
  def cons(a: A): List[A]  
}
```

Cheating Decl Variance

Consider the beginnings of an immutable list:

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trait List[+A] {  
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Challenge: add a cons method



```
trait List[+A] {  
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Cheating Decl Variance


Consider the beginnings of an immutable list:

```
trait List[+A] {  
  def head: A  
}
```

Challenge: add a cons method



```
trait List[+A] {  
  def head: A  
  def cons(a: A): List[A]  
}
```



```
trait List[+A] {  
  def head: A  
  def cons[AA >: A](a: AA): List[AA]  
}
```


Composition

```
trait List[+A] {  
  def head: A  
}
```

```
trait Ord[-A] {  
  def compare(x: A, y: A): Order  
}
```

What is the variance of:

```
type Foo[A] = List[List[A]]
```

```
type Bar[A] = Ord[Ord[A]]
```

```
type Baz[A] = List[Ord[A]]
```

```
type Qux[A] = Ord[List[A]]
```

Composition

- Scala rules:
 - covariance preserves variance
 - contravariant flips variance
 - invariant makes invariance

Composition

More generally...

Let \otimes be an associative binary relation between type params to a higher kinded type

\otimes	bi	+	-	inv
bi	bi	bi	bi	inv
+	bi	+	-	inv
-	bi	-	+	inv
inv	inv	inv	inv	inv

Composition

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trait List[+A] {  
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Composition

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covariant

```
type Bar[A] = Ord[Ord[A]]
```

```
type Baz[A] = List[Ord[A]]
```

```
type Qux[A] = Ord[List[A]]
```

Composition

```
trait List[+A] {  
  def head: A  
}
```

```
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}
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covariant

```
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```

covariant

```
type Baz[A] = List[Ord[A]]
```

```
type Qux[A] = Ord[List[A]]
```

Composition

```
trait List[+A] {  
  def head: A  
}
```

```
trait Ord[-A] {  
  def compare(x: A, y: A): Order  
}
```

What is the variance of:

```
type Foo[A] = List[List[A]]
```

covariant

```
type Bar[A] = Ord[Ord[A]]
```

covariant

```
type Baz[A] = List[Ord[A]]
```

contravariant

```
type Qux[A] = Ord[List[A]]
```

Composition

```
trait List[+A] {  
  def head: A  
}
```

```
trait Ord[-A] {  
  def compare(x: A, y: A): Order  
}
```

What is the variance of:

```
type Foo[A] = List[List[A]]
```

covariant

```
type Bar[A] = Ord[Ord[A]]
```

covariant

```
type Baz[A] = List[Ord[A]]
```

contravariant

```
type Qux[A] = Ord[List[A]]
```

contravariant


```
trait Function1[-A, +B] {  
  def apply(a: A): B  
}
```

Composition

What is the variance of A, B, and C in:

```
type F[A, B, C] = (A => B) => C
```

```
trait Function1[-A, +B] {  
  def apply(a: A): B  
}
```

Composition

What is the variance of A, B, and C in:

```
type F[A, B, C] = (A => B) => C
```

```
type F[A, B, C] = Function1[Function1[A, B], C]
```

```
trait Function1[-A, +B] {  
  def apply(a: A): B  
}
```

Composition

What is the variance of A, B, and C in:

```
type F[A, B, C] = (A => B) => C
```

```
type F[A, B, C] = Function1[Function1[A, B], C]
```

```
type F[A, B, C] = Function1[Function1[A, B], C]      contravariant
```

```
trait Function1[-A, +B] {  
  def apply(a: A): B  
}
```

Composition

What is the variance of A, B, and C in:

```
type F[A, B, C] = (A => B) => C
```

```
type F[A, B, C] = Function1[Function1[A, B], C]
```

```
type F[A, B, C] = Function1[Function1[A, B], C]      contravariant
```

A: - ⊗ - = +

B: - ⊗ + = -

C: +

```
scala> implicitly[F[Int, AnyVal, Int] <: F[AnyVal, Int, AnyVal]]  
res0: <: <[(Int => AnyVal) => Int, (AnyVal => Int) => AnyVal] = <function1>
```

...back to scodec

Mapping over a Decoder

```
trait Decoder[+A] {  
  def decode(bits: BitVector): String \/ (BitVector, A)  
  
  def map[B](f: A => B): Decoder[B] = ???  
}
```

Mapping over a Decoder

```
trait Decoder[+A] { self =>
  def decode(bits: BitVector): String \/ (BitVector, A)

  def map[B](f: A => B): Decoder[B] = new Decoder[B] {
    def decode(bits: BitVector) =
      self.decode(bits) map { case (rem, a) => (rem, f(a)) }
  }
}
```

Abstracting over mappability

```
trait Functor[F[_]] {  
  def map[A, B](fa: F[A])(f: A => B): F[B]  
}
```

Laws:

- identity:
 $\text{map fa identity} = \text{fa}$
- composition:
 $f: A \Rightarrow B, g: B \Rightarrow C$
 $\text{map (map fa f) g} = \text{map fa (g compose f)}$

Decoder Functor

```
trait Functor[F[_]] {  
  def map[A, B](fa: F[A])(f: A => B): F[B]  
}  
  
object Decoder {  
  implicit val functorInstance: Functor[Decoder] =  
    new Functor[Decoder] {  
      def map[A, B](decoder: Decoder[A])(f: A => B) =  
        decoder.map(f)  
    }  
}
```

Note that Decoder has more structure than just Functor...

```
trait Decoder[+A] { self =>
  def decode(bits: BitVector): String \/ (BitVector, A)

  def map[B](f: A => B): Decoder[B] = new Decoder[B] {
    def decode(bits: BitVector) =
      self.decode(bits) map { case (rem, a) => (rem, f(a)) }
  }

  def flatMap[B](f: A => Decoder[B]): Decoder[B] =
    new Decoder[B] {
      def decode(bits: BitVector) =
        self.decode(bits) flatMap { case (rem, a) =>
          f(a).decode(rem)
        }
    }
}
```

Models a dependency, where the next decoder is based on the previously decoded value

Decoder Monad

```
object Decoder {  
  
  def point[A](a: => A): Decoder[A] = new Decoder[A] {  
    private lazy val value = a  
    def decode(bits: BitVector) = \/.right((bits, value))  
  }  
  
  implicit val monadInstance: Monad[Decoder] =  
    new Monad[Decoder] {  
      def point[A](a: => A) = Decoder.point(a)  
      def bind[A, B](d: Decoder[A])(f: A => Decoder[B]) =  
        d.flatMap(f)  
    }  
}
```

Won't be discussing monads in this talk, but note that a monad gives rise to a functor via $\text{map } f = \text{bind}(\text{point} \text{ compose } f)$

Encoder Functor?

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
  
  def map[B](f: A => B): Encoder[B] = ???  
}
```

Encoder Functor?

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
  
  def map[B](f: A => B): Encoder[B] = new Encoder[B] {  
    def encode(value: B) = self.encode(???)  
  }  
}
```

Encoder Functor?

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
  
  def map[B](f: A => B): Encoder[B] = new Encoder[B] {  
    def encode(value: B) = self.encode(???)  
  }  
}
```

- We need a value of type A
- We have:
 - value of type B
 - function from A to B

Encoder Functor?

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
  
  def map[B](f: A => B): Encoder[B] = new Encoder[B] {  
    def encode(value: B) = self.encode(???)  
  }  
}
```

- We need a value of type A
- We have:
 - value of type B
 - function from A to B

Let's reverse the arrow
on the function...

Encoder ~~Functor~~

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
  
  def map[B](f: B => A): Encoder[B] = new Encoder[B] {  
    def encode(value: B) = self.encode(???)  
  }  
}
```


Encoder ~~Functor~~

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
  
  def map[B](f: B => A): Encoder[B] = new Encoder[B] {  
    def encode(value: B) = self.encode(f(value))  
  }  
}
```

Encoder ???

```
trait Encoder[A] {  
  def encode(value: A): String \/ BitVector  
  
  def contramap[B](f: B => A): Encoder[B] = new Encoder[B] {  
    def encode(value: B) = self.encode(f(value))  
  }  
}
```

Contravariant Functor

Abstracting over the ability to contramap gives:

```
trait Contravariant[F[_]] {  
  def contramap[A, B](fa: F[A])(f: B => A): F[B]  
}
```

Laws:

- identity:

$\text{contramap fa identity} = \text{fa}$

- composition:

$f: B \Rightarrow A, g: C \Rightarrow B$

$\text{contramap (contramap fa f) g} =$
 $\text{contramap fa (f compose g)}$

Contravariant Functor

- Contravariant Functor is just a Functor with “arrows” reversed
- Functor is also known as Covariant Functor
 - This abbreviation is common in both the programming community and in category theory

Contravariant Encoder

```
trait Contravariant[F[_]] {  
  def contramap[A, B](fa: F[A])(f: B => A): F[B]  
}  
  
object Encoder extends EncoderFunctions {  
  implicit val contraInstance: Contravariant[Encoder] =  
    new Contravariant[Encoder] {  
      def contramap[A, B](e: Encoder[A])(f: B => A) = e contramap f  
    }  
}
```

Codec

```
trait Codec[A] extends Decoder[A] with Encoder[A] {  
  // Lots of combinators for building new codecs  
}
```

- A codec support both encoding and decoding a value of type A
- Must be defined invariant in type A due to Decoder being covariant and encoder being contravariant

Codec

```
trait Codec[A] extends Decoder[A] with Encoder[A] {  
  // Lots of combinators for building new codecs  
}
```

- Let's add a map-like operation for converting a Codec[A] to a Codec[B]
- Note that given $c: \text{Codec}[A]$ and $f: A \Rightarrow B$, calling $c \text{ map } f$ yields a $\text{Decoder}[B]$
- Similarly, with $g: B \Rightarrow A$, $c \text{ contramap } g$ yields an $\text{Encoder}[B]$
- We want map-like to retain ability to encode and decode

Let the types guide us...

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def mapLike[B](???): Codec[B] = new Codec[B] {
    def encode(b: B): String \/ BitVector = ???
    def decode(bv: BitVector): String \/ (BitVector, B) = ???
  }
}
```


We want encoding to behave like the original codec, so we probably should reuse the original encode...

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def mapLike[B](???): Codec[B] = new Codec[B] {
    def encode(b: B): String \/ BitVector = self.encode(???)
    def decode(bv: BitVector): String \/ (BitVector, B) = ???
  }
}
```

We want encoding to behave like the original codec, so we probably should reuse the original encode...

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def mapLike[B](???): Codec[B] = new Codec[B] {
    def encode(b: B): String \/ BitVector = self.encode(???)
    def decode(bv: BitVector): String \/ (BitVector, B) = ???
  }
}
```

Need an A to pass to self.encode but we only have a B

Let's “dependency inject” the conversion

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def mapLike[B](g: B => A): Codec[B] = new Codec[B] {
    def encode(b: B): String \/ BitVector = self.encode(g(b))
    def decode(bv: BitVector): String \/ (BitVector, B) = ???
  }
}
```

Decode should behave like original decode but decoded value should be converted from a B to an A

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def mapLike[B](g: B => A): Codec[B] = new Codec[B] {
    def encode(b: B): String \/ BitVector = self.encode(g(b))
    def decode(bv: BitVector): String \/ (BitVector, B) =
      self.decode(bv).map { case (rem, a) => (rem, ???) }
  }
}
```

More dependency injection!

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def mapLike[B](f: A => B, g: B => A): Codec[B] = new Codec[B] {
    def encode(b: B): String \/ BitVector = self.encode(g(b))
    def decode(bv: BitVector): String \/ (BitVector, B) =
      self.decode(bv).map { case (rem, a) => (rem, f(a)) }
  }
}
```

```

trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def xmap[B](f: A => B, g: B => A): Codec[B] = new Codec[B] {
    def encode(b: B): String \/ BitVector = self.encode(g(b))
    def decode(bv: BitVector): String \/ (BitVector, B) =
      self.decode(bv).map { case (rem, a) => (rem, f(a)) }
  }
}

```

- To convert from `Codec[A]` to `Codec[B]`, we need a pair of functions $A \Rightarrow B$ and $B \Rightarrow A$
- Let's call this operation `xmap`

Invariant Functor

Abstracting over the ability to xmap gives:

```
trait InvariantFunctor[F[_]] {  
  def xmap[A, B](ma: F[A], f: A => B, g: B => A): F[B]  
}
```

Laws:

- identity:

$\text{xmap } fa \text{ identity identity} = fa$

- composition:

$f_1: A \Rightarrow B, g_1: B \Rightarrow A, f_2: B \Rightarrow C, g_2: C \Rightarrow B$

$\text{xmap } (\text{xmap } fa \text{ } f_1 \text{ } g_1) \text{ } f_2 \text{ } g_2 =$

$\text{xmap } fa \text{ } (f_2 \text{ compose } f_1) \text{ } (g_1 \text{ compose } g_2)$

Invariant Functor

- Invariant Functor is also known as Exponential Functor
- `xmap` is also known as `invmap`
- Every covariant functor is an invariant functor
 $\text{xmap } f \ g = \text{map } f$
- Every contravariant functor is an invariant functor
 $\text{xmap } f \ g = \text{contramap } g$

Codec

```
trait Codec[A] extends Decoder[A] with Encoder[A] {  
  def xmap[B](f: A => B, g: B => A): Codec[B] = ...  
}
```

Codec

```
trait Codec[A] extends Decoder[A] with Encoder[A] {  
  def xmap[B](f: A => B, g: B => A): Codec[B] = ...  
}
```

What about dependent codecs? Recall:

```
trait Decoder[+A] { self =>  
  def flatMap[B](f: A => Decoder[B]): Decoder[B] = ...  
  ...  
}
```

Codec#flatMap “forgets” how to encode

Codec

Can we define flatMap directly?

```
trait Codec[A] extends Decoder[A] with Encoder[A] {  
  def xmap[B](f: A => B, g: B => A): Codec[B] = ...  
  def flatMap[B](f: A => Codec[B]): Codec[B] = ???  
}
```

Codec

Straightforward to define decode:

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def xmap[B](f: A => B, g: B => A): Codec[B] = ...
  def flatMap[B](f: A => Codec[B]): Codec[B] = new Codec[B] {
    def encode(a: B): BitVector = ???
    def decode(bv: BitVector): String \/ (BitVector, B) =
      self.flatMap(f).decode(bv)
  }
}
```

Codec

Straightforward to define decode:

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def xmap[B](f: A => B, g: B => A): Codec[B] = ...
  def flatMap[B](f: A => Codec[B]): Codec[B] = new Codec[B] {
    def encode(a: B): BitVector = ???
    def decode(bv: BitVector): String \/ (BitVector, B) =
      self.flatMap(f).decode(bv)
  }
}
```

But impossible to define encode...

Codec

- We are trying to solve this domain specific problem:
- when decoding, first decode using a Decoder[A] and then use the decoded A value to determine how to decode the remaining bits in to a B
- when encoding, first encode using an Encoder[A] and then use that A to generate an Encoder[B] and encode that

Codec

We can implement this explicitly:

```
trait Codec[A] extends Decoder[A] with Encoder[A] { self =>
  def flatZip[B](f: A => Codec[B]): Codec[(A, B)] =
    new Codec[(A, B)] {
      override def encode(t: (A, B)) =
        Codec.encodeBoth(self, f(t._1))(t._1, t._2)
      override def decode(buffer: BitVector) = (for {
        a <- DecodingContext(self.decode)
        b <- DecodingContext(f(a).decode)
      } yield (a, b)).run(buffer)
    }
}
```

There are many other domain specific combinators
e.g., combining a `Codec[A]` and `Codec[B]` in
to a `Codec[(A, B)]`

Codec

```
trait Codec[A] extends Decoder[A] with Encoder[A] { ... }
```

- Syntactically, transforming codecs can be difficult
 - Can't use map/contramap/flatMap without forgetting behavior
- Can we incrementally convert a Codec[A] in to a Codec[B]?
- Intuition: don't forget stuff

GenCodec

```
trait GenCodec[-A, +B] extends Encoder[A] with Decoder[B] { ... }  
trait Codec[A] extends GenCodec[A, A] { ... }
```

- GenCodec lets the encoding type vary from the decoding type
- We want it to remember encoding behavior when transforming decoding behavior and vice-versa

GenCodec

```
trait GenCodec[-A, +B] extends Encoder[A] with Decoder[B] { self =>

  override def map[C](f: B => C): GenCodec[A, C] =
    new GenCodec[A, C] {
      def encode(a: A) = self.encode(a)
      def decode(bits: BitVector) =
        self.decode(bits).map { case (rem, b) => (rem, f(b)) }
    }

  override def contramap[C](f: C => A): GenCodec[C, B] =
    new GenCodec[C, B] {
      def encode(c: C) = self.encode(f(c))
      def decode(bits: BitVector) = self.decode(bits)
    }
}
```

GenCodec

Once transformations are done, we need
a way to convert back to a Codec

GenCodec

Once transformations are done, we need
a way to convert back to a Codec

```
trait GenCodec[-A, +B] extends Encoder[A] with Decoder[B] { self =>
  override def map[C](f: B => C): GenCodec[A, C] = ...
  override def contramap[C](f: C => A): GenCodec[C, B] = ...
  def fuse[AA <: A, BB >: B](implicit ev: BB ==> AA): Codec[BB] =
    new Codec[BB] {
      def encode(c: BB) = self.encode(ev(c))
      def decode(bits: BitVector) = self.decode(bits)
    }
}
```

Profunctor

Can we abstract out some form of `xmap`?

Profunctor

Can we abstract out some form of xmap?

```
trait Profunctor[F[_], _] { self =>

  def mapfst[A, B, C](fab: F[A, B])(f: C => A): F[C, B]

  def mapsnd[A, B, C](fab: F[A, B])(f: B => C): F[A, C]

  def dimap[A, B, C, D](fab: F[A, B])(f: C => A)(g: B => D): F[C, D] =
    mapsnd(mapfst(fab)(f))(g)
}
```

Profunctor

Can we abstract out some form of xmap?

```
trait Profunctor[F[_], _]] { self =>

  def mapfst[A, B, C](fab: F[A, B])(f: C => A): F[C, B]

  def mapsnd[A, B, C](fab: F[A, B])(f: B => C): F[A, C]

  def dimap[A, B, C, D](fab: F[A, B])(f: C => A)(g: B => D): F[C, D] =
    mapsnd(mapfst(fab)(f))(g)
}
```

- dimap is a generalization of xmap
- Note similarities of mapfst/contramap and mapsnd/map

Profunctor

- Profunctors abstract binary type constructors that are contravariant in first parameter and covariant in second parameter
- Also known as Difunctor (due to Meijer/Hutton)
- Most famous Profunctor is the single argument function

Correspondence?

Correspondence

- Disclaimer: IANACT (I am not a category theorist)
- Subtyping covariance and contravariance come from category theory
 - from a specific category where objects are types and morphisms are “is a subtype of”
- Functor typeclasses come from category theory
 - from specific categories where objects are types and morphisms are functions

Postulate

The subtyping transform binary relation manifests in functor typeclasses by considering derived functors from nested functors *

\otimes	bi	+	-	inv
bi	bi	bi	bi	inv
+	bi	+	-	inv
-	bi	-	+	inv
inv	inv	inv	inv	inv

* This may be completely false! IANACT

```
/** Boxed newtype for F[G[A]]. */  
case class Nested[F[_], G[_], A](value: F[G[A]])
```

Nested covariant functors yield a covariant functor

```
implicit def `+`[+] = +`[
  F[_]: Functor,
  G[_]: Functor
]: Functor[({type l[a] = Nested[F, G, a]})#l] =
  new Functor[({type l[a] = Nested[F, G, a]})#l] {
    def map[A, B](nested: Nested[F, G, A])(f: A => B): Nested[F, G, B] = {
      val fga = nested.value
      val fgb = fga.map((ga: G[A]) => ga.map(f))
      Nested(fgb)
    }
  }
```

Contravariant functor in a covariant functor yields a contravariant functor

```
implicit def `+[-] = -` [  
  F[_]: Functor,  
  G[_]: Contravariant  
]: Contravariant[({type l[a] = Nested[F, G, a]})#l] =  
  new Contravariant[({type l[a] = Nested[F, G, a]})#l] {  
    def contramap[A, B](nested: Nested[F, G, A])(f: B => A): Nested[F, G, B] = {  
      val fga = nested.value  
      val fgb = fga.map((ga: G[A]) => ga.contramap(f))  
      Nested(fgb)  
    }  
  }
```

Covariant functor in a contravariant functor yields a contravariant functor

```
implicit def `-[+]` = - `[
  F[_]: Contravariant,
  G[_]: Functor
]: Contravariant[({type l[a] = Nested[F, G, a]})#l] =
  new Contravariant[({type l[a] = Nested[F, G, a]})#l] {
    def contramap[A, B](nested: Nested[F, G, A])(f: B => A): Nested[F, G, B] = {
      val fga = nested.value
      val fgb = fga.contramap((gb: G[B]) => gb.map(f))
      Nested(fgb)
    }
  }
```

Nested contravariant functors yield a covariant functor

```
implicit def `-[-] = +` [
  F[_]: Contravariant,
  G[_]: Contravariant
]: Functor[({type l[a] = Nested[F, G, a]})#l] =
  new Functor[({type l[a] = Nested[F, G, a]})#l] {
    def map[A, B](nested: Nested[F, G, A])(f: A => B): Nested[F, G, B] = {
      val fga = nested.value
      val fgb = fga.contramap((gb: G[B]) => gb.contramap(f))
      Nested(fgb)
    }
  }
```


Nested contravariant functors yield a covariant functor

```
implicit def `-[-] = +` [  
  F[_]: Contravariant,  
  G[_]: Contravariant  
]: Functor[({type l[a] = Nested[F, G, a]})#l] =  
  new Functor[({type l[a] = Nested[F, G, a]})#l] {  
    def map[A, B](nested: Nested[F, G, A])(f: A => B): Nested[F, G, B] = {  
      val fga = nested.value  
      val fgb = fga.contramap((gb: G[B]) => gb.contramap(f))  
      Nested(fgb)  
    }  
  }
```

More at <https://github.com/mpilquist/variance-explorations/blob/master/variance.scala>

Questions?

Further Reading

- “Taming the Wildcards: Combining Definition- and Use-Site Variance” by Altidor
<http://cgi.di.uoa.gr/~smaragd/variance-pldi11.pdf>
- “Polarized Higher-Order Subtyping” by Steffen
<http://home.ifi.uio.no/msteffen/download/diss/diss.pdf>
- “Higher-Order Subtyping for Dependent Types” by Abel
<http://cs.ioc.ee/~tarmo/tsem11/abel-slides.pdf>

Further Reading

- “Rotten Bananas” by Kmett
<http://comonad.com/reader/2008/rotten-bananas/>
- “I love profunctors. They’re so easy.” by HU
<https://www.fpcomplete.com/school/to-infinity-and-beyond/pick-of-the-week/profunctors>
- “What’s up with Contravariant?”
http://www.reddit.com/r/haskell/comments/1vc0mp/whats_up_with_contravariant/