On the extension of PEERS element for quadrilateral elasticity mixed formulation

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1 Introduction

2 Reduced symmetry formulation

First of all, we recall the equations governing the linear elastic problems. Let $\Omega \in R^2$

$$
\begin{aligned}
\operatorname{div} \boldsymbol{\sigma} = \boldsymbol{f} \quad & \text{on } \Omega \,, \\
\boldsymbol{\varepsilon} = \nabla^{\mathrm{s}} \boldsymbol{u} \quad & \text{on } \Omega \,, \\
\boldsymbol{u} = \bar{\boldsymbol{u}} \quad & \text{on } \Gamma_D \,, \\
\boldsymbol{\sigma} \cdot \boldsymbol{n} = \boldsymbol{t} \quad & \text{on } \Gamma_N \,,
\end{aligned}
\tag{1}
$$

3 PEERs element

4 Numerical example

In this section we report some examples using the presented formulation to proven the good behaviour. All examples are studied in nearly incompressible limit.

4.1 Square problem

First example is a unit square domain with homogeneous Dirichlet boundary conditions and we the exact solution is

$$
u_1 = \cos(\pi x) \sin(2\pi y), \quad u_2 = \sin(\pi x) \cos(\pi y). \tag{2}
$$

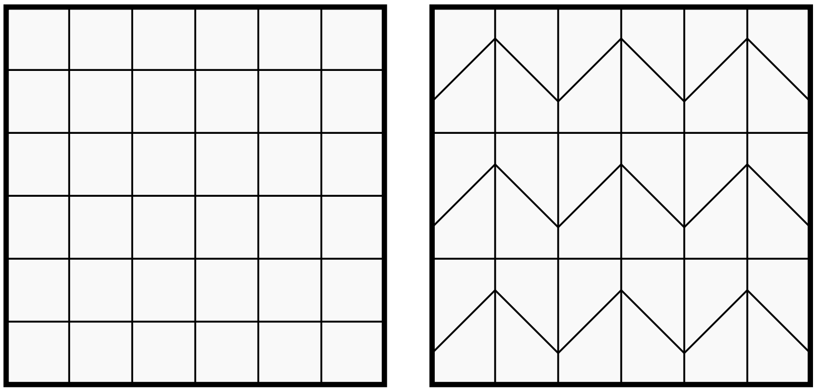
The Lamé constant are fix to $\lambda = 123$ and $\mu = 79.3$. By imposition of the previously exact solution one obtain for the body force f

$$
\begin{aligned}
f_1 &= -\pi^2 \cos(\pi x) \sin(\pi y) \left(\lambda + \mu + 2\lambda \cos(\pi y) + 12\mu \cos(\pi y)\right), \\
f_2 &= -\pi^2 \sin(\pi x) \left(\lambda \cos(\pi y) + 3\mu \cos(\pi y) + 2\lambda \left(2\cos(\pi y)^2 - 1\right) + 2\mu \left(2\cos(\pi y)^2 - 1\right)\right)
\end{aligned}
\tag{3}
$$

The problem is study using two type of mesh, first of all using a square mesh and before using a trapezoidal mesh. The two different types of meshes are shown in Figures 2(a) and 2(b).

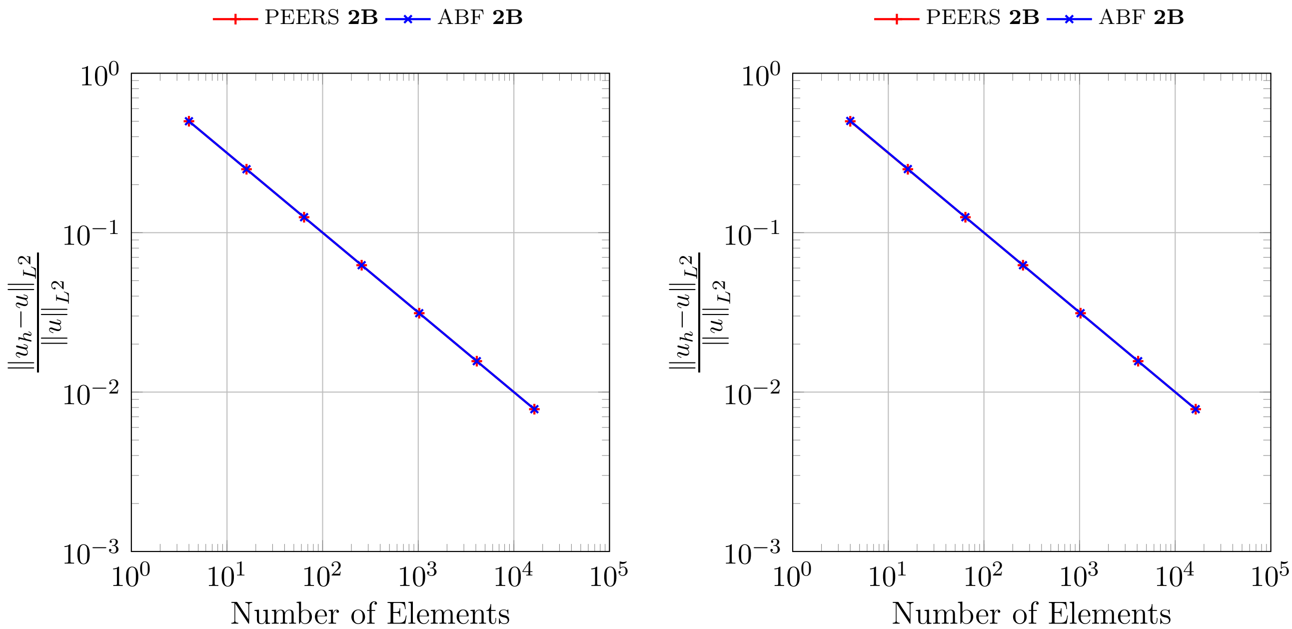
4.2 Cantilever beam problem

Now we consider the beam with length $L = 10$ and height $l = 2$ as we shown in Figure 3. The Young modulus is set equal to $E = 1500$ and the Poisson $\nu = 0.4999$. The beam are fixed in the bottom left corner and subjected to a distributed load with $f = 300$ on the right edge as shown in Figure 3. We use to model the beam with two types of mesh: regular and trapezoidal as in the previous example (see Figures 2(a) and 2(b)).



(a) Regular mesh (b) Trapezoidal mesh

Figura 1: Square Problem



(a) Regular mesh (b) Trapezoidal mesh

Figura 2: Error in L^2–norm of square problem

4.3 Cook's membrane

The final example is the Cook's membrane. That is a typical benchmark and consist of a beam with vertex: $(0,0)$, $(48,44)$, $(48,60)$ and $(0,44)$. The left vertical edge is clamped and the right vertical edge is subjected to the vertical distributed forces with resultant $F = 100$ as shown in Figure 4. The material properties are taken to be $E = 250$ and $\nu = 0.4999$, so that a nearly incompressible response is obtained. We take into account the case of uniform meshes and the case of random distorted meshes (see Figure). We report in Figures 5(a) and 5(b), the vertical displacement of the point A versus the number of element per side for different PEERS and ABF elements in the case of regular mesh. In the case of random distorted mesh the same results are shown in Figures 6(a) and 6(b). It can be observe that the standard solution obtained

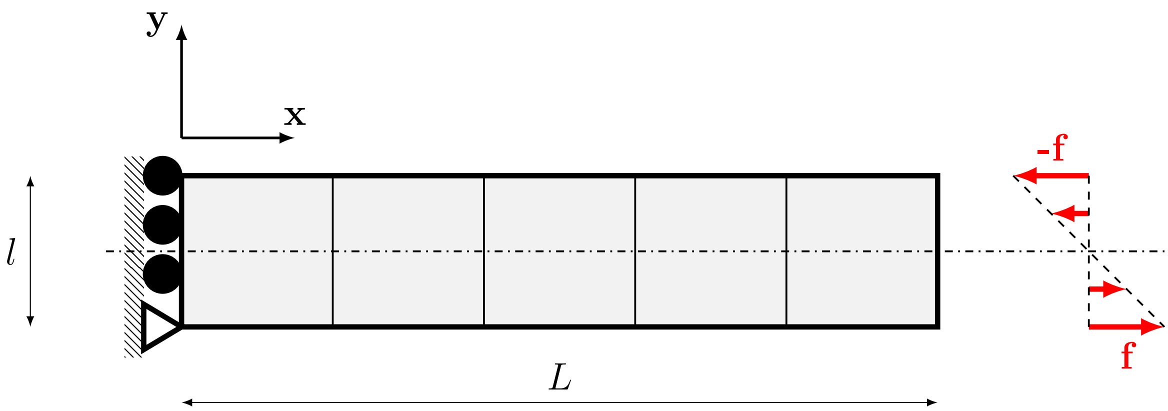


Figura 3: Cantilever Beam: Geometry problems

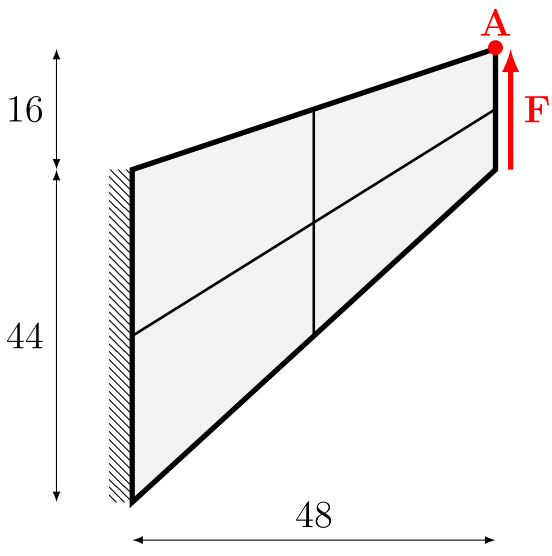
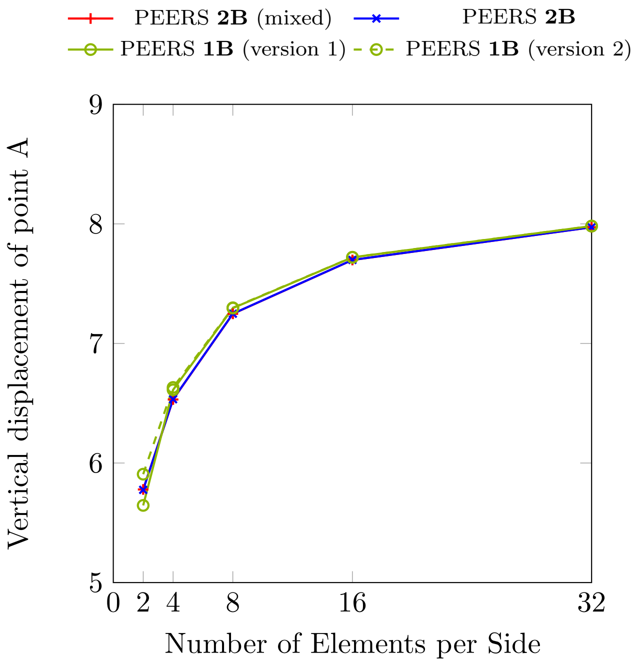
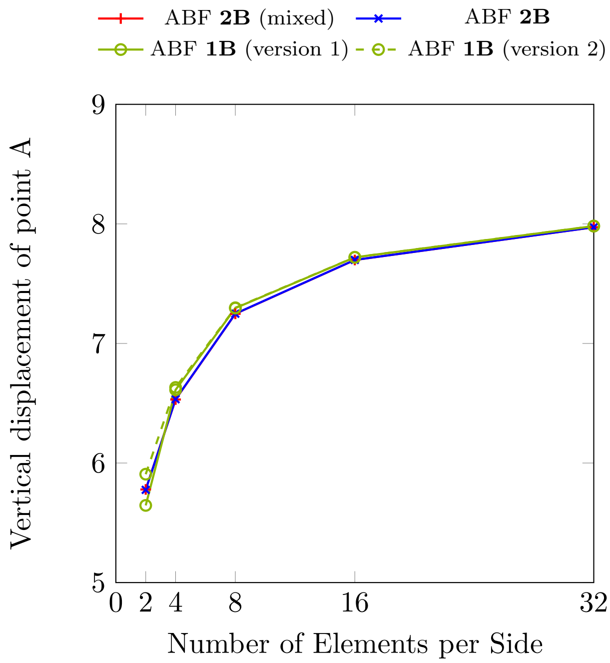


Figura 4: Cook's Membrane Geometry

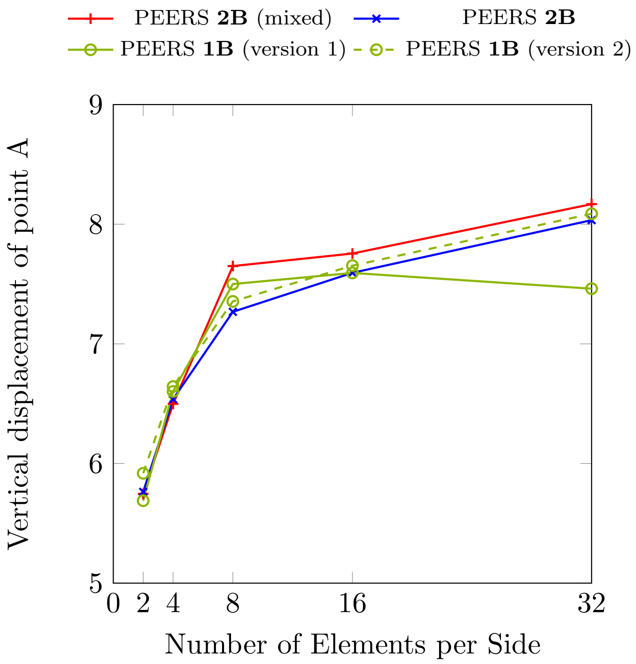


(a) PEERS Element

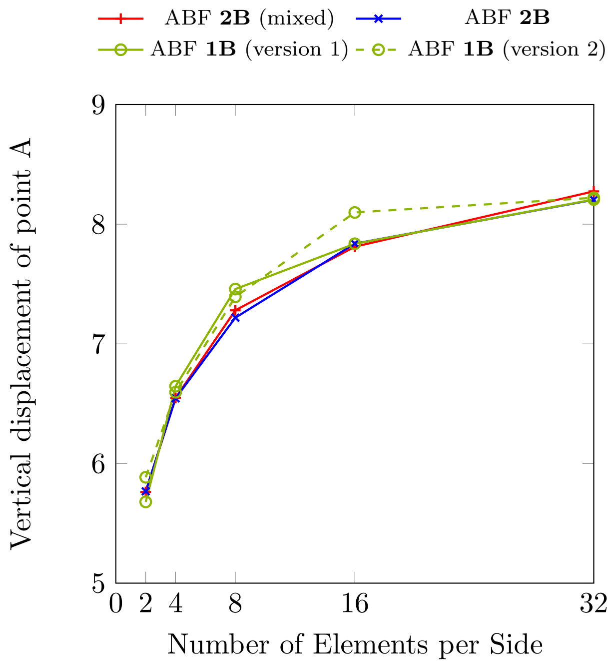


(b) ABF Element

Figura 5: Cook's Membrane (regular mesh): Vertical Displacement of Point A vs. Element per Side



(a) PEERS Element



(b) ABF Element

Figura 6: Cook's Membrane (random distorted mesh): Vertical Displacement of Point A vs. Element per Side