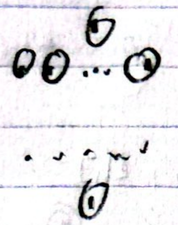
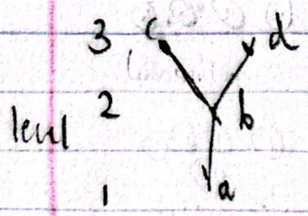


1. no Y exists as only depth of 3 with $[n]$ being the only vertex at distance 2

2. add any vertex since have at least 2 0's guaranteed to be $\preceq$ to at least 2



Consider $F = \{\emptyset, \binom{n}{n-1}, [n]\}$, $|F| = n+2$

1. First consider that F contains only 3 distinct levels, so the 3rd level must contain the 2 disjoint branches however, the 3rd level contains only the $[n]$, so a Y is not possible

2. Observe that the element must be between levels 1,2 so F', now will span 4 levels, making $\emptyset$ or e the root, if e is the root, d, then the same issue arises in F, so $\emptyset$ = root = a, since $\forall s, s \in B[n], \emptyset \preceq s$, $\emptyset \preceq e$, let $b = e$, now to find c,d the disjoint branches, we have only 2 levels remaining, since $\forall s \in B[n], s \preceq [n]$. c,d must be in $\binom{n}{n-1}$, so to be Y e must be related to 2 elements in $\binom{n}{n-1}$ Consider that since e falls between levels 1,2 meaning $1 \le |e| \le n-2$, so as represented as a bit, it contains anywhere from one 1 to $n-2$ 1's, let e contain i 1's, $1 \le i \le n-2$ consider that e will have $n-i$ 0's, so e will have $2 \le J \le n-1$ #0's $= J$. since we know e will have ≥ 2 0's, and $\binom{n}{n-1}$ elements will have $n-1$ 1's and 1 0 (we know $e \preceq$ to some $e' \in \binom{n}{n-1}$ if e' contains all the 1's in e) we know e will be $\preceq$ to J e''s $\in \binom{n}{n-1}$. as for $e \preceq e'$, it must match up on i places (1's) for the J other places (0's) it does matter for e so the number of e''s with the ith places fixed to one is the number of permutations with 1 0 left over or $\binom{J}{J-1} = J$ since $J \ge 2$ we know $e \preceq$ at least 2 e's so we have $a(\emptyset) \preceq b(e) \preceq c,d \in \binom{n}{n-1}$ and disjoint as same level

) Improved lower bound for Y?

$sat^*(n, V_k) \leq (k-1)(n-1)+2,\ k \geq 2.\ k \leq n$

~~Consider $F = \{\emptyset, [n], C_1, \ldots, C_{k-1}\}$~~

~~so for $a_1 = \emptyset, a_2 = \{1\}, \ldots, a_{n+1} = [n],\ b_1 = \emptyset, b_2 = \{j\}, b_{n+1} = [n]$~~

~~$a_i \subseteq a_j,\ b_i \subseteq b_j.\ j > i$~~

~~so $|F| =$~~

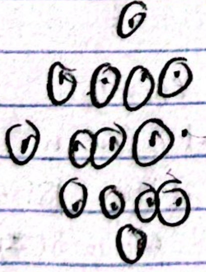
Consider $F = \{\emptyset, [n], C_1, \ldots, C_{k-1}\}$, where C_i is a full chain

so $\forall C_i, C_j \in C_i,\ C_i \| C_j$

1. Consider any k vertices, V_k, by PH principle, there must be at least 2 vertices in the same chain, by def of F $V_1 \| V_2$ therefore $V_k \notin F$

2. now consider any F', the new element e must appear between the 1st and last layers, since kth, no layer is full, by PH there must now be a layer with k vertices, by def two set is disjoint, now consider $\emptyset$ as the root as $\forall S \in B_n, \emptyset \subseteq S$, so V_k

Consider $k > n$?



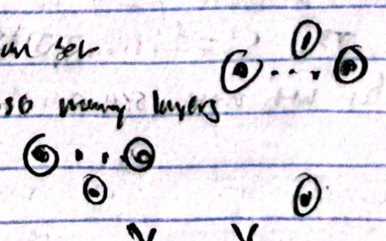
V_5 on B_4, if $k > n$, take all layers $< k$.

then for each layer $\geq k$, take $k-1$ vertices to create $k-1$ full chains?

expand result to B_k

Y_k can see

we just so many layers



prove that ~~Y_{kn}~~ Y_k cannot fit on B_n if $k \geq n$

~~V+1~~, $|F| = 2n$, same as 2(2

1 ... n NAh
1

for V_k consider the chains, $C_1 \cdots C_{k-1}$ ($k \le n+1$)

n	1	2	n-2
{1,2,...,n-1}	{2,3,...,n}	{3,4,...,1}	{n-1,1,2,...,n-3}
...	...	...	...
{1,2,3}	{2,3,4}	{3,4,5}	{n-1,1,2}
{1,2}	{2,3}	{3,4} ...	{n-1,1}
{1}	{2}	{3}	{n-1}

Chains are disjoint, k-1 on each layer, room for n chains

each distinct

at $k=n$, $n-1$ chains in F, n disjoint by PH each in one chain

at $k=n+1$, n chains in F. $n+1$ disjoint, n chains ✓

at $k=n+2$? $n+2$ disjoint

n	1	2
1,...,n-1	2,...,n	3,...,n,1

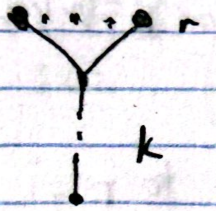
~~consider the n possible full chains, each with 1 element missing since there are n possible elements in set, n distinct sets size n-1. For each consider the element before~~

Consider the n possible singletons, {1}, ..., {n}, each disjoint as each element is distinct, now consider i+1, for n consider wraparound (1), these $\subseteq$'s are distinct as if they |S| < n and were the same, by this process the same parts would have to be $\neq$, =?

You can continue this process creating distinct $\subseteq$'s of the same size until [n], let $S = \{i, \dots, j\}$, $S' = \{i, \dots, j, j+1\}$

$S \subseteq S'$ is fully/connected, ~~sum~~ by with next sequential element

$Y_{k,r}$, assuming $r \le n$, just take layers from top if $r \ge n$



r n, take $\binom{n}{r-1}$ layer, but to ensure connection need center at $n-r$ layer if not lower, just take full layers?

k establish center chain, take first $k-1$ layers then final center as F' to get k ll vertices

assuming $r \le n$

$|F| = \sum_{i=0}^{k} \binom{n}{i} + n+1$

$r > n$

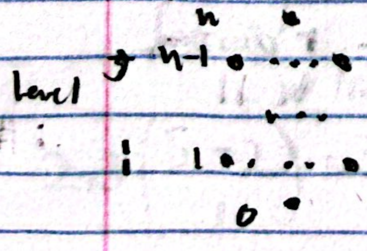
$|F| = \sum_{i=0}^{k} \binom{n}{i} +$ sum top layers until $\binom{n}{j} \ge n$?

Results

1. $2C_2 = 6$ by PH
2. # of connections between levels
3. $Y_{2,2}$
4. V_k, D_k, V'_k , $1 \le k \le n-1$
5. $Y_{k,r}$?

$Y_{k,2} \le \sum_{i=0}^{k} \binom{n}{i} + n+1$

connections between levels =



$\binom{n-i}{j-i}$ where i = level, $n-i$ # of 0's
$j > i$ and j = level, $j-i$ # new 1's
$j - i \ge 1$

[n]
...
{i,j}
{i}
∅

$Y_{2,r}$? Cannot find top r vertices as have to watch group of r disjoint and the center has to connect to the r vertices

r<n just top layer need vertex that connects to r
if r=n cannot use top layer as no $S \in B_{[n]}$ connects to all n, must use lower layer
r≥n use lower layers, worry disjointness

need center in ≤ n−r layer, take last n−r−1 layers
$Y_{2,r} = \sum_{i=0}^{r-1}\binom{n}{i}+1$, take first r−1 layers surround center connects n-elements connecting to last layer? r

1 [n] •
n $\binom{n}{n-1}$ • ... •
2 $\binom{n}{n-2}$ • ... • $\binom{n-(n-2)}{1}=\binom{2}{1}=2$
3
4 $\binom{n}{n-3}$ • ... • $\binom{n-(n-3)}{2}=\binom{3}{2}=3$

n

$\binom{n}{k}$ are max when $\binom{n}{\lfloor \frac{n}{2}\rfloor}$ to get most want n to be most so bottom layer as soon as $\binom{n-i}{i-\lfloor \frac{i}{2}\rfloor}$ can reach r, stop taking layers
finding r disjoint vertices in top layers? proving disjoint and does not exist in F? to

Stop taking layers as soon vertices can connect r disjoint, must be in one layer as vertices above will be ⊂ to below?

$\binom{n-i}{i-\lfloor \frac{i}{2}\rfloor}$ fastest growing relations, once value hits or exceeds r, cut loop the layer and below is the same for F', n−i (remove layer from top iterating on, $i-\lfloor \frac{i}{2}\rfloor$ is several layer above n−i the n−i e's can ⊂ to

$\binom{n-i}{i-\lfloor \frac{i}{2} \rfloor}$ $\quad i=1 \quad \binom{n-1}{1-0} = \binom{n-1}{1} = n-1$

$i=2 \quad \binom{n-2}{2-1} = \binom{n-2}{1} = n-2$

$i=3 \quad \binom{n-3}{3-1} = \binom{n-3}{2}$

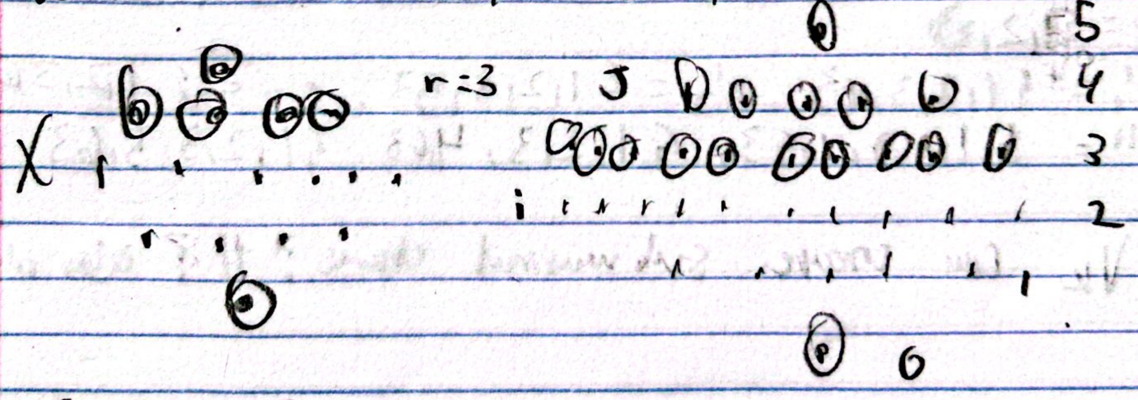
understand formula, if it works looking for max relations between chosen layers, bottom layer will connect most. then middle layer or taken should have most connections but need to express formula

consider if disjoint group size r between chosen layers, possible, if so how if not proof?

$\binom{i}{i-\lfloor \frac{i}{2} \rfloor}$ i layers from top $i=0$ $[n]$, $i=1$ $\binom{n}{n-1}$

$i-\lfloor \frac{i}{2} \rfloor$ middle layer of taken layers

X since r disjoint and need to be connect must fall on same layer or different then ||, since ~~...~~ looking at top layer do not have to worry about other



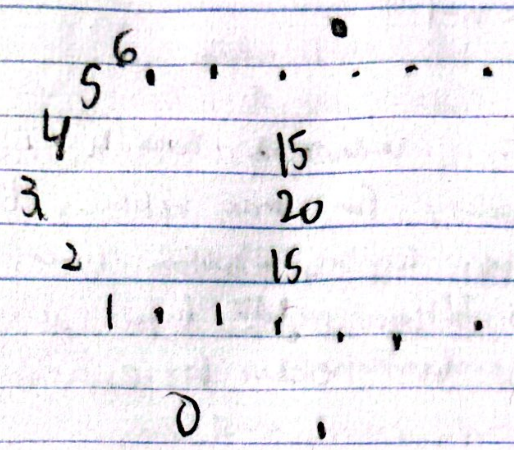
$\binom{n-i}{j-i}$

$2,3 \quad \binom{5-2}{1} = 3$

$2,4 \quad \binom{3}{2} = 3$

$3,4 \quad \binom{2}{1} = 2$

r = 3,4



$\binom{6}{4} = \frac{6!}{4!\,2!} = 6\cdot 5$

$\binom{6}{3} =$

4,5 $\binom{2}{1} = 2$

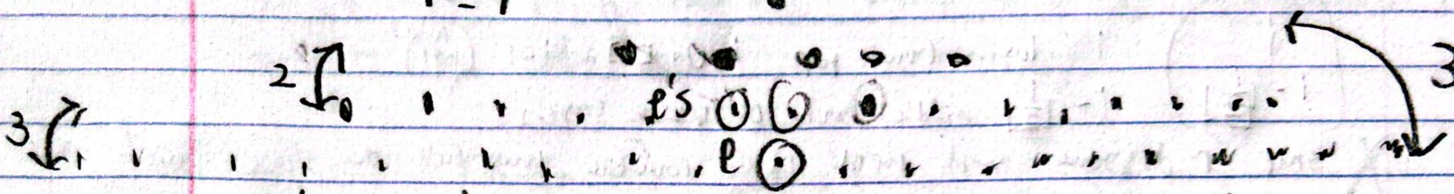
3,4 $\binom{3}{1} = 3$

3,5 $\binom{3}{2} = 3$

2,4 $\binom{4}{2} = 6$

2,5 $\binom{4}{3} = 4$

r = 4



if trying to find r disjoint vertices. can't take all on one layer, as any additional will be ||

$e = \{1,2,3\}$

$e'_1 = \{1,2,3,4\}$, $e'_2 = \{1,2,3,5\}$, $e'_3 = \{1,2,3,6\}$

$e'' = \{1,2,3,4,5\}$, $\{1,2,3,4,6\}$, $\{1,2,3,5,6\}$

V_k can create sub maximal chains? # ? also algorithm?